Physics 9 — Friday, October 19, 2018

- Turn in HW5. Pick up HW6 handout.
- For this coming Monday, read Giancoli ch14 (heat).
- For next Wednesday, read Giancoli ch15 (thermodynamics).
- I found a way to run both Odeon and CATT-Acoustic on MacOS without a virtual machine! Stay tuned.

recap: most important results for fluids

pressure: $P = F/A$. $1\ \mathrm{Pa} = 1\ \mathrm{N/m^2}$. $1\ \mathrm{atm} = 101323\ \mathrm{Pa} = 760\ \mathrm{mm\text{-}Hg}$.

Pascal's principle: if an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.

Archimedes' principle: the buoyant force on an object immersed (or partially immersed) in a fluid equals the weight of the fluid displaced by that object.

Equation of continuity: $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

Bernoulli's equation (neglects viscosity, assumes constant density):

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

We argued that Bernoulli's equation represents conservation of mechanical energy for a fluid in which frictional forces can be neglected, while the continuity equation represents conservation of mass for an incompressible fluid. Let's work through a problem together using these two equations.

Suppose that to heat a home, an HVAC system pumps hot water in a single loop (no branches or "tees" in the pipe) that flows past all of the baseboard radiators in the house.

If the water is pumped at a speed of 0.50 $\mathrm{m/s}$ through a 4.0 cm (diameter) pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.0 cm (diameter) pipe on the third floor, 10.0 m above? (Neglect viscosity.)

Let's figure out the upstairs flow speed first. It is

(A) 0.125 m/s
(B) 0.25 m/s
(C) 0.50 m/s
(D) 1.00 m/s
(E) 2.00 m/s
(F) 4.00 m/s

Suppose that to heat a home, an HVAC system pumps hot water in a single loop (no branches or "tees" in the pipe) that flows past all of the baseboard radiators in the house.

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Now that we have upstairs speed = 2.0 m/s, upstairs pressure is:

(A) 0.53 atm
(B) 1.05 atm
(C) 1.53 atm
(D) 2.01 atm
(E) 2.98 atm
(F) 3.51 atm

In this case, would the answer have changed much if we had neglected the change in speed?

At what speed does water flow from a small hole at the bottom of a very wide, 4.9-meter-deep storage tank filled with water?

(A) 1.2 m/s

(B) 3.5 m/s

(C) 4.9 m/s

(D) 6.9 m/s

(E) 9.8 m/s

(F) 13.9 m/s

Use Bernoulli's equation to estimate the air pressure P_{storm} inside a category 5 hurricane, where the wind speed is 300 km/h (83 m/s). Let $P_{\mathrm{atm}} = 101325\ \mathrm{N/m^2}$ denote normal atmospheric pressure, and let $\rho = 1.2\ \mathrm{kg/m^3}$ be the density of air.

(A) $P_{\mathrm{storm}} + \frac{1}{2}\rho(0)^2 + \rho g(0) = P_{\mathrm{atm}} + \frac{1}{2}\rho(83\ \frac{\mathrm{m}}{\mathrm{s}})^2 + \rho g(0)$

(B) $P_{\mathrm{storm}} + \frac{1}{2}\rho(0)^2 + \rho g(0) = P_{\mathrm{atm}} + \frac{1}{2}\rho(0)^2 + \rho g(83\ \mathrm{m})$

(C) $P_{\mathrm{atm}} + \frac{1}{2}\rho(0)^2 + \rho g(0) = P_{\mathrm{storm}} + \frac{1}{2}\rho(83\ \frac{\mathrm{m}}{\mathrm{s}})^2 + \rho g(0)$

(D) $P_{\mathrm{atm}} + \frac{1}{2}\rho(0)^2 + \rho g(0) = P_{\mathrm{storm}} + \frac{1}{2}\rho(0)^2 + \rho g(83\ \mathrm{m})$

If the wind blows horizontally at speed $v = 10\ \mathrm{m/s}$ over a house (that's about 22 mph), what is the net force on a flat roof of area $A = 100\ \mathrm{m}^2$ (that's about 1100 sqft)? For the density of air, use $\rho = 1.3\ \mathrm{kg/m}^3$.

What is the speed at which water flows from a faucet if the pressure head is 20 meters? (In other words the height of the nearby water tower is 20 meters above the height of the faucet.) Neglect all friction or other dissipative effects. Use $g = 10 \mathrm{~m/s^2}$.

(a) 5 m/s (b) 10 m/s (c) 14 m/s (d) 20 m/s

What is the volume rate of flow of water from a 2 cm-diameter faucet if the pressure head is 20 meters? (In other words the height of the nearby water tower is 20 meters above the height of the faucet.) Neglect all friction or other dissipative effects. Use $g = 10\ \mathrm{m/s^2}$. Use $\pi \approx 3$.

