Physics 9 — Wednesday, October 17, 2018

- HW5 due Friday.
- For last Wednesday, you read PTFP ch2 (atoms & heat)
- For Monday, you read PTFP ch9 (invisible light)
- For today you read Giancoli ch13 (temperature & kinetic theory)
- HW help sessions: Wed 4–6pm DRL **4C2** (Bill), Thu 6:30–8:30pm DRL 2C8 (Grace)
- If you'd like to do some extra-credit reading on Architectural Acoustics, you can read one or more chapters of a nicely illustrated (more drawings, less text) textbook I have (by Egan). For each chapter you read, you can collect extra credit by writing a few paragraphs (1–2 pages) to summarize what you learned from the chapter. Email me if you're interested.
- I found a way to run both Odeon and CATT-Acoustic on MacOS without a virtual machine! Stay tuned.

Equation of continuity

Expresses the fact that mass is not accumulating anywhere, for a steady flow of fluid. Time rate of mass flowing past a given surface of cross-sectional area A is constant:

$$\frac{\Delta m}{\Delta t} = \text{constant}$$

$$\frac{\Delta m}{\Delta t} = \rho\ A\ \frac{\Delta x}{\Delta t} = \rho\ A\ v = \text{constant}$$

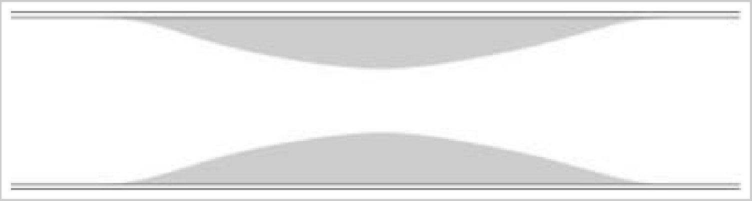
And if density ρ is constant (i.e. incompressible fluid), then

$$A\ v = \text{constant} \quad \Rightarrow \quad A_1 v_1 = A_2 v_2$$

For example, the water running out of your faucet is accelerating (gravity), so the area must decrease as the speed increases.

Another example: river flows at higher speed where it is narrower.

Water flows through an old plumbing pipe that is partially blocked by mineral deposits along the wall of the pipe. Through which part of the pipe is the fluid speed largest?

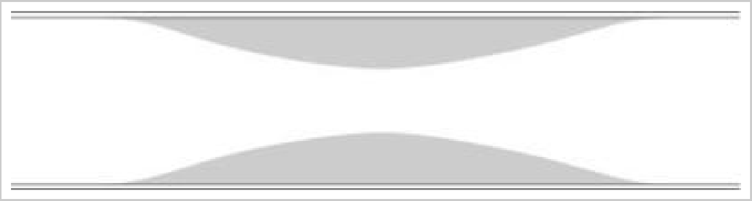


(A) Fastest in the narrow part.

(B) Fastest in the wide part.

(C) The speed is the same in both parts.

Water flows through an old plumbing pipe that is partially blocked by mineral deposits along the wall of the pipe. Through which part of the pipe is the flux (volume of water per unit time) largest?



(A) Fastest in the narrow part.

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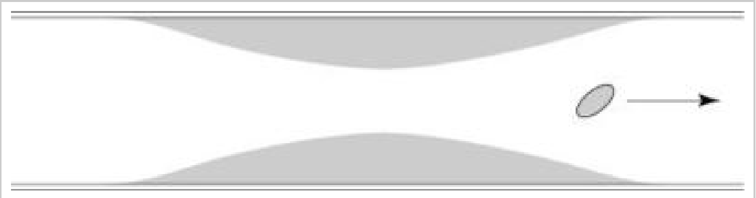
A water-borne insect drifts along with the flow of water through a pipe that is partially blocked by deposits. As the insect drifts from the narrow region to the wider region, it experiences



(A) a forward acceleration (it speeds up)

(B) a backward acceleration (it slows down)

(C) no acceleration (its speed and direction do not change)

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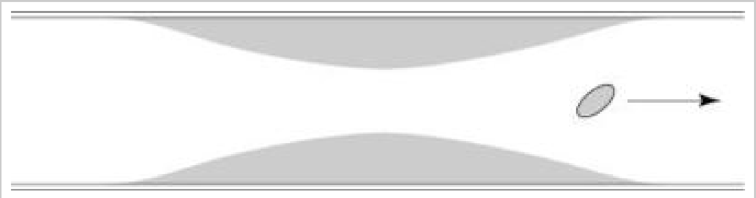

(A) a forward acceleration (it speeds up)

(B) a backward acceleration (it slows down)

(C) no acceleration (its speed and direction do not change)

If you said that the speed changes, now ask yourself: Is there a net force (per unit area) that causes this change in speed?

