Physics 9 — Wednesday, October 3, 2018

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- If you'd like to do some extra-credit reading on Architectural Acoustics, you can read one or more chapters of a nicely illustrated (more drawings, less text) textbook I have (by Egan). For each chapter you read, you can collect extra credit by writing a few paragraphs (1–2 pages) to summarize what you learned from the chapter. Email me if you're interested.
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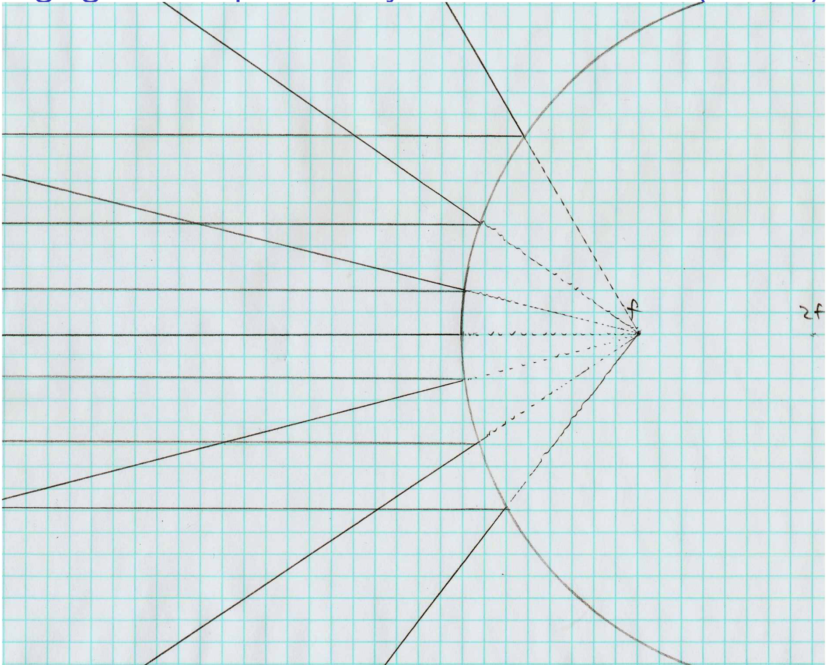
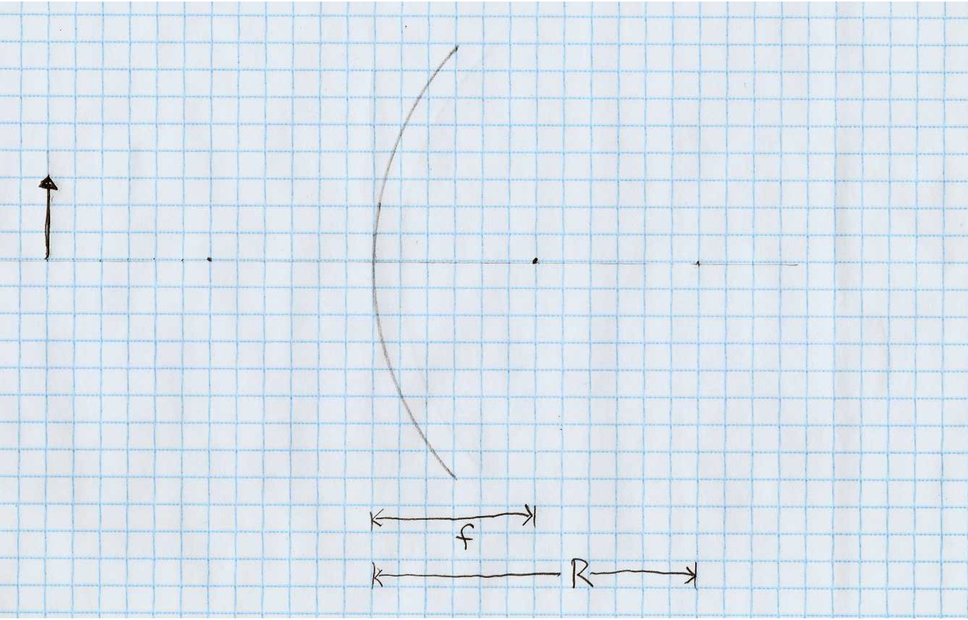
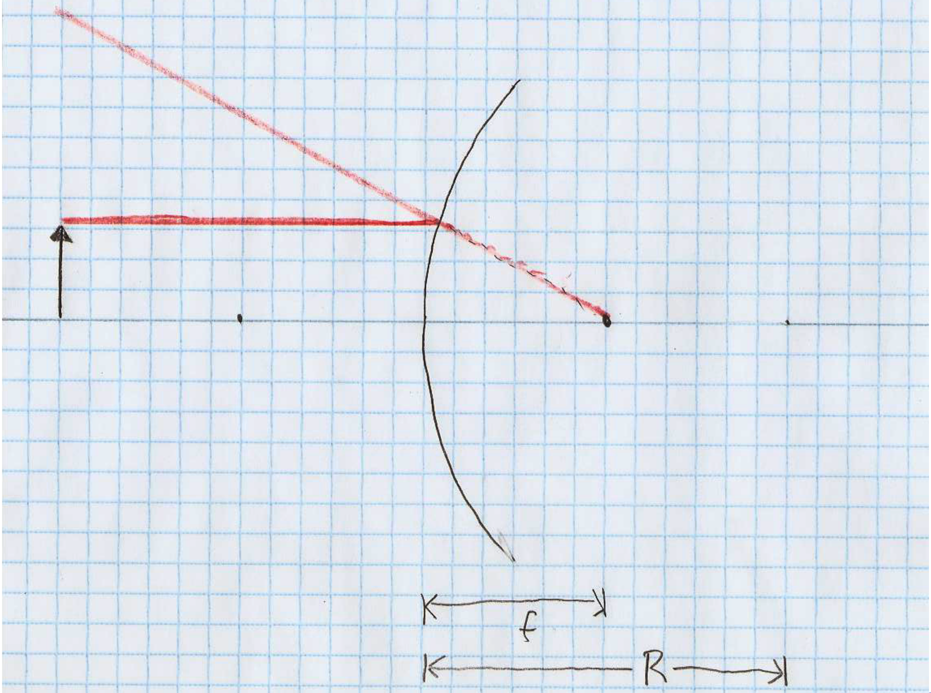
Diverging mirror: parallel rays "focus" on RHS (virtual)

