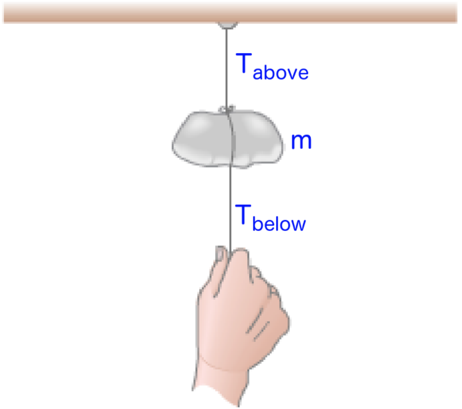
Physics 9 — Wednesday, September 12, 2018

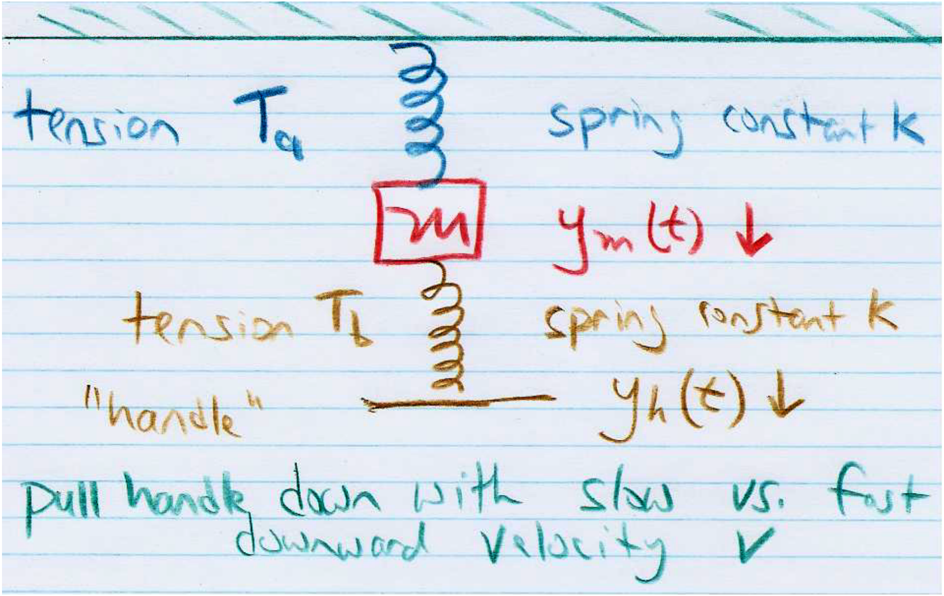
- HW #1 due Friday in class. Remember online feedback page.
- HW help sessions: Wed 4–6pm DRL 4C6 (Bill), Thu 6:30–8:30pm DRL 2C8 (Grace)
- For today, you read Giancoli ch12 (sound)
- Be sure to **click reload** on the "reading response" page, as I have updated next week's reading & questions.
- Course web page is `positron.hep.upenn.edu/physics9`
- Slides, etc.: `positron.hep.upenn.edu/physics9/files`
- My jury duty has been postponed until summer, so I will not need to miss any classes next week. Whew!
- On Friday, 9/21, recent Penn ARCH/Music grad Davis Butner will speak about his developing career in arch. acoustics.
- On Monday, 9/24, Terry Tyson from Acentech Acoustics will speak with us. If you're curious, here's an article by him:

`https://insulation.org/io/articles/good-design-for-architectural-acoustics/`

I worked out an analogue of this problem using two **springs**

A stone hangs by a fine thread from the ceiling, and a section of the same thread dangles from the bottom of the stone (Fig. 4–36). If a person gives a sharp pull on the dangling thread, where is the thread likely to break: below the stone or above it? What if the person gives a slow and steady pull? Explain your answers.