This one is trickier

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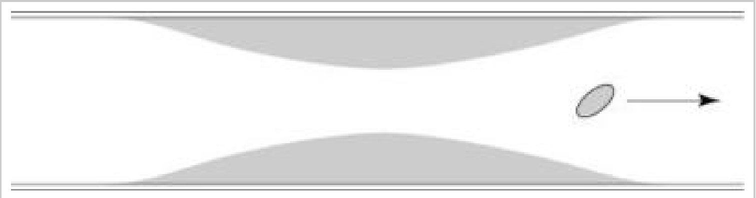

(A) an increase in water pressure ($P_{\text{wide}} > P_{\text{narrow}}$)
(B) no change in water pressure ($P_{\text{wide}} = P_{\text{narrow}}$)
(C) a decrease in water pressure ($P_{\text{wide}} < P_{\text{narrow}}$)

Hint: is the speed the same or different? Is there a net force (per unit area) that causes this change in speed?

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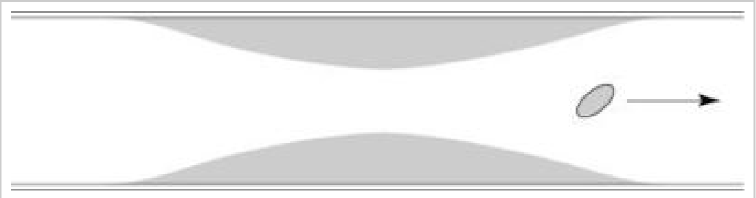

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Hint: is the speed the same or different? Is there a net force (per unit area) that causes this change in speed?

Stop to ponder the net force acting on the insect when insect is just upstream of vs. just downstream of the narrow region.

We've been looking for a while now at the equation

$$\Delta P = -\rho g \Delta y$$

which says that the pressure in a stationary incompressible fluid increases in proportion to depth.

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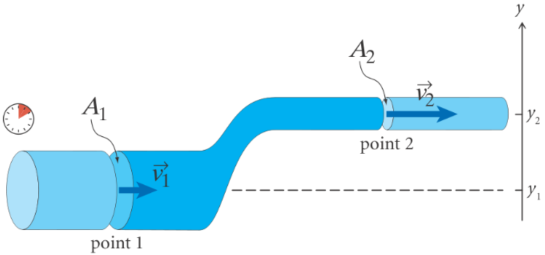
which says that the pressure in a stationary incompressible fluid increases in proportion to depth.

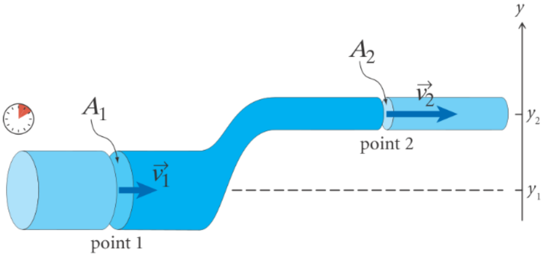
We just saw in the insect-in-pipe example that pressure seems to increase when flow speed is slower, and pressure seems to decrease when flow speed is faster.

We can use "Bernoulli's equation" to make sense of both of these facts, by considering **conservation of mechanical energy** for an incompressible fluid.



Consider the mechanical energy of a little chunk of fluid flowing through a pipe, from point 1 to point 2.

If $v_2 > v_1$, what form of mechanical energy has increased?



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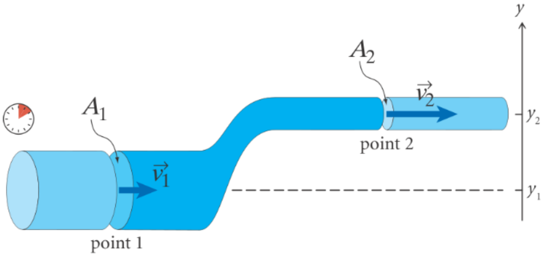
If $y_2 > y_1$, what form of mechanical energy has increased?



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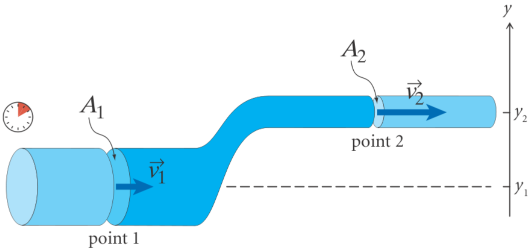
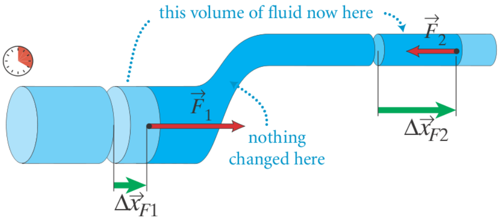
If $v_2 > v_1$, what form of mechanical energy has increased?