(a) 6 $\mathrm{cm^3/s}$ (b) 0.6 $\mathrm{L/s}$ (c) 6 $\mathrm{L/s}$ (d) 6 $\mathrm{m^3/s}$

What is the volume rate of flow of water from a 2 cm-diameter faucet if the pressure head is 20 meters? (In other words the height of the nearby water tower is 20 meters above the height of the faucet.) Neglect all friction or other dissipative effects. Use $g = 10\ \mathrm{m/s^2}$. Use $\pi \approx 3$.

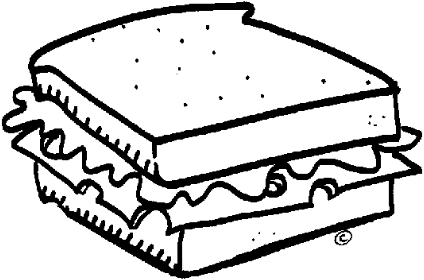
(a) $6\ \mathrm{cm^3/s}$ (b) $0.6\ \mathrm{L/s}$ (c) $6\ \mathrm{L/s}$ (d) $6\ \mathrm{m^3/s}$

This seems unrealistically fast, even though a 20 meter (2 atm "gauge pressure," or 3 atm absolute pressure) pressure head is not unrealistic. What are we neglecting?!

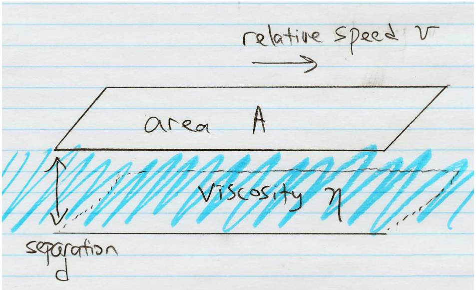
Viscosity

Viscosity η measures how molasses-like a fluid is! Viscosity describes the frictional drag force experienced e.g. between two parallel surfaces with area A, separation d, and relative velocity v

$$F = \frac{\eta \, A \, v}{d}$$



Picture a molasses sandwich . . .



Viscosity

$$F = \frac{\eta\ A\ v}{d}$$

Viscosity implies that something trying to move through the liquid experiences a frictional force, which dissipates mechanical energy.

Remember Wednesday's demo: steel ball sinking in cylinder of STP (molasses-like additive for engine oil) vs. sinking in water vs. free-falling in air.

Digression: terminal velocity

$$F = \frac{\eta \; A \; v}{d}$$

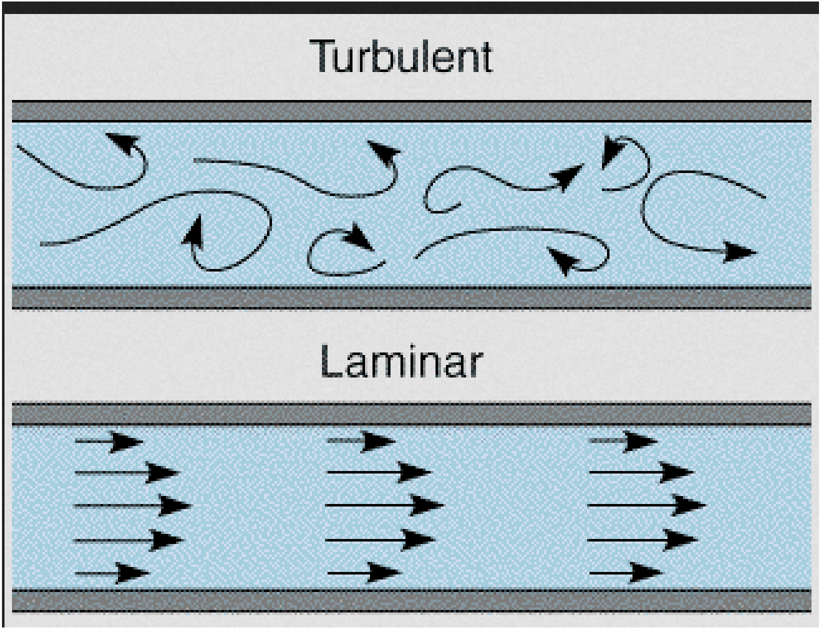
Notice that $F_{\rm drag} \propto v$, i.e. the drag force increases with higher speed, unlike the sliding friction we are used to from mechanics.

An object falling under the constant force of gravity will reach **terminal velocity** when $F_{\rm drag} = F_{\rm gravity}$.