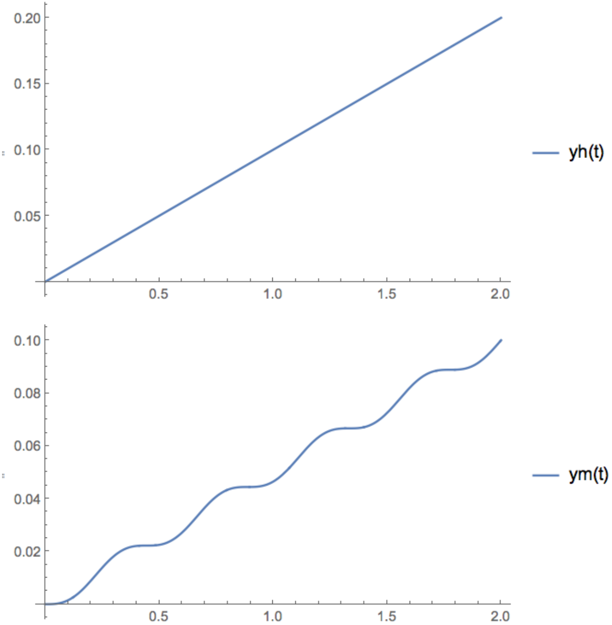
Then I solved for the motion using Mathematica, for slow pull vs. for fast pull.

```
ClearAll["Global`*"];
Manipulate[
 tmax = 0.2 / v;
 m = 1.0;
 g = 9.8;
 k = 100.0;
 ym0 = m g / k;
 yh[t_] := ym0 + v t;
 Ta[t_] := (ym[t] - ym0) k + m g;
 Tb[t_] := (yh[t] - ym[t]) k;
 soln = NDSolve[{
    ym[0] == ym0,
    ym'[0] == 0,
    m ym''[t] == m g + Tb[t] - Ta[t]
   }, ym[t], {t, 0, tmax}];
 ymsoln[t_] := ym[t] /. soln;
 Column[
  {Plot[{ymsoln[t] - ym0, yh[t] - yh[0]}, {t, 0, tmax},
    PlotLegends → {"ym(t)", "yh(t)"}],
   Plot[((m g + Tb[t] - Ta[t]) / m) /. soln, {t, 0, tmax},
    PlotLegends → "acceleration of block m"],
   Plot[{Ta[t] /. soln, Tb[t] /. soln}, {t, 0, tmax},
    PlotLegends → "Expressions"]}],
 {v, 0.1, 5.0}]
```

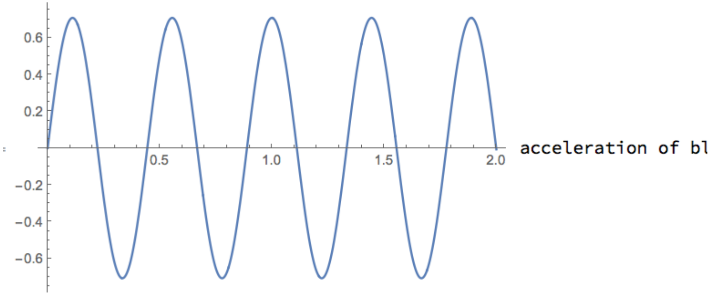
This version makes an interactive demo with a slider bar. I conveniently pick $m = 1\,\mathrm{kg}$ and $k = 100\,\mathrm{N/m}$.

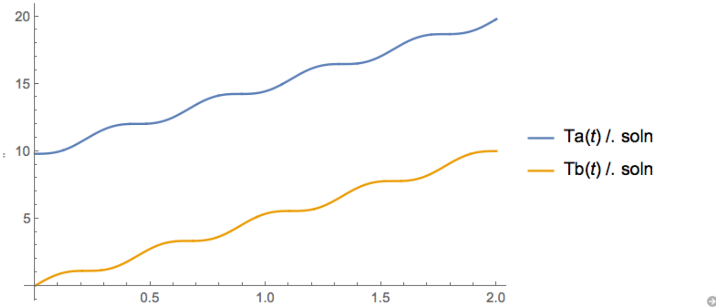
```
ClearAll["Global`*"];
v = 0.1; (* pull very slowly *)
tmax = 0.2 / v;
m = 1.0;
g = 9.8;
k = 100.0;
ym0 = m g / k;
yh[t_] := ym0 + v t;
Ta[t_] := (ym[t] - ym0) k + m g;
Tb[t_] := (yh[t] - ym[t]) k;
soln = NDSolve[{
     ym[0] == ym0,
     ym'[0] == 0,
     m ym''[t] == m g + Tb[t] - Ta[t]
   }, ym[t], {t, 0, tmax}];
ymsoln[t_] := ym[t] /. soln;
Plot[yh[t] - yh[0], {t, 0, tmax}, PlotLegends → {"yh(t)"}]
Plot[ymsoln[t] - ym0, {t, 0, tmax}, PlotLegends → {"ym(t)"}]
Plot[((m g + Tb[t] - Ta[t]) / m) /. soln, {t, 0, tmax},
 PlotLegends → "acceleration of block m"]
Plot[{Ta[t] /. soln, Tb[t] /. soln}, {t, 0, tmax},
 PlotLegends → "Expressions"]
```