If $y_2 > y_1$, what form of mechanical energy has increased?

Suppose the combined kinetic + potential energy of this chunk of fluid has increased from point 1 to point 2. So some external agent has done work to provide this energy.

Work is force times displacement. What force (per unit area) could perform this work on that chunk of fluid?





$$(P_1A_1)(v_1\Delta t) - (P_2A_2)(v_2\Delta t) = \frac{1}{2}m(v_2^2 - v_1^2) + mg(y_2 - y_1)$$

$$(P_1 - P_2)(\text{Volume}) = (P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}m(v_2^2 - v_1^2) + mg(y_2 - y_1)$$

Pressure in a fluid depends both on depth and on speed. To analyze this, we consider mechanical energy of the fluid.

Notice PV has units of energy, and of course work has units of energy. (Work = force × displacement.)

$$(\text{pressure})(\text{volume}) = \left(\frac{\text{force}}{\text{area}}\right)(\text{area} \times \text{length}) = (\text{force})(\text{length})$$

If a difference in pressure causes a volume of fluid to be displaced, that pressure difference is doing work on the fluid, thus changing the fluid's energy.

In the absence of friction (a.k.a. "viscosity,"), we expect mechanical energy to add up. So we set the work done (due to the pressure difference) equal to the change in mechanical energy: $(\text{work}) = \Delta\,(\text{energy})$.

$$(P_1 - P_2)V = \frac{1}{2}m(v_2^2 - v_1^2) + mg(y_2 - y_1)$$

$$(P_1 - P_2)V = \frac{1}{2}m(v_2^2 - v_1^2) + mg(y_2 - y_1)$$

rearranging terms, we get

$$P_1 V + \frac{1}{2}mv_1^2 + mgy_1 = P_2 V + \frac{1}{2}mv_2^2 + mgy_2$$

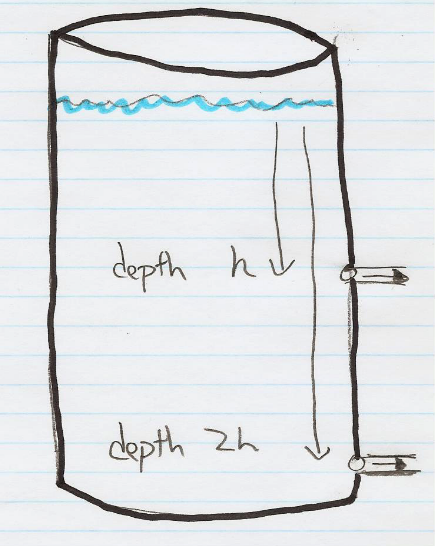
If we divide all terms by volume, we get *Bernoulli's equation:*

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

or in this more compact form:

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}}$$

Strictly speaking, Bernoulli's equation only works if the fluid is incompressible, if the flow is steady and non-turbulent, and if there is no dissipation of energy by friction. But it's handy for estimation even in cases where it is not strictly valid.



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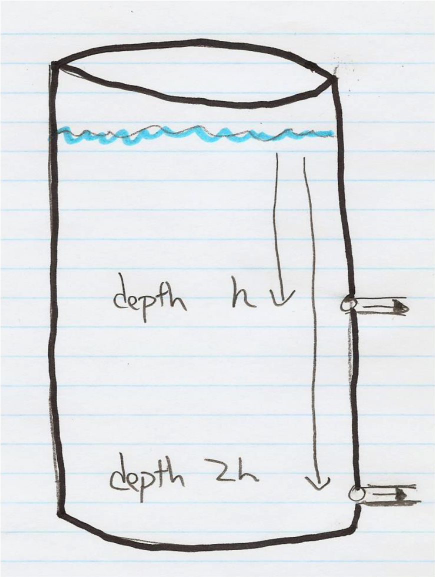


A large, water-filled cylinder has two horizontal spigots. Spigot 1 is at depth h, and spigot 2 is at depth $2h$. If I neglect the small downward motion of the top surface of the water, what I can I say about the horizontal velocity v_x of the water emerging from each spigot?