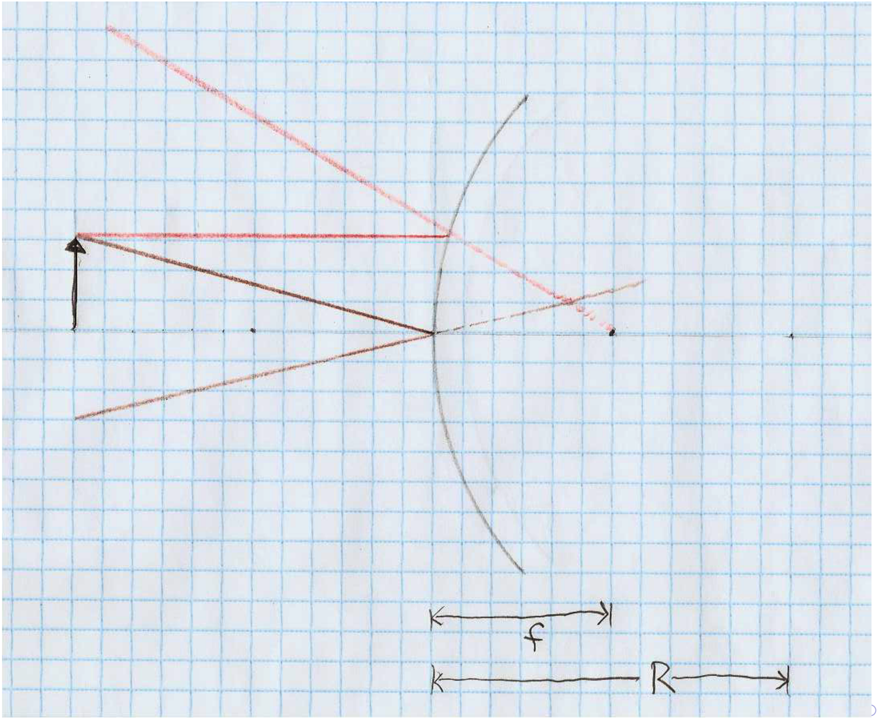


RHS car mirror: "objects in mirror are closer than they appear"



How do we draw the "principal rays" to find the image here?


f
R


f
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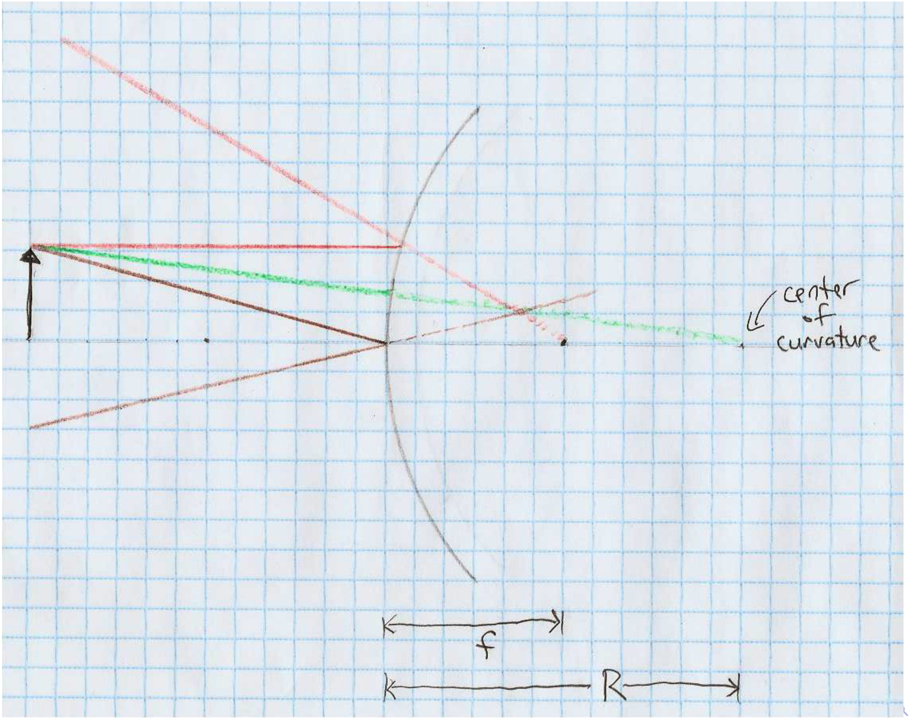
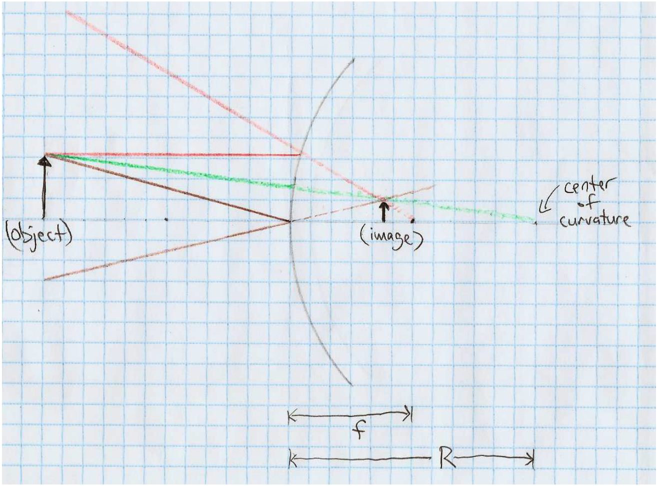