This version moves the handle very slowly: 0.1 meter/second



You see the handle moving slowly (0.1 m/s) and the handle following along at one-half the displacement of the handle.





The block's acceleration shows a small jiggling motion. The "above" tension is always mg larger than the "below" tension.

```
ClearAll["Global`*"];
v = 10; (* pull very quickly *)
tmax = 0.2 / v;
m = 1.0;
g = 9.8;
k = 100.0;
ym0 = m g / k;
yh[t_] := ym0 + v t;
Ta[t_] := (ym[t] - ym0) k + m g;
Tb[t_] := (yh[t] - ym[t]) k;
soln = NDSolve[{
     ym[0] == ym0,
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    }, ym[t], {t, 0, tmax}];
ymsoln[t_] := ym[t] /. soln;
Plot[yh[t] - yh[0], {t, 0, tmax}, PlotLegends → {"yh(t)"}]
Plot[ymsoln[t] - ym0, {t, 0, tmax}, PlotLegends → {"ym(t)"}]
Plot[((m g + Tb[t] - Ta[t]) / m) /. soln, {t, 0, tmax},
 PlotLegends → "acceleration of block m"]
Plot[{Ta[t] /. soln, Tb[t] /. soln}, {t, 0, tmax},
 PlotLegends → "Expressions"]
```
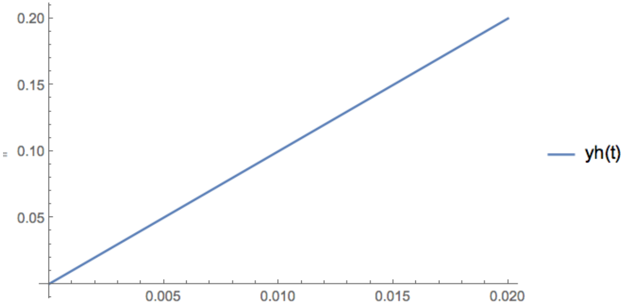
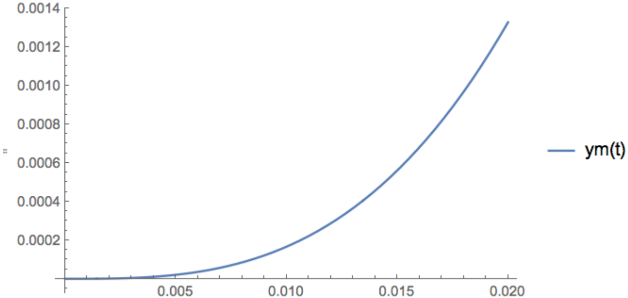
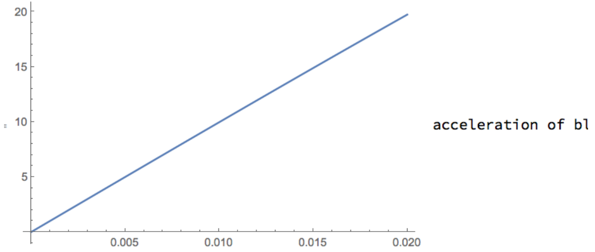
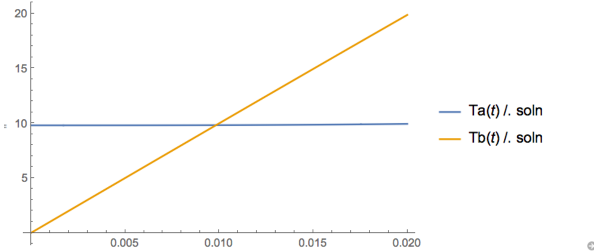

This version moves the handle very quickly: 10 meters/second





Handle moves 10 m/s. Block's motion is tiny (less than 1% of handle's motion), but notice that block is accelerating.