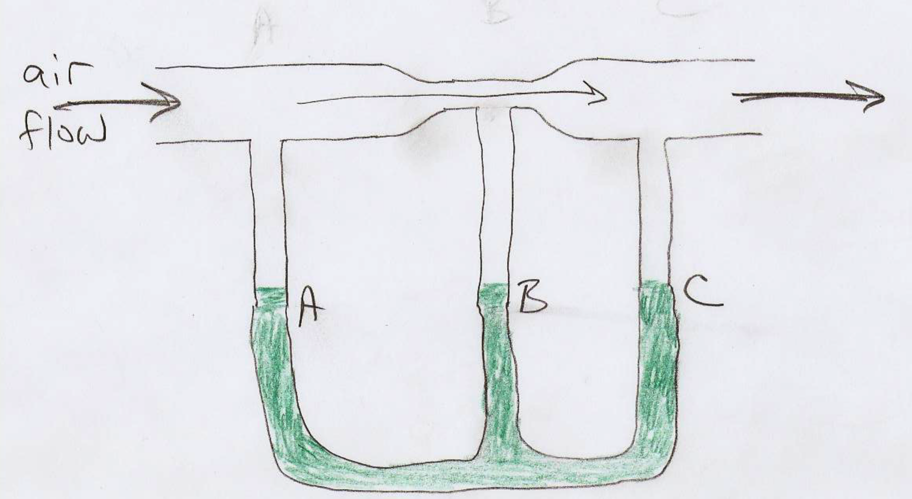
(A) $v_{2x} = \frac{1}{2} v_{1x}$

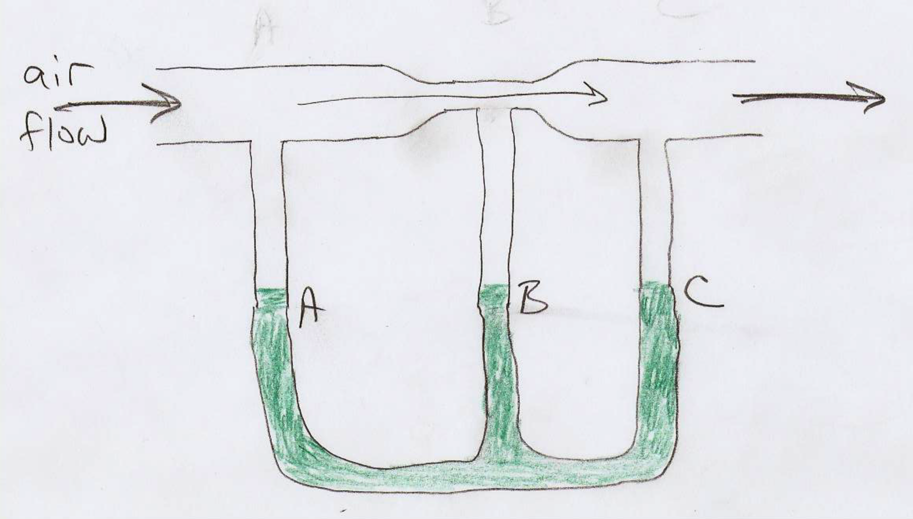
(B) $v_{2x} = \sqrt{2} v_{1x}$

(C) $v_{2x} = v_{1x}$

(D) $v_{2x} = 2 v_{1x}$

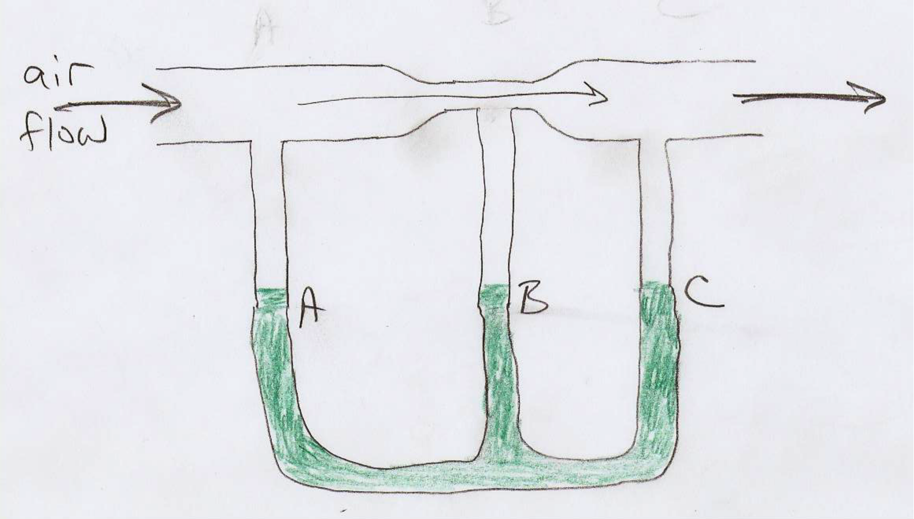
This experiment combines the "equation of continuity" with Bernoulli's equation. When I open the valve, compressed air will flow from left to right through the horizontal tube. The tube is wide on the left and right, but narrow in the middle. Where is the speed largest? Where is the pressure lowest? How will height of the green liquid respond to changes in pressure in the horizontal tube?