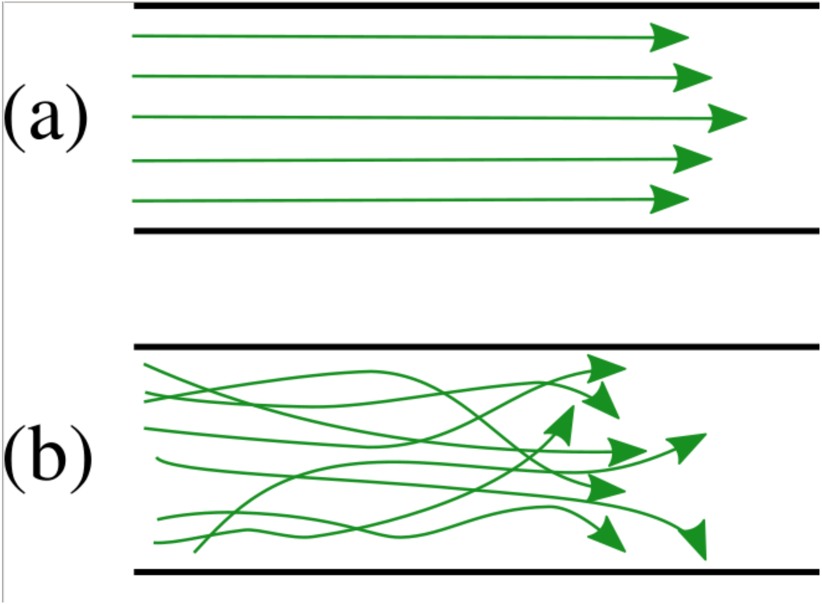
$$\left(\frac{\eta A}{d}\right) \; v = mg \quad \Rightarrow \quad v_{\rm terminal} = \frac{mg}{(\eta A/d)}$$

In free-fall, everything falls with constant acceleration g. (Recall feather & penny dropped in evacuated cylinder.)

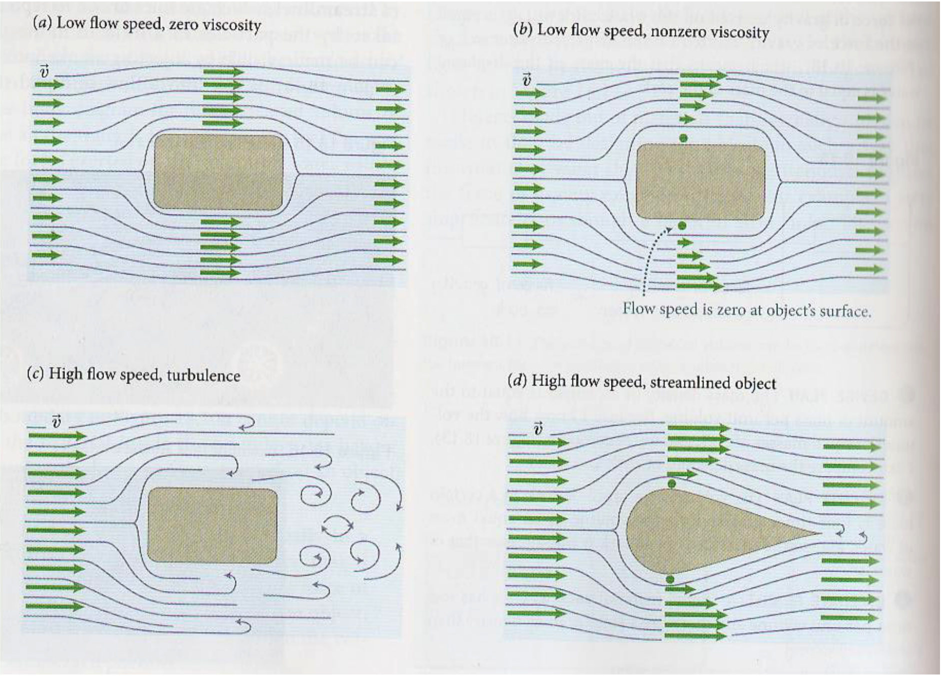
But for two falling objects of same size and shape, the more massive object has larger terminal velocity. Experience with friction & viscosity is why Newton's laws can be counter-intuitive.



Turbulent flow is noisy; laminar flow is quiet.



Which flow is laminar?



(a) Low flow speed, zero viscosity
(b) Low flow speed, nonzero viscosity
Flow speed is zero at object's surface.
(c) High flow speed, turbulence
(d) High flow speed, streamlined object
$\vec{v}$

Digression: v vs. v^2 drag forces (laminar vs. turbulent)

If the fluid flows smoothly around the obstacle (laminar flow), then

$$F_{\text{drag}} \propto v$$

But if there is turbulent flow around the obstacle, then

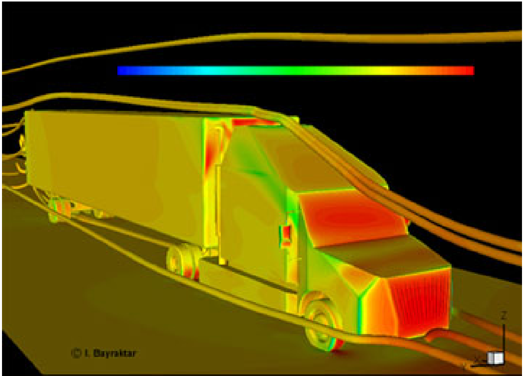
$$F_{\text{drag}} \propto v^2$$

If a falling object feels $F_{\text{drag}} = (\text{constant}) \times v^2$, then

$$v_{\text{terminal}} = \sqrt{\frac{mg}{(\text{constant})}}$$

For example, falling coffee filters experience air drag $\propto v^2$, so the air flow around them must be somewhat turbulent.