The block accelerates more and more as bottom spring stretches. Top spring's tension stays fixed at mg. Bottom spring's tension increases rapidly as handle's motion stretches bottom spring. Bottom spring "breaks" first, as bottom spring's tension is larger than top spring's tension, once block's acceleration is larger than g.

On Monday, we saw standing waves appear on the long vibrating taut string (fundamental frequency $\approx$ 15 Hz), and we heard the effect of standing waves on a guitar string. In each case,

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{m/L}}$$

How would we describe, **mathematically**, the shapes of the fundamental mode, the second harmonic, the third harmonic, etc.?

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How would we describe, **mathematically**, the shapes of the fundamental mode, the second harmonic, the third harmonic, etc.?

For the *n*th mode, the wave function (which describes the vertical displacement of each little piece of the string) is

$$D(x,t) = A\,\sin(\frac{n\pi}{L})\,\cos(2\pi f_n t)$$

You don't need to remember this, but talk through with your neighbor whether this expression makes sense for a standing wave on a string whose two ends are clamped: $D(0,t) = D(L,t) = 0$.

Question

Suppose that a guitar string is tuned to play at 330 Hz. Now suppose I replace the string with another string that is made of the same material, has the same tension, but has three times the diameter of the original string? What will the new fundamental frequency be?

(A) 36 Hz

(B) 110 Hz

(C) 191 Hz

(D) 330 Hz

(E) 572 Hz

(F) 990 Hz

(G) 2970 Hz

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By the way, the standard 6-string guitar tuning is 82 Hz (E_2), 110 Hz (A_2), 147 Hz (D_3), 196 Hz (G_3), 247 Hz (B_3), 330 Hz (E_4)

- On Monday, we quietly made an abrupt transition from talking about "wave pulses" to talking about sinusoidal ("harmonic") waves, without any explanation.
- Frequency is a fundamental idea for vibrations, waves, and sound. To our ear, frequency corresponds to pitch. To a vibrating object, like a pendulum, frequency is number of cycles per second.
- Mathematically, frequency is a parameter of a sinusoidal function:

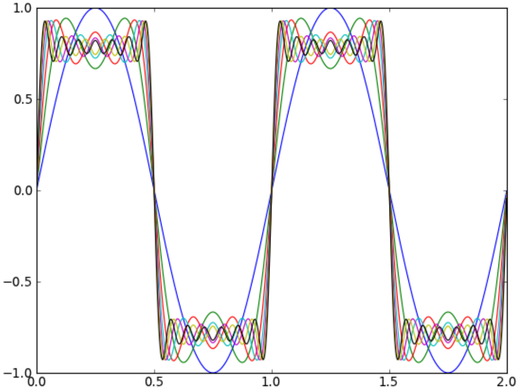
$$x(t) = A \sin(2\pi f t)$$

- Motion that is "purely sinusoidal" (a.k.a. "simple harmonic motion") contains only a single frequency. So the sine function represents a kind of simplicity: a building block.
- An amazing math result called Fourier's theorem tells us how to build up more complicated functions by adding up sine functions of different frequencies.

- For example, we can build up a square wave that repeats itself once every T seconds by adding up

$$\sin(\frac{2\pi t}{T}) + \frac{1}{3}\sin(\frac{6\pi t}{T}) + \frac{1}{5}\sin(\frac{10\pi t}{T}) + \frac{1}{7}\sin(\frac{14\pi t}{T}) + \cdots$$

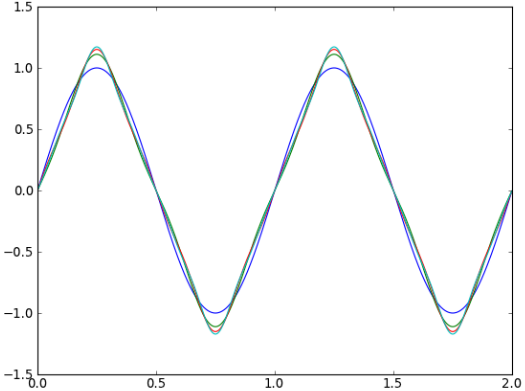
- So a "110 Hz square wave" is the same as a 110 Hz sine plus $\frac{1}{3}$ of a 330 Hz sine plus $\frac{1}{5}$ of a 550 Hz sine plus . . . .
- Here I graph just the first term, then the sum of the first two terms, then the sum of the first three terms, and so on.