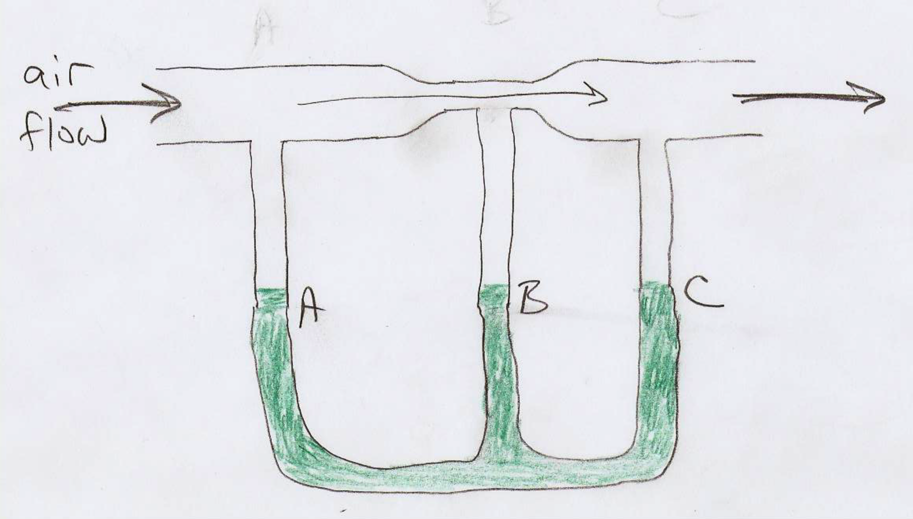
Once I turn on the air flow, the horizontal speed of the flowing air will be

(A) fastest above tube B
(B) slowest above tube B
(C) the same above all tubes



Once I turn on the air flow, the air pressure above the green liquid will be

(A) lowest in tube B
(B) highest in tube B
(C) the same in all tubes



Once I turn on the air flow, the height of the green liquid will be

(A) lowest in tube B

(B) highest in tube B

(C) the same in all tubes

What gauge pressure in the water mains is necessary if a firehose is to spray water to a height of 10 meters? (Use $g = 10\ \mathrm{m/s^2}$ and ignore all friction or other dissipative effects.)

(a) $10^3\ \mathrm{N/m^2}$ (b) $10^4\ \mathrm{N/m^2}$ (c) $10^5\ \mathrm{N/m^2}$ (d) $10^6\ \mathrm{N/m^2}$

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How many atmospheres is this?

If the "gauge pressure" is 1 atm, what is the absolute pressure?

recap: most important results for fluids

pressure: $P = F/A$. $1\ \text{Pa} = 1\ \text{N/m}^2$. $1\ \text{atm} = 101323\ \text{Pa} = 760\ \text{mm-Hg}$.

Pascal's principle: if an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.

Archimedes' principle: the buoyant force on an object immersed (or partially immersed) in a fluid equals the weight of the fluid displaced by that object.

Equation of continuity: $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

Bernoulli's equation (neglects viscosity, assumes constant density):

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

We argued that Bernoulli's equation represents conservation of mechanical energy for a fluid in which frictional forces can be neglected, while the continuity equation represents conservation of mass for an incompressible fluid. Let's work through a problem together using these two equations.

Suppose that to heat a home, an HVAC system pumps hot water in a single loop (no branches or "tees" in the pipe) that flows past all of the baseboard radiators in the house.

If the water is pumped at a speed of 0.50 $\mathrm{m/s}$ through a 4.0 cm (diameter) pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.0 cm (diameter) pipe on the third floor, 10.0 m above? (Neglect viscosity.)

Let's figure out the upstairs flow speed first. It is

(A) 0.125 m/s

(B) 0.25 m/s

(C) 0.50 m/s

(D) 1.00 m/s

(E) 2.00 m/s

(F) 4.00 m/s

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Now that we have upstairs speed = 2.0 m/s, upstairs pressure is:

(A) 0.53 atm
(B) 1.05 atm
(C) 1.53 atm
(D) 2.01 atm
(E) 2.98 atm
(F) 3.51 atm

In this case, would the answer have changed much if we had neglected the change in speed?

At what speed does water flow from a small hole at the bottom of a very wide, 4.9-meter-deep storage tank filled with water?

(A) 1.2 m/s

(B) 3.5 m/s

(C) 4.9 m/s

(D) 6.9 m/s

(E) 9.8 m/s

(F) 13.9 m/s

Use Bernoulli's equation to estimate the air pressure P_{storm} inside a category 5 hurricane, where the wind speed is 300 km/h (83 m/s). Let $P_{\mathrm{atm}} = 101325 \ \mathrm{N/m^2}$ denote normal atmospheric pressure, and let $\rho = 1.2 \ \mathrm{kg/m^3}$ be the density of air.