center
of
curvature
f
R



RHS car mirror: diverging mirror, with object typically (but not necessarily) far away. Image is upright but smaller than object (smaller $\implies$ brain thinks it is farther away). [Also used as wide-angle security mirror, or for seeing around blind corners.]

Let's analyze same situation using equations.

Same mirror/lens equation still works for diverging mirror:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \qquad \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Solve for d_i and h_i:

$$d_i = \frac{d_o f}{d_o - f} \qquad h_i = \frac{h_o f}{f - d_o}$$

Tricky: for diverging mirror, $\boxed{f = -\frac{1}{2}R}$ (i.e. $f < 0$)

Now plug in $d_o = 2|f| = -2f$, to match our drawing. Normally the object is pretty far away from a wide-angle mirror.

What do we get for d_i and for h_i ?

Is the image "upright" or "inverted?" Is the image "real" or "virtual?"

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \qquad \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

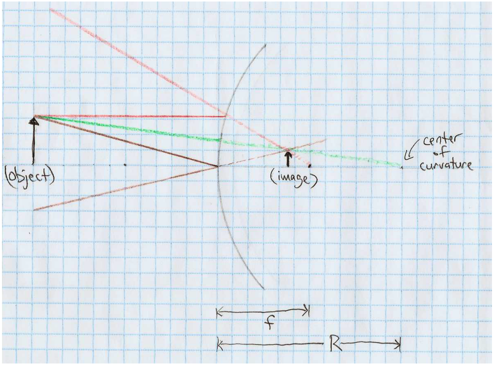
Solve for d_i and h_i:

$$d_i = \frac{d_o f}{d_o - f} \qquad h_i = \frac{h_o f}{f - d_o}$$

For diverging mirror, $f = -\frac{1}{2}R$ (i.e. $f < 0$)

Plug in $d_o = 2|f| = -2f$. Then $\boxed{d_i = \frac{2}{3}f = -\frac{2}{3}|f|}$, and $\boxed{h_i = \frac{1}{3}h_o}$.

$h_i > 0$ means "upright," and $d_i < 0$ means "virtual."



Diverging mirror:
$f = -R/2$.

Distant object: $d_o = 2|f|$ in our example.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \qquad\qquad \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Solve for d_i and h_i:

$$d_i = \frac{d_o f}{d_o - f} \qquad\qquad h_i = \frac{h_o f}{f - d_o}$$

Now plug in $d_o = 2|f|$: $\quad d_i = \frac{2}{3} f = -\frac{2}{3}|f|, \quad h_i = \frac{1}{3} h_o$
You see an upright, shrunken image, behind the mirror. Virtual!

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \qquad \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Solve for d_i and h_i:

$$d_i = \frac{d_o f}{d_o - f} \qquad h_i = \frac{h_o f}{f - d_o}$$

For diverging mirror, $f = -\frac{1}{2}R \quad$ (i.e. $f < 0$)

By the way, do you ever get an inverted image with a diverging mirror? Do you ever get a real image?

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \qquad \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Solve for d_i and h_i:

$$d_i = \frac{d_o f}{d_o - f} \qquad h_i = \frac{h_o f}{f - d_o}$$

For diverging mirror, $f = -\frac{1}{2}R$ (i.e. $f < 0$)

By the way, do you ever get an inverted image with a diverging mirror? Do you ever get a real image?

$$d_i = -\frac{d_o|f|}{d_o + |f|} \qquad h_i = \frac{h_o|f|}{d_o + |f|}$$

So for any $d_o > 0$, you always get an upright image that appears to be behind the lens ("virtual"), with a diverging mirror. And the image is never larger than the object.

We'll look at this summary table again after we do lenses.

Lens & mirror summary (light always enters from LHS):

converging lens	$f > 0$	$d_i > 0$ is RHS	$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$
diverging lens	$f < 0$	$d_i > 0$ is RHS	$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$
converging mirror	$f > 0$	$d_i > 0$ is LHS	$f = R/2$
diverging mirror	$f < 0$	$d_i > 0$ is LHS	$f = -R/2$

Horizontal locations of object, image (beware of sign conventions!):

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Magnification (image height / object height):

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Lenses: $R_{1,2} > 0$ for "outie" (convex), < 0 for "innie" (concave).

Real image: $d_i > 0$. Virtual image: $d_i < 0$. Real image means light really goes there. Virtual: rays converge where light doesn't go.