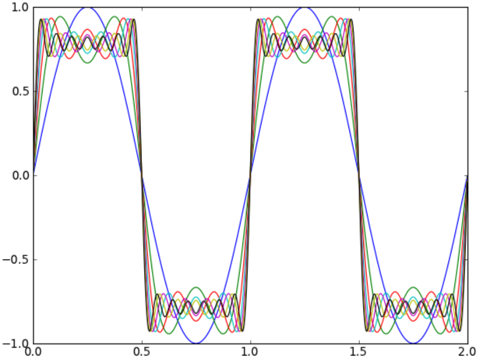
- Similarly, we can build up a triangle wave that repeats itself once every T seconds by adding up

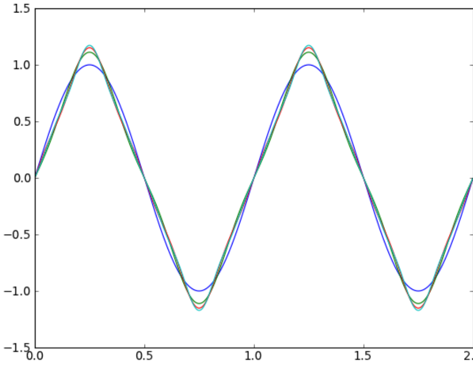
$$\sin(\frac{2\pi t}{T}) - \frac{1}{9}\sin(\frac{6\pi t}{T}) + \frac{1}{25}\sin(\frac{10\pi t}{T}) - \frac{1}{49}\sin(\frac{14\pi t}{T}) + \cdots$$

- So a "110 Hz triangle wave" equals a 110 Hz sine minus $\frac{1}{9}$ of a 330 Hz sine plus $\frac{1}{25}$ of a 550 Hz sine minus ....



- A "110 Hz square wave" equals a 110 Hz sine plus $\frac{1}{3}$ of a 330 Hz sine plus $\frac{1}{5}$ of a 550 Hz sine plus ....
- A "110 Hz triangle wave" equals a 110 Hz sine minus $\frac{1}{9}$ of a 330 Hz sine plus $\frac{1}{25}$ of a 550 Hz sine minus ....
- You can hear the difference between a sine, a square, and a triangle. All have the same **fundamental** frequency, but they contain different **overtone** mixtures.
- These overtones are one facet of "timbre" or "tonal quality."



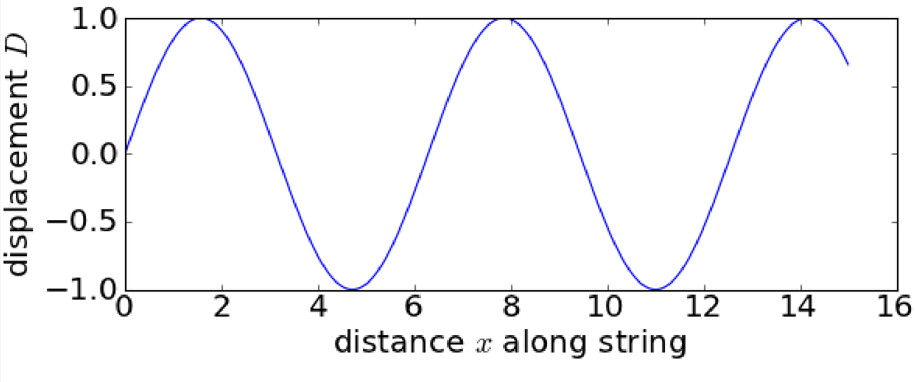


- So sine waves are building blocks.
- If we understand how sinusoidal waves travel down the wave machine, then we can analyze more complicated waves by decomposing them into sinusoidal components.
- That's why we like sine waves: they are the simplest waves, with which we can build up any other kind of wave we like.
- Mathematically, the reason this works is that Hooke's Law allows us to use the "principle of linear superposition."
- When you hear someone mention the "frequency content" of a wave or a vibration, she means the relative proportions of the many sinusoidal components. "Bass" $\rightarrow$ more low-frequency content; "treble" $\rightarrow$ more high-frequency content.