(A) $P_{\mathrm{storm}} + \frac{1}{2}\rho(0)^2 + \rho g(0) = P_{\mathrm{atm}} + \frac{1}{2}\rho(83 \ \frac{\mathrm{m}}{\mathrm{s}})^2 + \rho g(0)$

(B) $P_{\mathrm{storm}} + \frac{1}{2}\rho(0)^2 + \rho g(0) = P_{\mathrm{atm}} + \frac{1}{2}\rho(0)^2 + \rho g(83 \ \mathrm{m})$

(C) $P_{\mathrm{atm}} + \frac{1}{2}\rho(0)^2 + \rho g(0) = P_{\mathrm{storm}} + \frac{1}{2}\rho(83 \ \frac{\mathrm{m}}{\mathrm{s}})^2 + \rho g(0)$

(D) $P_{\mathrm{atm}} + \frac{1}{2}\rho(0)^2 + \rho g(0) = P_{\mathrm{storm}} + \frac{1}{2}\rho(0)^2 + \rho g(83 \ \mathrm{m})$

If the wind blows horizontally at speed $v = 10\ \mathrm{m/s}$ over a house (that's about 22 mph), what is the net force on a flat roof of area $A = 100\ \mathrm{m}^2$ (that's about 1100 sqft)? For the density of air, use $\rho = 1.3\ \mathrm{kg/m^3}$.

What is the speed at which water flows from a faucet if the pressure head is 20 meters? (In other words the height of the nearby water tower is 20 meters above the height of the faucet.) Neglect all friction or other dissipative effects. Use $g = 10\ \mathrm{m/s}^2$.

(a) 5 m/s (b) 10 m/s (c) 14 m/s (d) 20 m/s

What is the volume rate of flow of water from a 2 cm-diameter faucet if the pressure head is 20 meters? (In other words the height of the nearby water tower is 20 meters above the height of the faucet.) Neglect all friction or other dissipative effects. Use $g = 10 \text{ m/s}^2$. Use $\pi \approx 3$.

(a) $6 \text{ cm}^3/\text{s}$ (b) 0.6 L/s (c) 6 L/s (d) $6 \text{ m}^3/\text{s}$

What is the volume rate of flow of water from a 2 cm-diameter faucet if the pressure head is 20 meters? (In other words the height of the nearby water tower is 20 meters above the height of the faucet.) Neglect all friction or other dissipative effects. Use $g = 10\ \mathrm{m/s^2}$. Use $\pi \approx 3$.

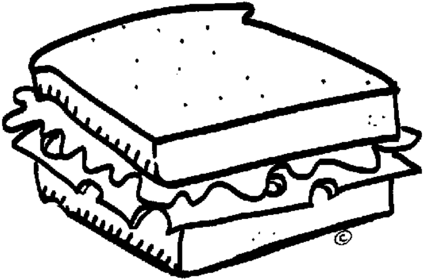
(a) 6 $\mathrm{cm^3/s}$ (b) 0.6 $\mathrm{L/s}$ (c) 6 $\mathrm{L/s}$ (d) 6 $\mathrm{m^3/s}$

This seems unrealistically fast, even though a 20 meter (2 atm "gauge pressure," or 3 atm absolute pressure) pressure head is not unrealistic. What are we neglecting?!

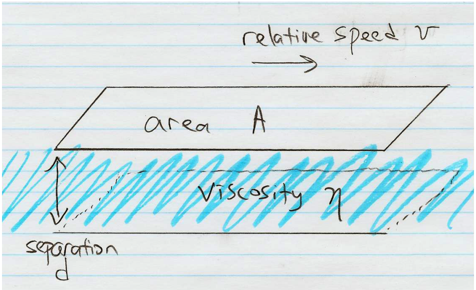
Viscosity

Viscosity η measures how molasses-like a fluid is! Viscosity describes the frictional drag force experienced e.g. between two parallel surfaces with area A, separation d, and relative velocity v

$$F = \frac{\eta\, A\, v}{d}$$



Picture a molasses sandwich ...



Viscosity

$$F = \frac{\eta \ A \ v}{d}$$

Viscosity implies that something trying to move through the liquid experiences a frictional force, which dissipates mechanical energy.

Demo: steel ball sinking in cylinder of STP (molasses-like additive for engine oil) vs. sinking in water.