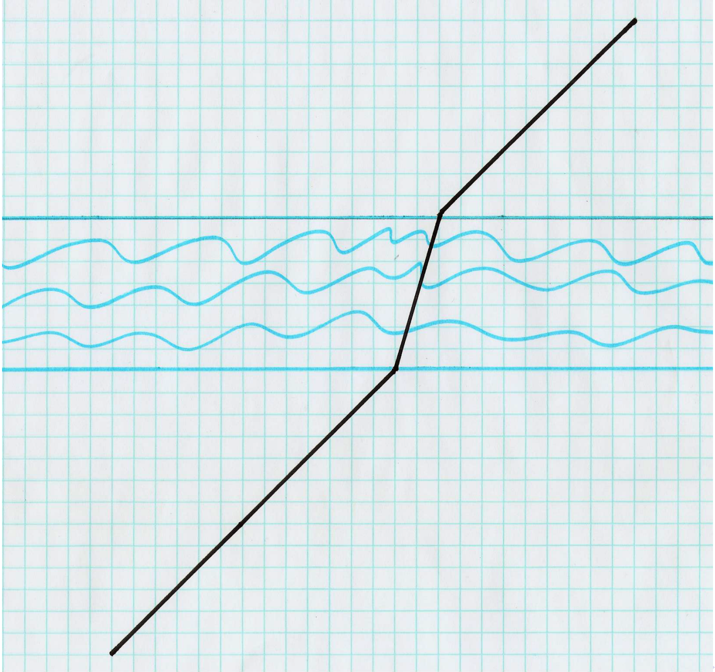
Refraction

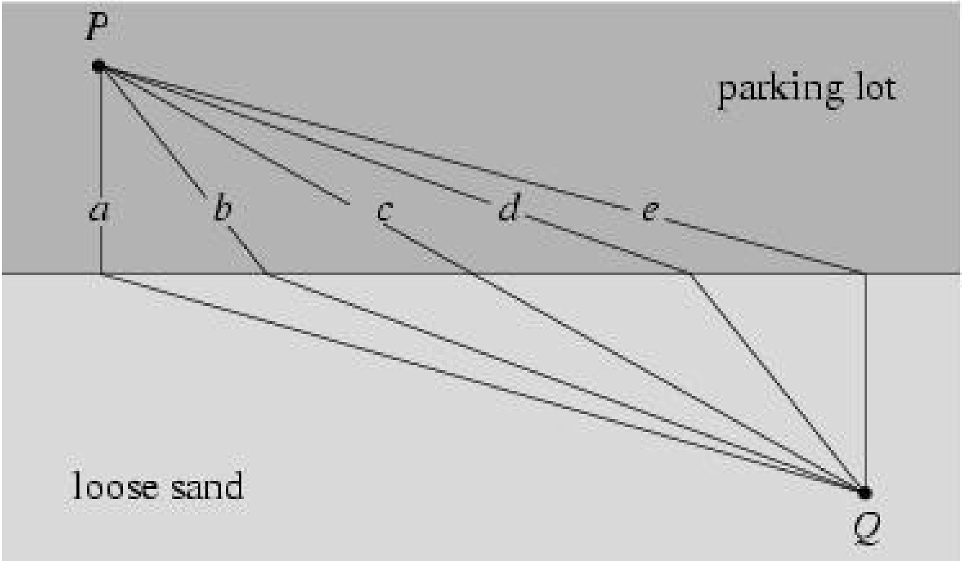
Light travels at different speeds in different media. In denser media, induced polarization of atoms cancels out more of the EM wave's electric field, which slows down the wave propagation. We saw earlier what happens when a sound wave meets a boundary between two media: same frequency, different wavelength, some transmission, some reflection.

In a medium with **index of refraction** n, light propagates at a speed c/n. For glass, $n \approx 1.5$. for water $n = 1.33$. So light moves $2/3$ as fast in glass ($3/4$ as fast in water) as it does in air. (For air, $n \approx 1.0003$. Light is a bit slower in cooler, denser air.)

Light chooses the *fastest* path between two points, accounting for the different speeds in different media. If you're walking diagonally to a river, what does your path do where it crosses the river? It changes angle! (A bit more distance on land in exchange for less distance through water.)

I hope you remember this picture even if you forget the equations.





You want to run as quickly as possible from point Q on the sandy beach to your car at point P on the paved parking lot. You can run about $2\times$ as fast on pavement as on sand. Which path takes the least time?

There is also Richard Muller's easy-to-remember statement that "The refracted wave bends toward the slower medium."

His analogy is two friends walking with arms linked. If one person changes speed, the pair always turns toward the slower person.

With a volunteer (Richard?), this is easy to demonstrate!

How much the wave turns when it crosses a boundary between two media is quantified using Snell's law: next page.

(Does anyone remember the equation for Snell's law? It involves n_1, n_2, θ_1, θ_2, and a trig function)

Snell's law

Consequence of "principle of least time."

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Light's angle w.r.t. the surface normal is smaller in the slower medium (larger n) and larger in the faster medium (smaller n).

(Illustrate with lasers & glass block, prism, etc.)

“Blackboard optics”

- mirrors & rectangular block
- Converging & diverging lens
- Notice that paths that converge all take same time!
- Measure f for converging lens
- Ray tracing with laser: $d_o = 2f, \quad d_o = 3f, \quad d_o = \frac{3}{2}f$
- Big lens: $f \approx 40\ \mathrm{cm}$. Try $d_o = 2f$.