- So let's figure out how we would mathematically represent a sinusoidal wave traveling down a taut string, as in the top row of this animation:

`http://www.acs.psu.edu/drussell/Demos/superposition/standing.gif`

Suppose at time $t = 0$ the string (or the wave machine) looks like this: displacement as a function of position x measured from the left side of the string.



How can we write the displacement $D(x)$?

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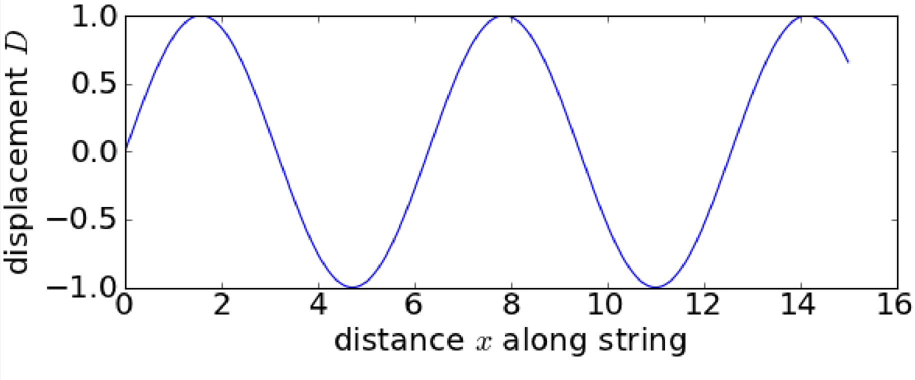
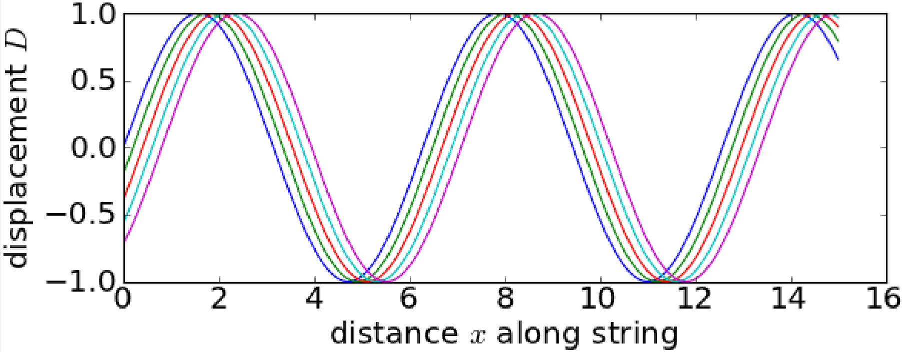


How can we write the displacement $D(x)$?

In this graph, $D(x) = sin(x)$. For more general amplitude A and wavelength λ,

$$D(x) = A \sin(\frac{2\pi x}{\lambda})$$

Now we want to make the sine wave travel from left to right at speed v. So the "zero" of the sine that was initially at $x = 0$ has moved, after time t, to $x = vt$.



The **wave function** $D(x, t)$ gives you the displacement, at time t, of the piece of string whose equilibrium position is at location x along the length of the string.

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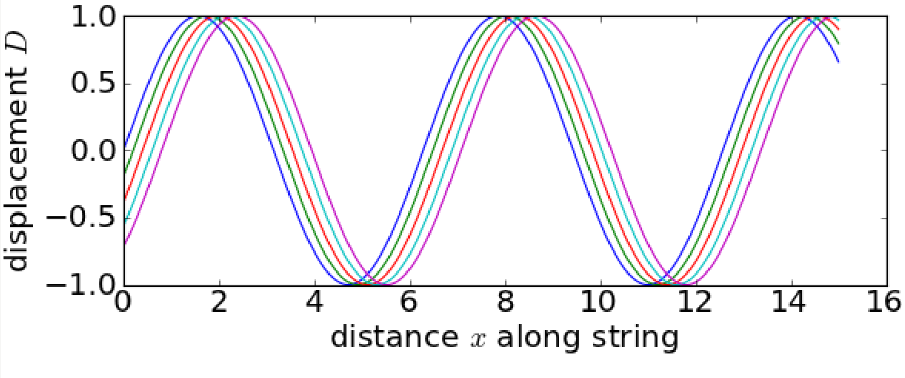