Digression: terminal velocity

$$F = \frac{\eta\ A\ v}{d}$$

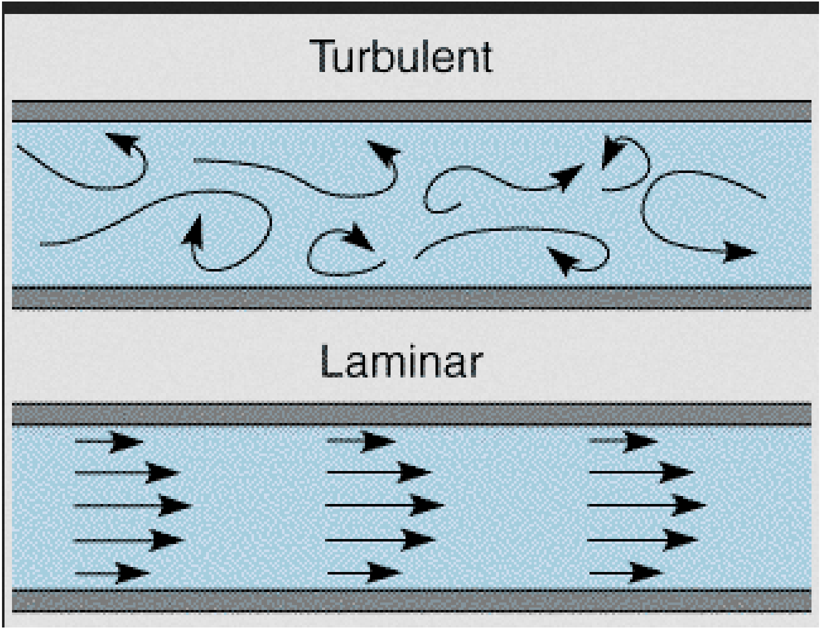
Notice that $F_{\text{drag}} \propto v$, i.e. the drag force increases with higher speed, unlike the sliding friction we are used to from mechanics.

An object falling under the constant force of gravity will reach **terminal velocity** when $F_{\text{drag}} = F_{\text{gravity}}$.

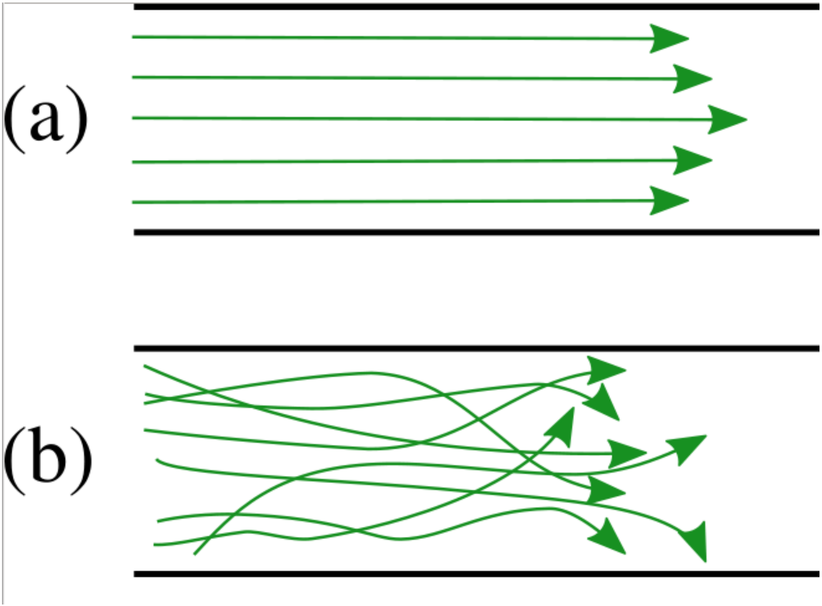
$$\left(\frac{\eta A}{d}\right)\ v = mg \quad \Rightarrow \quad v_{\text{terminal}} = \frac{mg}{(\eta A/d)}$$

In free-fall, everything falls with constant acceleration g. (Recall feather & penny dropped in evacuated cylinder.)

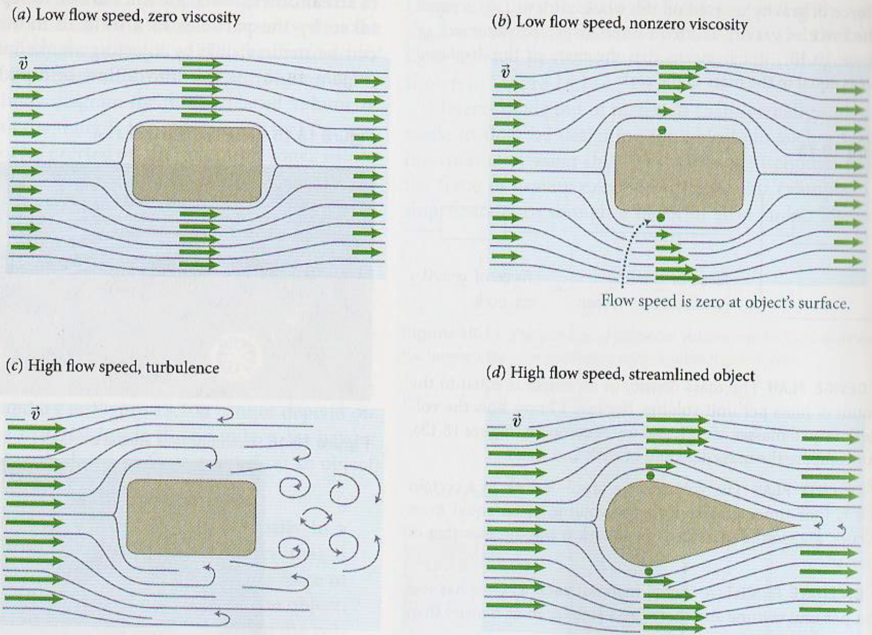
But for two falling objects of same size and shape, the more massive object has larger terminal velocity. Experience with friction & viscosity is why Newton's laws can be counter-intuitive.