Notice turbulent air flow behind truck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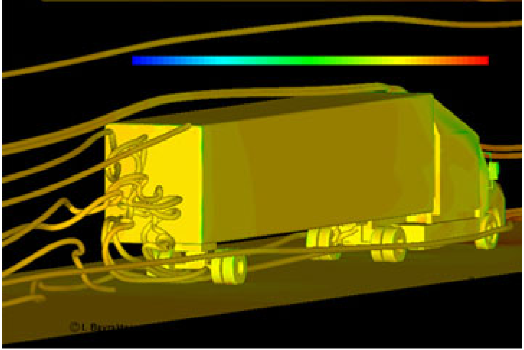

Aerodynamic side skirts beneath truck trailers improve fuel economy by 5–10%, by keeping air flow laminar underneath truck. Also notice the smooth shape of top of cab.

All this is to keep that v^2 contribution as small as possible!

Poiseuille's equation

Viscosity also impedes the flow of a fluid through a conduit: imagine trying to flow molasses through a narrow tube.

For **laminar flow**, flow rate Q ($\frac{\text{volume}}{\text{time}}$, i.e. $\frac{\text{m}^3}{\text{s}}$) is given by

$$Q = \frac{\pi R^4 \, (P_1 - P_2)}{8 \eta L}$$

(Derivation at end of Eric Mazur's fluids chapter (18), if you're curious. Once the flow is fast enough to be turbulent, this equation no longer works. It only works when the flow is quite slow.)

- longer pipe $\to$ slower flow
- higher viscosity $\to$ slower flow
- higher input pressure $\to$ faster flow
- wider pipe $\to$ **hugely faster flow**

Q: What happens to flow rate if I double the tube radius, keeping everything else the same? Let's try it!

Heat

- Let's move on to heat!
- What is heat? Does anyone remember a definition?
- What are the three methods by which thermal energy is transferred into or out of a system?

Heat

- Let's move on to heat!
- What is heat? Does anyone remember a definition?
- What are the three methods by which thermal energy is transferred into or out of a system?
- Which method is this (demo)?
- If you double the temperature of an object, what happens to the rate at which that object radiates thermal energy?
- When making that calculation, in what units do you need to measure temperature?

Radiation

Light from the Sun travels to Earth mostly through empty space. So the thermal energy transfer is clearly not by conduction or by convection. It must be by radiation (Q = heat):

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = e\sigma A T^4$$

where σ is the *Stefan-Boltzmann constant*

$$\sigma = 5.67 \times 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}}$$

e is the *emissivity*, $0 \leq e \leq 1$, A is surface area, and T is temperature (kelvin).

Radiation

The Sun is 150 million kilometers ($R_{es} = 1.5 \times 10^{11}$ m) from Earth, has a surface temperature $T_{\mathrm{sun}} = 5780$ K, and a radius of 696 thousand kilometers ($R_{\mathrm{sun}} = 6.96 \times 10^{8}$ m). Taking $e \approx 1$,

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = e\sigma A T^4 = \sigma \left(4\pi R_{\mathrm{sun}}^2\right) T_{\mathrm{sun}}^4$$

$$= (5.67 \times 10^{-8} \ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}})(4\pi)(6.96 \times 10^{8}\ \mathrm{m})^2(5780\ \mathrm{K})^4$$

$$= 3.85 \times 10^{26}\ \mathrm{W}$$

Let's call that $P_{\mathrm{sun}} = 3.85 \times 10^{26}\ \mathrm{W}$, the power output of the Sun.

btw, what process do you suppose generates all that power?

Intensity at Earth of sun's radiation

Other things being equal, do you expect the Sun's radiation to be more intense when you are very close to the Sun than when you are very far away from the Sun? Or does it not matter how far away you are? (I mean, suppose you go $10\times$ farther away.)

If we know the total power radiated by the Sun, how can we determine the **intensity** of solar radiation (measured in $\mathrm{W/m^2}$, i.e. power per unit area) at some distance r away from the Sun?

Hint: we saw the same rule for sound waves spreading out into open air, where there are no reflecting surfaces.

Intensity at Earth of Sun's radiation

Remember that for sound waves, we saw that the intensity (measured in $\mathrm{W/m^2}$) at a distance r goes like

$$\frac{\mathrm{power}}{4\pi r^2}$$

The same rule works for light (and more generally for radiated power). The intensity of the Sun's radiation reaching Earth is

$$\mathrm{intensity} = \frac{P_{\mathrm{sun}}}{4\pi R_{es}^2} = \frac{3.85\times 10^{26}\ \mathrm{W}}{(4\pi)(1.5\times 10^{11}\ \mathrm{m})^2} = 1360\ \mathrm{W/m^2}$$

This is the so-called "solar constant."

It turns out that not all of this 1360 $\mathrm{W/m^2}$ is absorbed by Earth.
If it were, Earth would look dark when observed from space!