The **wave function** $D(x, t)$ gives you the displacement, at time t, of the piece of string whose equilibrium position is at location x along the length of the string.

$$D(x) = A\,\sin(\frac{2\pi x}{\lambda}) \;\rightarrow\; D(x,t) = A\,\sin(\frac{2\pi(x - vt)}{\lambda})$$

- Mazur §16.6: The wave function $D(x,t)$ for a harmonic wave traveling from left to right is

$$D(x,t) = A\,\sin(\frac{2\pi(x - vt)}{\lambda})$$

which we can rewrite as

$$D(x,t) = A\,\sin(\frac{2\pi x}{\lambda} - 2\pi f t) = A\,\sin(kx - \omega t)$$

using $f = v/\lambda$, $\omega = 2\pi f$, and $k = 2\pi/\lambda$. ω is called the "angular frequency" and k is called the "wave number."

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$$D(x,t) = A\sin(\frac{2\pi x}{\lambda} + 2\pi ft) = A\sin(kx + \omega t)$$

- When two harmonic traveling waves of same amplitude, same freq., and opposite direction meet, they form a standing wave:

$$D(x,t) = A\sin(kx-\omega t) + A\sin(kx+\omega t) = 2A\sin(kx)\cos(\omega t)$$

$$D(x,t) = 2A\ \sin(\frac{2\pi x}{\lambda})\cos(2\pi ft)$$

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$$D(x,t) = 2A\ \sin(\frac{2\pi x}{\lambda})\cos(2\pi f t)$$

which we saw on the clamped string, with $\lambda_n = 2L/n$.

http://positron.hep.upenn.edu/p9/files/wave3py.pyde

- Standing wave (I dropped the factor of 2 out front):

$$D(x,t) = A \ \sin(\frac{2\pi x}{\lambda}) \cos(2\pi f t)$$

- This expression can represent standing waves on a guitar string of length L.
- We need $D(x = 0, t) = 0$ and $D(x = L, t) = 0$, because both ends of the string are immobilized by the bridge & nut.
- The first is easy: $\sin(0) = 0$.
- When else is the sine function equal to zero?

`http://positron.hep.upenn.edu/p9/files/wave6py.pyde`

`http://www.acs.psu.edu/drussell/Demos/superposition/standing.gif`

If we have a “string” (or a wave machine) of length L, on which the speed of wave propagation is v, and the string is immobilized at $x = 0$ and at $x = L$,

- What are the possible wavelengths for standing waves?
- What are the possible frequencies for standing waves?
- It takes about 2.5 s for a wave to go from end to end of the wave machine. The machine is about 2 m long. What is the wave speed?

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- What are the possible wavelengths for standing waves?
- What are the possible frequencies for standing waves?
- It takes about 2.5 s for a wave to go from end to end of the wave machine. The machine is about 2 m long. What is the wave speed?
- $n(\lambda/2) = L$. $\lambda = 2L/n$. $f = v/\lambda$. $f = vn/(2L) = (n/2)(v/L)$. $v \approx 0.8\ \mathrm{m/s}$. $v/L \approx 0.4\ \mathrm{Hz}$. $f_n = n(0.2\ \mathrm{Hz})$, where n is the number of half-waves that fit on the string.
- Next, ask yourself how can you describe the motion of the piece of the wave machine at position x from the left side.

- Standing wave:

$$D(x,t) = A\ \sin(\frac{2\pi x}{\lambda})\cos(2\pi ft)$$

- From this expression, what is the "wave speed?" (Suppose we know λ and we know f.)
- What is the vertical displacement, as a function of time, of the piece of the wave machine at position x?
- Tricky: what is the vertical **velocity**, as a function of time, of the piece of the wave machine at position x?

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$$v_y(x,t) = -(2\pi f)A\ \sin(\frac{2\pi x}{\lambda})\sin(2\pi f t)$$

Sound waves in room-temperature air travel at a wave speed of 343 m/s. (Much more on sound in the next two weeks!)