The usual mirror/lens equation still works!

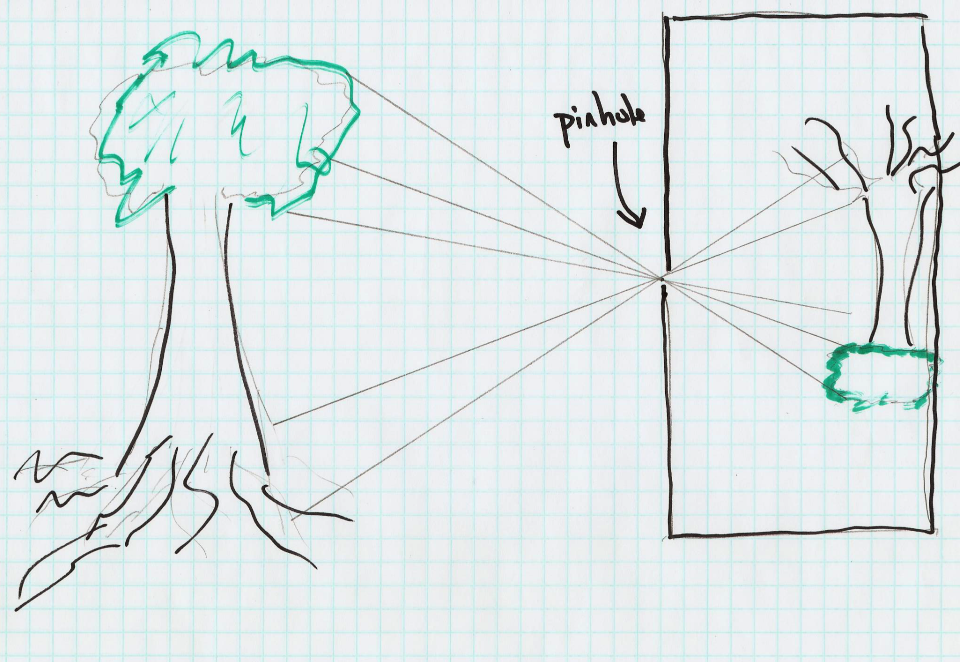
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \qquad\qquad \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Solve for d_i and h_i:

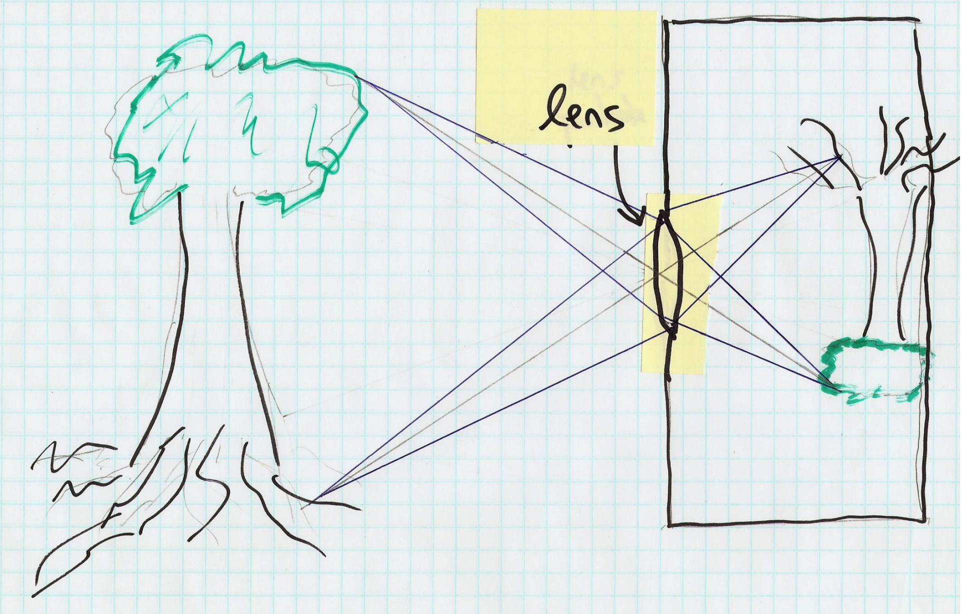
$$d_i = \frac{d_o f}{d_o - f} \qquad\qquad h_i = \frac{h_o f}{f - d_o}$$

But now $d_i > 0$ (“real image”) means the **far side** of the lens!