Turbulent flow is noisy; laminar flow is quiet.



Which flow is laminar?


(a) Low flow speed, zero viscosity
(b) Low flow speed, nonzero viscosity
Flow speed is zero at object's surface.
(c) High flow speed, turbulence
(d) High flow speed, streamlined object

Digression: v vs. v^2 drag forces (laminar vs. turbulent)

If the fluid flows smoothly around the obstacle (laminar flow), then

$$F_{\mathrm{drag}} \propto v$$

But if there is turbulent flow around the obstacle, then

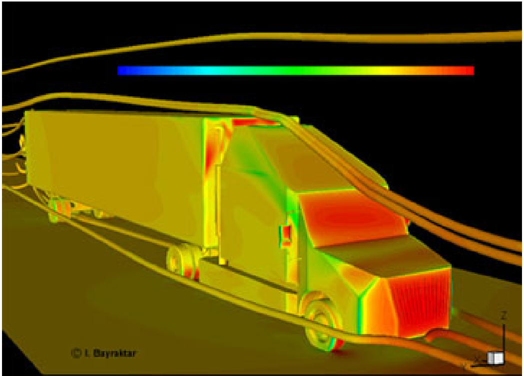
$$F_{\mathrm{drag}} \propto v^2$$

If a falling object feels $F_{\mathrm{drag}} = (\mathrm{constant}) \times v^2$, then

$$v_{\mathrm{terminal}} = \sqrt{\frac{mg}{(\mathrm{constant})}}$$

For example, falling coffee filters experience air drag $\propto v^2$, so the air flow around them must be somewhat turbulent.

Notice turbulent air flow behind truck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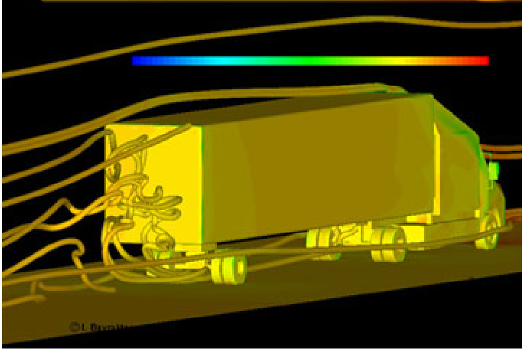

Aerodynamic side skirts beneath truck trailers improve fuel economy by 5–10%, by keeping air flow laminar underneath truck. Also notice the smooth shape of top of cab.

All this is to keep that v^2 contribution as small as possible!

Poiseuille's equation

Viscosity also impedes the flow of a fluid through a conduit: imagine trying to flow molasses through a narrow tube.

For **laminar flow**, flow rate Q ($\frac{\text{volume}}{\text{time}}$, i.e. $\frac{\text{m}^3}{\text{s}}$) is given by

$$Q = \frac{\pi R^4 \left(P_1 - P_2\right)}{8\eta L}$$

(Derivation at the end of Mazur's chapter on fluids.)

- longer pipe → slower flow
- higher viscosity → slower flow
- higher input pressure → faster flow
- wider pipe → **hugely faster flow**

Q: What happens to flow rate if I double the tube radius, keeping everything else the same? Let's try it!

Physics 9 — Monday, October 15, 2018

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- For today, you read PTFP ch9 (invisible light)
- For Wed, read Giancoli ch13 (temperature & kinetic theory)
- If you'd like to do some extra-credit reading on Architectural Acoustics, you can read one or more chapters of a nicely illustrated (more drawings, less text) textbook I have (by Egan). For each chapter you read, you can collect extra credit by writing a few paragraphs (1–2 pages) to summarize what you learned from the chapter. Email me if you're interested.
- I found a way to run both Odeon and CATT-Acoustic on MacOS without a virtual machine! Stay tuned.