Digression: About how long does it take for a pulse of sound (maybe a clap of thunder or the sound of a baseball bat hitting a ball) to travel 1 km?

What about a mile (1.61 km $\approx$ (5/3) km)?

What about one foot ((1/3.28) meter, or (1/5280) mile)?

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What about one foot ((1/3.28) meter, or (1/5280) mile)?

Sound waves in air travel 1 km in 3 s, 1 mile in 5 s,
1 foot in 1 millisecond.

Sound waves in room-temperature air travel at a wave speed of 343 m/s. At a frequency of about 34 Hz (near the lower end of the range of frequencies people can hear), what is the wavelength?

(Young human ears can hear roughly 20 Hz — 20 kHz.)

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What is the wavelength at 17150 Hz, which is close to the (roughly) 20 kHz upper range for young human ears?

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What is the wavelength at 17150 Hz, which is close to the (roughly) 20 kHz upper range for young human ears?

It turns out that the conventional telephone network only transmits sounds in the frequency range 300 Hz — 3400 Hz. What's the wavelength (for sound in air) at 343 Hz? At 3430 Hz?

If I take an organ pipe whose length is $L = 0.25\ \mathrm{m}$ (that's 25 cm) and let both ends be open to the atmosphere, what does the fundamental standing wave look like?

- Where are the displacement nodes and antinodes?
- Where are the pressure nodes and antinodes?
- What is the largest allowed wavelength?
- What is the fundamental frequency?
- What happens if instead I close off one end!

Somewhat out of order, but useful for one homework problem: if you have a traveling wave

$$D(x, t) = A \sin(\frac{2\pi x}{\lambda} - 2\pi f t)$$

what is the vertical velocity vs. time of the "particle" at x?

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$$v_y(x,t) = -(2\pi f)A\cos(\frac{2\pi x}{\lambda} - 2\pi ft)$$

So what is the maximum up-and-down speed of a "particle" of the wave machine? (Or in the case of the homework it is a "human wave" at a football game.)

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$$v_y(x,t) = -(2\pi f)A\,\cos(\frac{2\pi x}{\lambda} - 2\pi f t)$$

So what is the maximum up-and-down speed of a "particle" of the wave machine? (Or in the case of the homework it is a "human wave" at a football game.)

$$v_{\max} = (2\pi f)A$$

Also somewhat out of order, but useful for the homework:

- What stays the same and what changes as a harmonic (i.e. sinusoidal) wave crosses the boundary from one medium to another (e.g. different mass per unit length)?
- Does the wave speed stay the same?
- Does the frequency stay the same?
- Does the wavelength stay the same?

Longitudinal waves are harder to visualize than transverse waves.

Notice that the peaks of displacement are $\lambda/4$ (i.e. 90°) out of phase with the peaks of compression and rarefaction.

Notice that at a free end (like the open end of a flute or an organ pipe), the displacement changes maximally (displacement *antinode*), while the compression stays unchanged (compression *node*).

Notice that at a fixed end (like the closed end of a tube), the displacement is zero (displacement *node*), while the compression varies maximally (compression *antinode*).

Displacement and compression in air $\leftrightarrow$ displacement and slope on a vibrating string.

http://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

Physics 9 — Wednesday, September 12, 2018

- HW #1 due Friday in class. Remember online feedback page.
- HW help sessions: Wed 4–6pm DRL 4C6 (Bill), Thu 6:30–8:30pm DRL 2C8 (Grace)
- For today, you read Giancoli ch12 (sound)
- Be sure to **click reload** on the "reading response" page, as I have updated next week's reading & questions.
- Course web page is `positron.hep.upenn.edu/physics9`
- Slides, etc.: `positron.hep.upenn.edu/physics9/files`
- My jury duty has been postponed until summer, so I will not need to miss any classes next week. Whew!
- On Friday, 9/21, recent Penn ARCH/Music grad Davis Butner will speak about his developing career in arch. acoustics.
- On Monday, 9/24, Terry Tyson from Acentech Acoustics will speak with us. If you're curious, here's an article by him:

`https://insulation.org/io/articles/good-design-for-architectural-acoustics/`