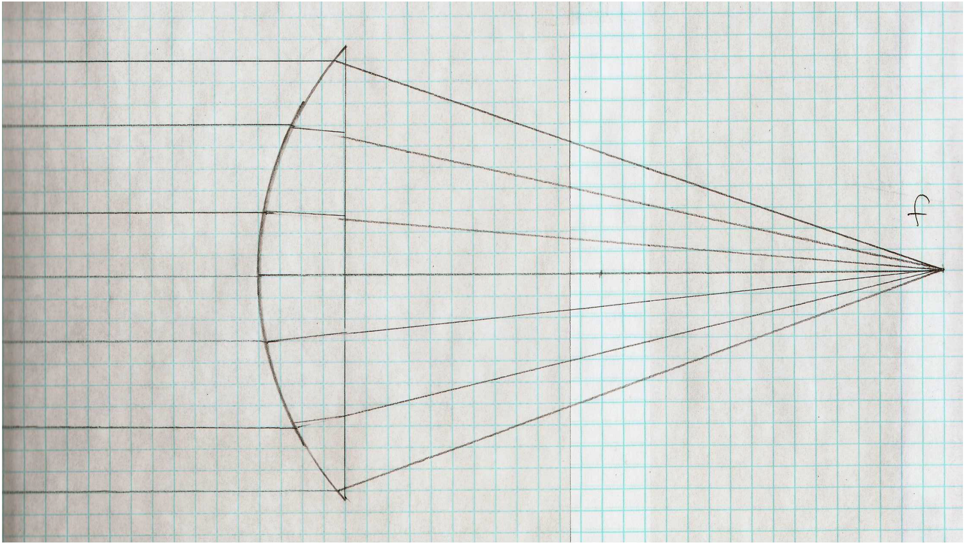
Pinhole camera



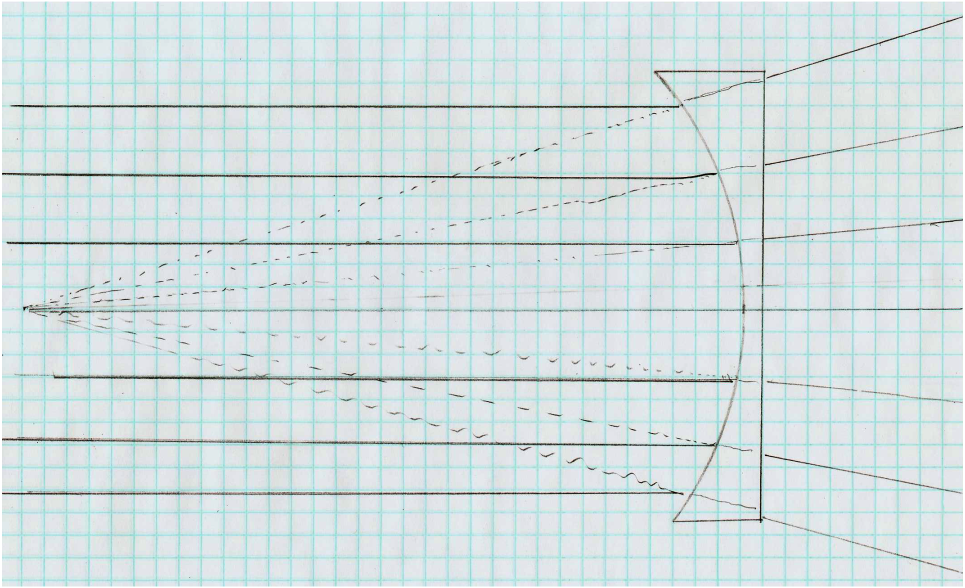
Pinhole $\rightarrow$ lens: collect *much* more light at each point on image. Light $\sim$ area $\sim \pi r^2$



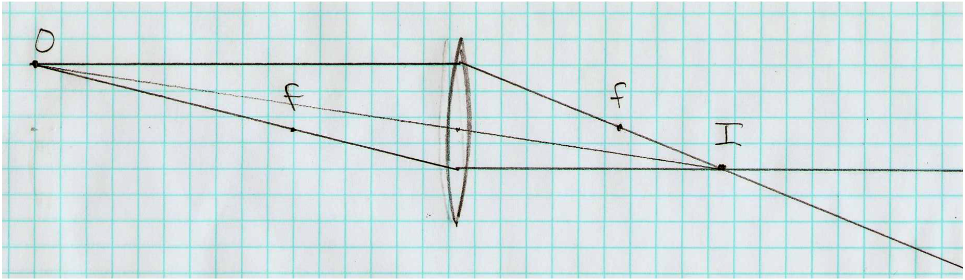
Converging lens: parallel rays focus on RHS ($f > 0$)



Diverging lens: parallel rays “focus” on LHS ($f < 0$)



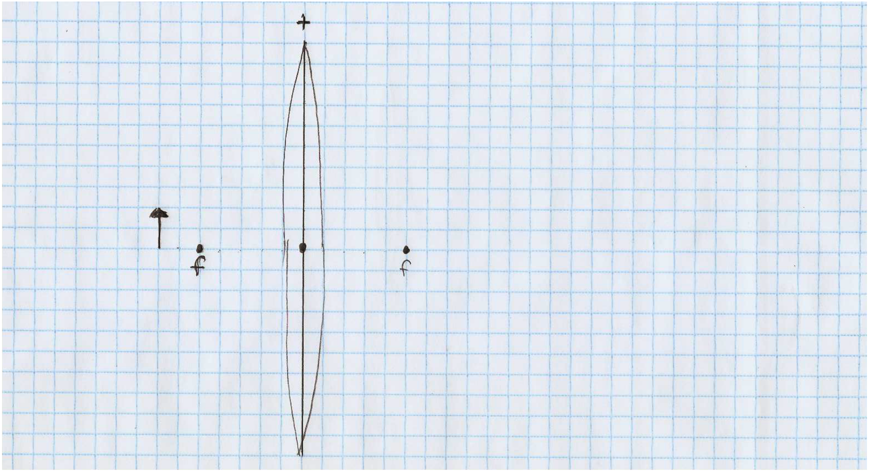
The trick for finding the image point graphically: draw the easy-to-draw rays ("principal rays"), and see where they meet.



For thin converging lens, what comes in parallel to axis on LHS must pass through focus on RHS. What passes through focus on LHS must exit parallel to axis on RHS. What passes through the point at center of lens does not bend.