$\sim$ 30% of incoming sunlight is reflected by clouds, water, land, etc.

If Earth absorbs 70% of the 1360 $\mathrm{W/m^2}$ of solar intensity incident at Earth, what is the total solar power absorbed by Earth? (Hint: how large a shadow would you see on a very large screen set up just behind Earth?)

(A) $1360\ \mathrm{W/m^2} \times \pi R_{\mathrm{earth}}^2$

(B) $0.70 \times 1360\ \mathrm{W/m^2} \times \pi R_{\mathrm{earth}}^2$

(C) $1360\ \mathrm{W/m^2} \times 4\pi R_{\mathrm{earth}}^2$

(D) $0.70 \times 1360\ \mathrm{W/m^2} \times 4\pi R_{\mathrm{earth}}^2$

Heat from Sun absorbed by Earth per unit time:

$$\frac{\mathrm{d}Q_{\mathrm{absorbed}}}{\mathrm{d}t} = (70\%)(\pi R_{\mathrm{earth}}^2)(1360\ \mathrm{W/m^2}) = 1.2 \times 10^{17}\ \mathrm{W}$$

That's an enormous amount of power. It is about 9000$\times$ as large as the 13 TW currently consumed by all of human industry, transportation, heating, etc. ($\frac{1}{4}$ of which is used by the USA.)

Earth absorbs 1.2×10^{17} W of energy per unit time from the Sun's rays. Where does that energy go?

(A) The heat from the Sun is stored in the oceans, which (being mostly water!) have a very large heat capacity.

(B) Most sunlight is converted by photosynthesis to grow plants.

(C) Because Earth's temperature is (very nearly) constant over time, we must have $Q_{\mathrm{in}} \approx Q_{\mathrm{out}}$, so Earth must radiate very nearly all of that energy back out into space.

Heat absorbed by Earth per unit time:

$$\frac{\mathrm{d}Q_{\mathrm{absorbed}}}{\mathrm{d}t} = (70\%)(\pi R_{\mathrm{earth}}^2)(1360\ \mathrm{W/m^2}) = 1.2 \times 10^{17}\ \mathrm{W}$$

For Earth to remain at a constant temperature, it must be emitting heat out into space at the same rate (on average) at which it absorbs heat from the Sun.

$$\frac{\mathrm{d}Q_{\mathrm{emitted}}}{\mathrm{d}t} = e\sigma A T^4 = \sigma\ (4\pi R_{\mathrm{earth}}^2)\ (T_{\mathrm{earth}})^4 = 1.2 \times 10^{17}\ \mathrm{W}$$

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$$\Rightarrow \quad T_{\mathrm{earth}} \approx 255\ \mathrm{K}$$

that's $-18^\circ\mathrm{C}$ ($0^\circ\mathrm{F}$). Brrr! That's what you would get if there were no atmosphere to insulate Earth, like a blanket (or a greenhouse). Averaged over seasons & area, $T \approx 288\ \mathrm{K}$ ($+15^\circ\mathrm{C}$, $60^\circ\mathrm{F}$) is the actual temperature.

Suppose I leave my sleek, black iPhone$^{\mathrm{(TM)}}$ out in the mid-day sunlight. What happens?

A Nothing of any consequence happens.

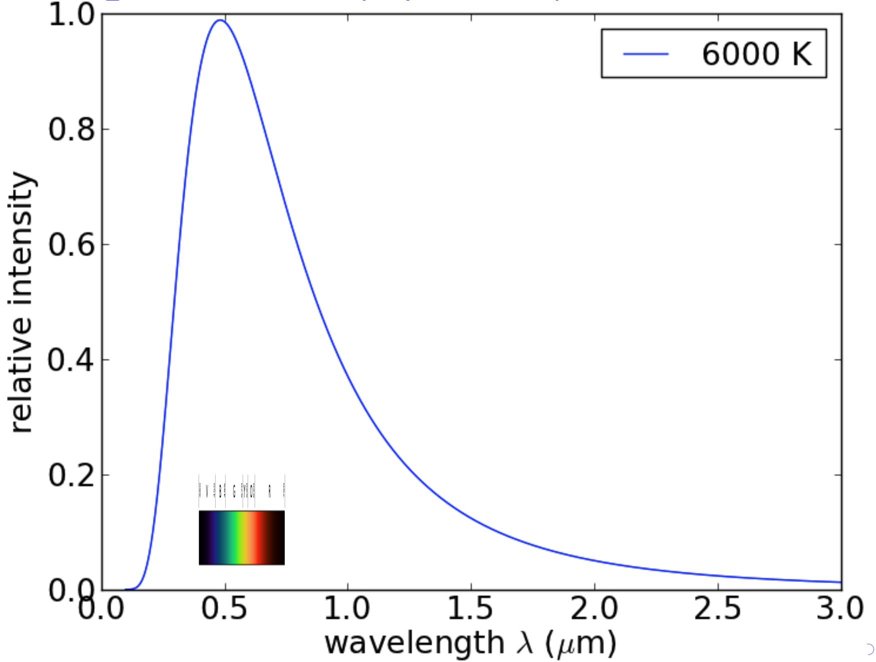
B The iPhone heats up until it reaches $\pi/2$ times the outdoor temperature (in kelvins), then remains at constant T.