(There are analogous tricks for mirrors, thin diverging lenses, but thin converging lens is most useful case to remember.)

An object is placed 7 m to the left of a converging lens having focal length $f = 5\ \mathrm{m}$. Where does the image form?



Let's try the equations first this time.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad , \quad \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad \Rightarrow \quad d_i = \frac{d_o f}{d_o - f} \quad , \quad h_i = \frac{h_o f}{f - d_o}$$

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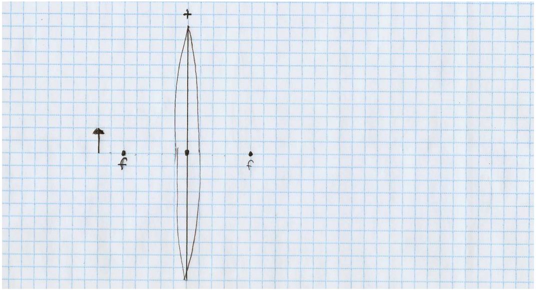

Object and image distances:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Magnification:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Image distance d_i is

(A) $+17.5\ \mathrm{m}$ (B) $-17.5\ \mathrm{m}$ (C) $+35\ \mathrm{m}$ (D) $-35\ \mathrm{m}$ (E) $+5\ \mathrm{m}$

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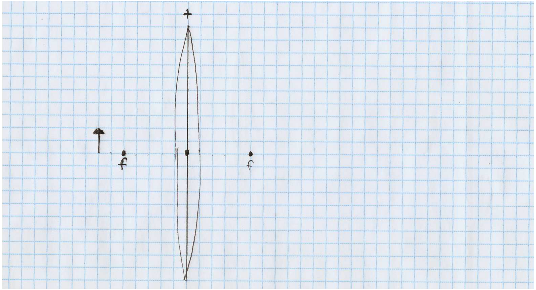


Object and image distances:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

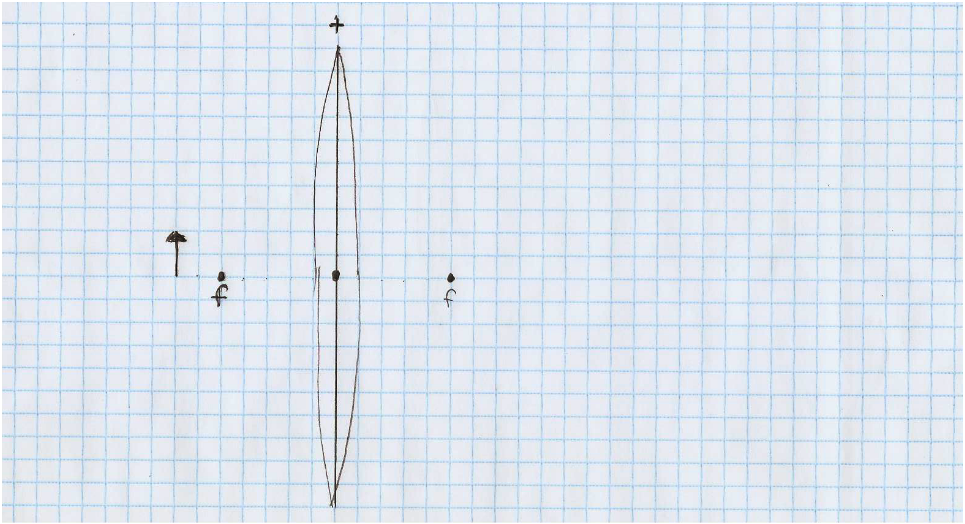
Magnification:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Magnification m is:

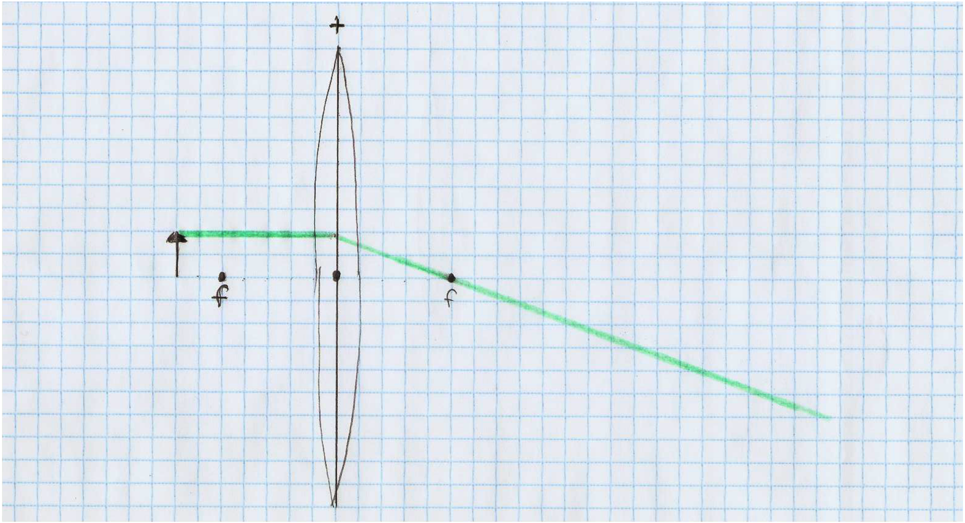
(A) $+0.4$ (B) -0.4 (C) $+2.5$ (D) -2.5 (E) $+5.0$

An object is placed 7 m to the left of a converging lens having $f = 5$ m. Where does the image form?