C The iPhone continues to heat up forever, until finally its plastic case melts and its microchips revert to amorphous silicon.

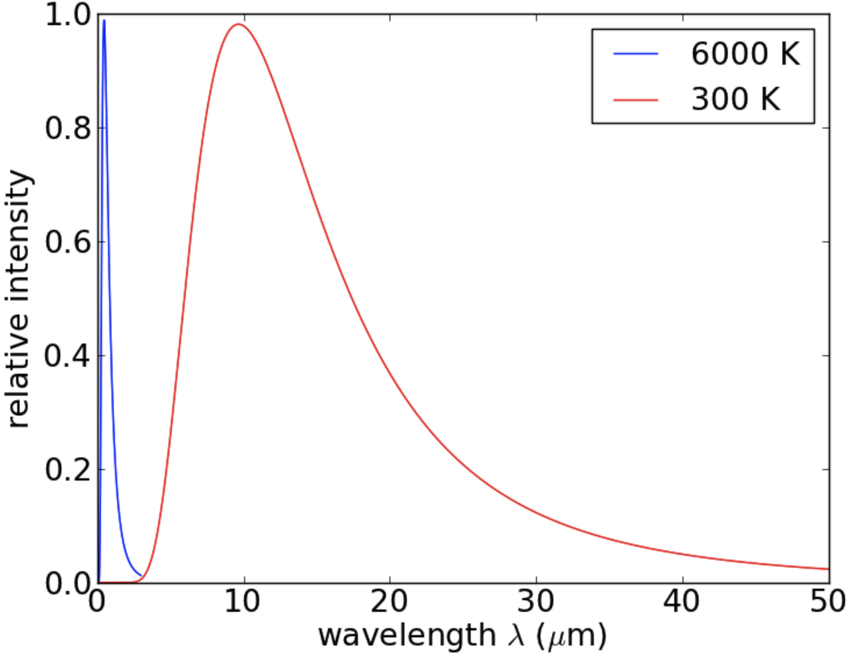
D The iPhone's temperature initially rises. As T increases, the rate $\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t$ of heat transferred out of the iPhone increases (because radiation $\propto T^4$, while conduction $\propto T - T_{\mathrm{ambient}}$, and convection also increases as $T - T_{\mathrm{ambient}}$ increases). Eventually, T stabilizes at the point where $\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t = \mathrm{d}Q_{\mathrm{in}}/\mathrm{d}t$.

(Put black object in the "sun." Try varying the distance.)

Wavelength distribution (“spectrum”) for 6000 K



Wavelength distribution ("spectrum") for 300 K



After my iPhone's temperature stabilizes in the mid-day sunlight, I then cover it up with an upside-down (transparent) glass bowl. Now what happens?

A The temperature stays the same as before.

B The temperature is reduced, because the transparent glass bowl blocks out a lot of the Sun's light.

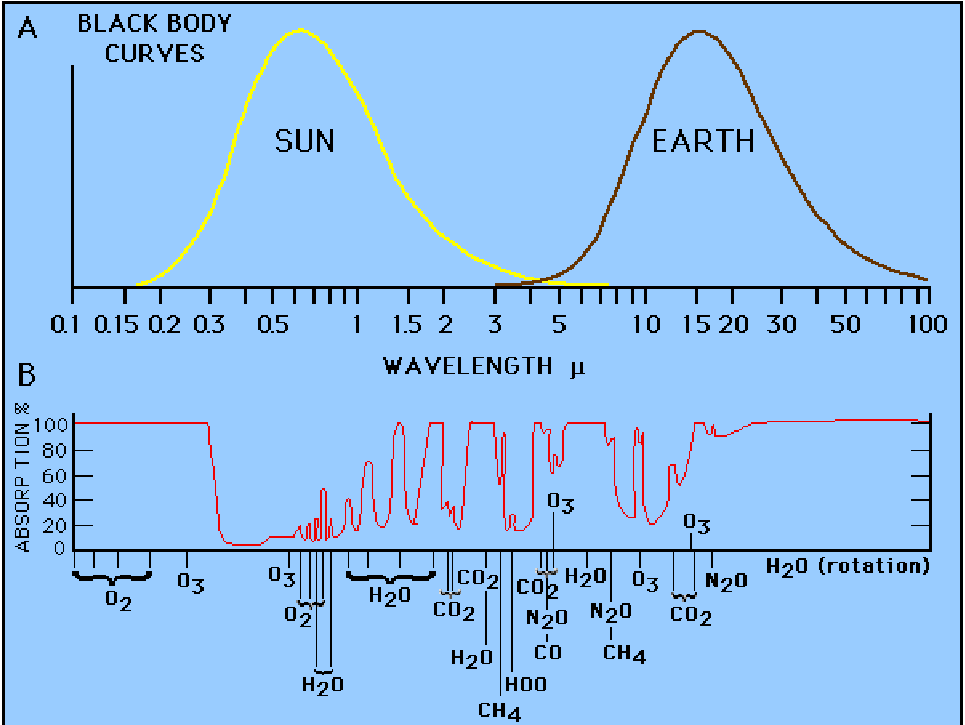
C The temperature increases, because the Sun's light can still come in at about the same rate, but it is harder now for the heat to escape. ($\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t$ is lower for a given T because of the insulating layer of glass.) Eventually, T stabilizes at the new, higher point where $\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t = \mathrm{d}Q_{\mathrm{in}}/\mathrm{d}t$.

(Put glass beaker over black object & thermometer.)

Let's work out what happens if we surround Earth with an insulating layer that (for a given T) cuts $\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t$ in half.

(You know that water vapor is one such insulator: it is cooler in the morning after a clear night, and warmer after a cloudy night. The weather report sometimes mentions "radiational cooling" after a clear night. Water vapor acts something like the glass roof of a greenhouse.)

It turns out that the more jiggly molecules in the atmosphere (e.g. $\mathrm{H_2O}$ vapor, $\mathrm{CO_2}$, $\mathrm{CH_4}$, which have many "degrees of freedom") are transparent to incoming visible light but absorb and re-emit outgoing infrared radiation. Dense clouds can, of course, block both incoming and outgoing radiation.


A
BLACK BODY CURVES
SUN
EARTH
0.1 0.15 0.2 0.3 0.5 1 1.5 2 3 5 10 15 20 30 50 100
WAVELENGTH μ
B
ABSORPTION %
100
80
60
40
20
0
O2
O3
O3
O2
H2O
H2O
CO2
CO2
H2O
CH4
HOO
O3
CO2
N2O
CO
H2O
N2O
CH4
O3
O3
CO2
N2O
H2O (rotation)

The incoming power is still

$$\frac{\mathrm{d}Q_{\mathrm{absorbed}}}{\mathrm{d}t} = (70\%)(\pi R_{\mathrm{earth}}^2)(1360\ \mathrm{W/m^2}) = 1.2 \times 10^{17}\ \mathrm{W}$$

But if the atmosphere absorbs the IR and re-scatters it half upward and half downward, then the power radiated by Earth's surface must double, so that the net power leaving the top of the atmosphere equals the power coming in from the Sun:

$$\frac{\mathrm{d}Q_{\mathrm{emitted}}}{\mathrm{d}t} = e\sigma A T^4 = 2 \times 1.2 \times 10^{17}\ \mathrm{W}$$

$$\Rightarrow \quad T_{\mathrm{earth}} = (2)^{1/4} \times 255\ \mathrm{K} = 303\ \mathrm{K}$$

That's closer to the right answer (which is $15^\circ\mathrm{C} = 288\ \mathrm{K}$), but too hot. If instead of trapping 50%, the atmosphere instead traps 39% of Q_{out}, we get 288 K at the surface.

More effective insulating blanket $\rightarrow$ warmer surface.

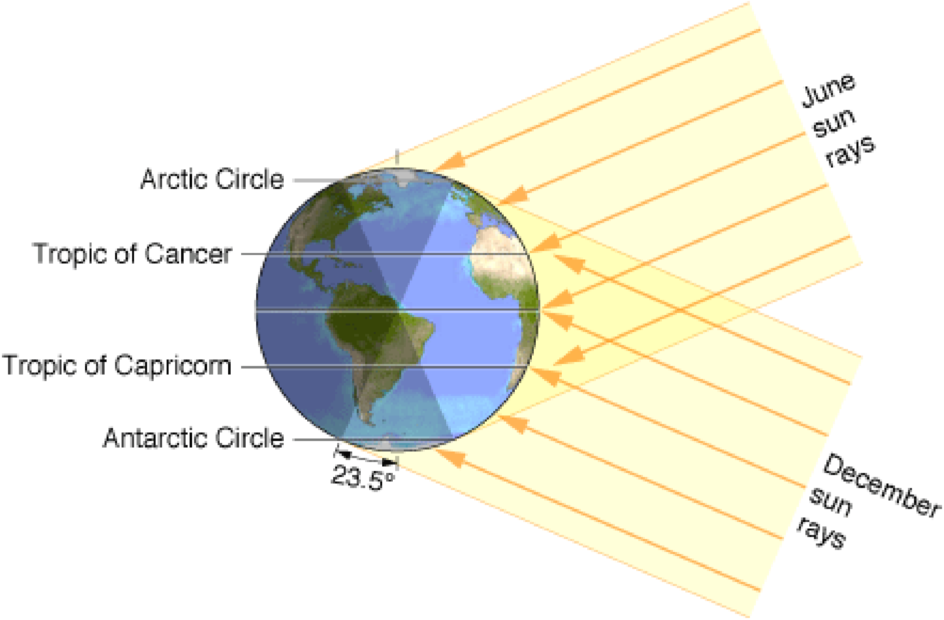
One more thing about the Sun's rays . . .