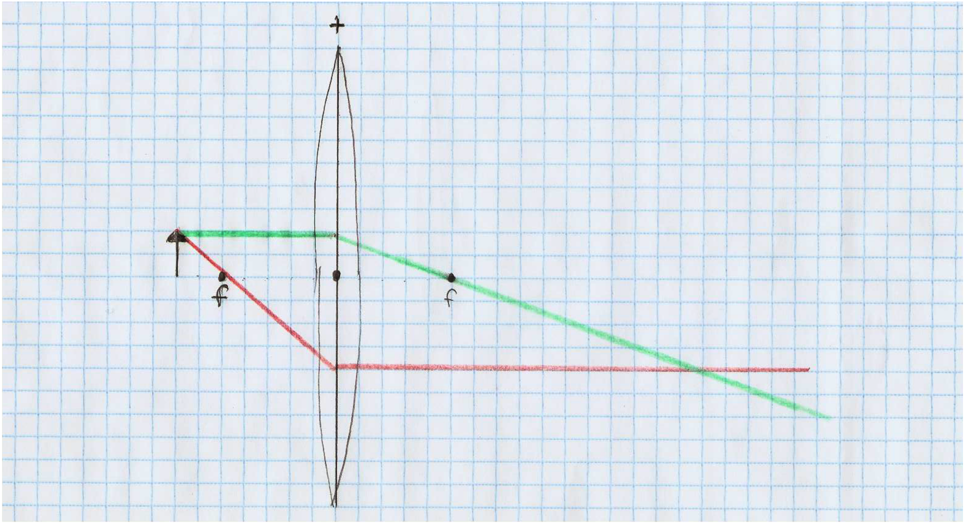
Where are the principal rays? Start with one that is horizontal on the left side of the lens.

An object is placed 7 m to the left of a converging lens having $f = 5\ \mathrm{m}$. Where does the image form?



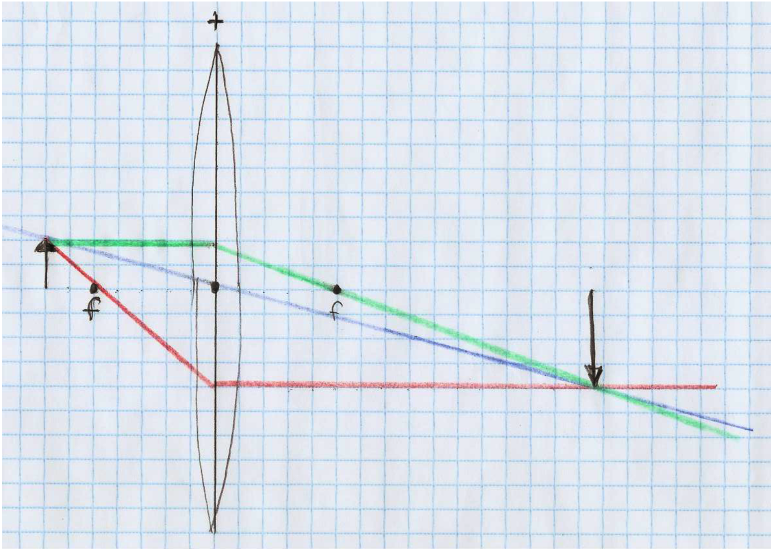
Next principal ray: passing through left focus.

An object is placed 7 m to the left of a converging lens having $f = 5\ \mathrm{m}$. Where does the image form?

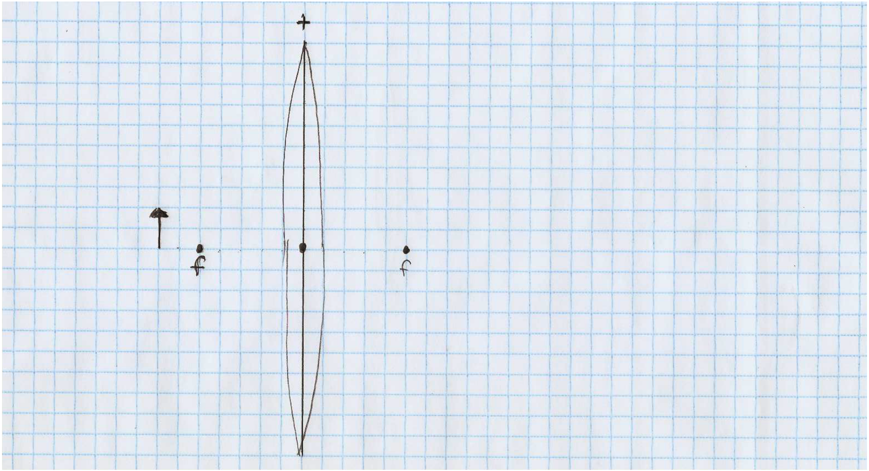


Next principal ray: passing through center of (thin) lens.

An object is placed 7 m to the left of a converging lens having $f = 5\ \mathrm{m}$. Where does the image form?



Math gave $d_i = +17.5\ \mathrm{m}$, $m = -2.5$. The graph qualitatively agrees, but my thick colored pencils missed the mark slightly.



$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad , \quad \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad \Rightarrow \quad d_i = \frac{d_o f}{d_o - f} \quad , \quad \frac{h_i}{h_o} = \frac{f}{f - d_o}$$

For what values of d_o is the image inverted/upright? Real/virtual?

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