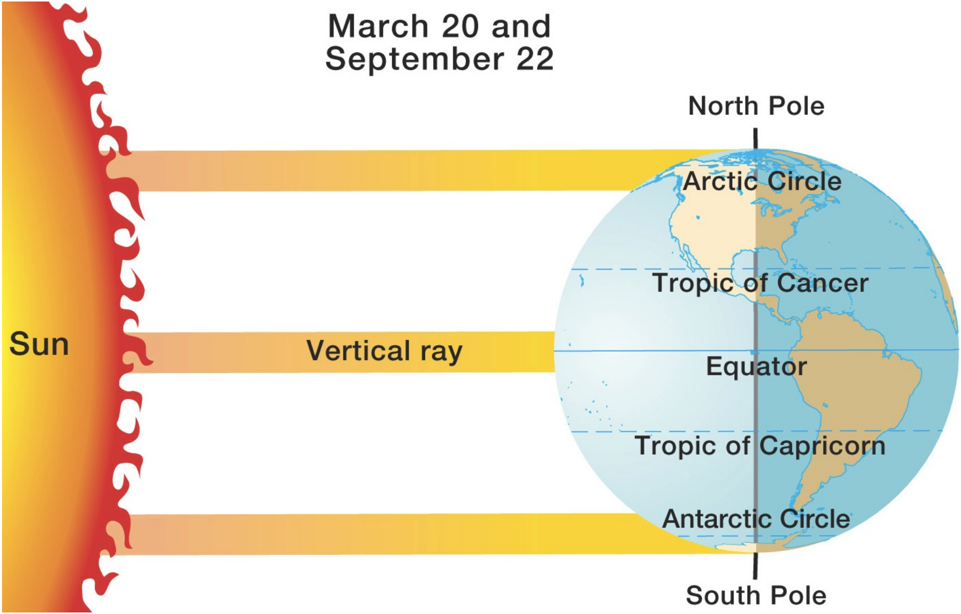
Remember that Sun's intensity scales like

$$\text{intensity} \quad \propto \quad \cos\theta$$

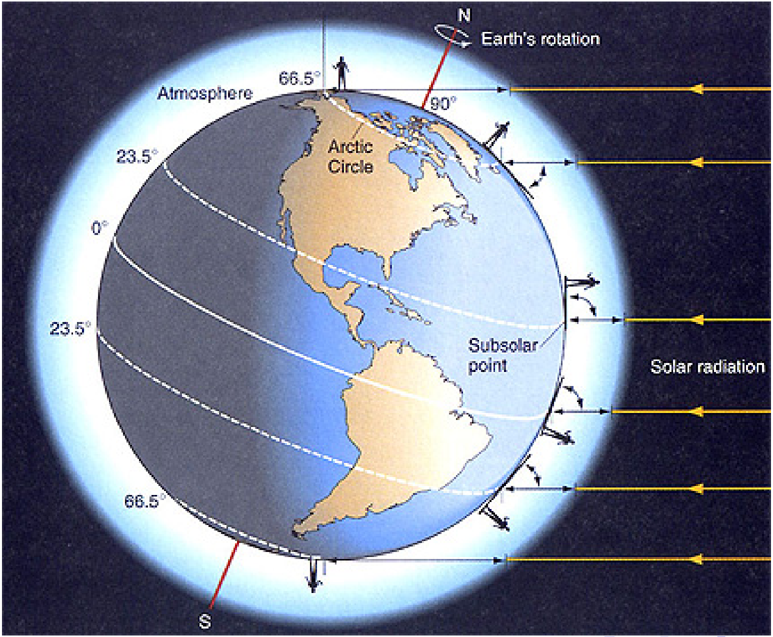
where θ is the angle between the sun's rays and the surface normal.

In your senior studio work, you may see this in Ecotect!


Arctic Circle
Tropic of Cancer
Tropic of Capricorn
Antarctic Circle
23.5°
June sun rays
December sun rays


March 20 and
September 22
North Pole
Arctic Circle
Tropic of Cancer
Sun
Vertical ray
Equator
Tropic of Capricorn
Antarctic Circle
South Pole

What month is this?



About how much solar power is incident on one square meter of flat, level ground near Philadelphia, at noon on March 21? (Note that 70% $\times$ 1350 $\mathrm{W/m^2} \approx$ 1000 W.)

(A) 1000 watts $\times$ $\sin(40^\circ) \approx$ 640 watts

(B) 1000 watts $\times$ $\cos(40^\circ) \approx$ 770 watts

(C) 1000 watts $\times$ $\tan(40^\circ) \approx$ 840 watts

(D) 1000 watts

(E) 1350 watts

About how much solar power is incident on one square meter of a vertical, south-facing window near Philadelphia, at noon on March 21? (Note that $70\% \times 1350 \text{ W/m}^2 \approx 1000 \text{ W}$.)

(A) 1000 watts $\times$ sin(40°) $\approx$ 640 watts

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