Physics 9 — Monday, October 22, 2018

- For today, you read Giancoli ch14 (heat).
- For Wednesday, read Giancoli ch15 (thermodynamics).
- HW6 due Friday.
- Are there a few people interested in coming along with me to Acentech's "3D Listening" demo next Monday, 4pm, on 17th Street?
- I found a way to run both Odeon and CATT-Acoustic on MacOS without a virtual machine! Stay tuned.

Poiseuille's equation

Viscosity also impedes the flow of a fluid through a conduit: imagine trying to flow molasses through a narrow tube.

For **laminar flow**, flow rate Q ($\frac{\mathrm{volume}}{\mathrm{time}}$, i.e. $\frac{\mathrm{m}^3}{\mathrm{s}}$) is given by

$$Q = \frac{\pi R^4 \, (P_1 - P_2)}{8 \eta L}$$

(Derivation at end of Eric Mazur's fluids chapter (18), if you're curious. Once the flow is fast enough to be turbulent, this equation no longer works. It only works when the flow is quite slow.)

- longer pipe → slower flow
- higher viscosity → slower flow
- higher input pressure → faster flow
- wider pipe → **hugely faster flow**

Q: What happens to flow rate if I vary (×1.47) the tube radius, keeping everything else the same? Let's try it!

Heat

- Let's move on to heat!
- What is heat? Does anyone remember a definition?
- What are the three methods by which thermal energy is transferred into or out of a system?

Heat

- Let's move on to heat!
- What is heat? Does anyone remember a definition?
- What are the three methods by which thermal energy is transferred into or out of a system?
- Which method is this (demo)?
- If you double the temperature of an object, what happens to the rate at which that object radiates thermal energy?
- When making that calculation, in what units do you need to measure temperature?

Radiation

Light from the Sun travels to Earth mostly through empty space. So the thermal energy transfer is clearly not by conduction or by convection. It must be by radiation (Q = heat):

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = e\sigma A T^4$$

where σ is the *Stefan-Boltzmann constant*

$$\sigma = 5.67 \times 10^{-8} \ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}}$$

e is the *emissivity*, $0 \leq e \leq 1$, A is surface area, and T is temperature (kelvin).

Radiation

The Sun is 150 million kilometers ($R_{es} = 1.5 \times 10^{11}\ \mathrm{m}$) from Earth, has a surface temperature $T_{\mathrm{sun}} = 5780\ \mathrm{K}$, and a radius of 696 thousand kilometers ($R_{\mathrm{sun}} = 6.96 \times 10^{8}\ \mathrm{m}$). Taking $e \approx 1$,

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = e\sigma A T^4 = \sigma\ (4\pi R_{\mathrm{sun}}^2)\ T_{\mathrm{sun}}^4$$

$$= (5.67 \times 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}})(4\pi)(6.96 \times 10^{8}\ \mathrm{m})^2(5780\ \mathrm{K})^4$$

$$= 3.85 \times 10^{26}\ \mathrm{W}$$

Let's call that $P_{\mathrm{sun}} = 3.85 \times 10^{26}\ \mathrm{W}$, the power output of the Sun.

btw, what process do you suppose generates all that power?

Intensity at Earth of sun's radiation

Other things being equal, do you expect the Sun's radiation to be more intense when you are very close to the Sun than when you are very far away from the Sun? Or does it not matter how far away you are? (I mean, suppose you go $10\times$ farther away.)

If we know the total power radiated by the Sun, how can we determine the **intensity** of solar radiation (measured in W/m^2, i.e. power per unit area) at some distance r away from the Sun?

Hint: we saw the same rule for sound waves spreading out into open air, where there are no reflecting surfaces.

Intensity at Earth of Sun's radiation

Remember that for sound waves, we saw that the intensity (measured in $\mathrm{W/m^2}$) at a distance r goes like

$$\frac{\mathrm{power}}{4\pi r^2}$$

The same rule works for light (and more generally for radiated power). The intensity of the Sun's radiation reaching Earth is

$$\mathrm{intensity} = \frac{P_{\mathrm{sun}}}{4\pi R_{es}^2} = \frac{3.85 \times 10^{26}\ \mathrm{W}}{(4\pi)(1.5 \times 10^{11}\ \mathrm{m})^2} = 1360\ \mathrm{W/m^2}$$

This is the so-called "solar constant."

It turns out that not all of this 1360 $\mathrm{W/m^2}$ is absorbed by Earth.
If it were, Earth would look dark when observed from space!

$\sim$ 30% of incoming sunlight is reflected by clouds, water, land, etc.

If Earth absorbs 70% of the 1360 $\mathrm{W/m^2}$ of solar intensity incident at Earth, what is the total solar power absorbed by Earth? (Hint: how large a shadow would you see on a very large screen set up just behind Earth?)

(A) $1360\ \mathrm{W/m^2} \times \pi R_{\mathrm{earth}}^2$

(B) $0.70 \times 1360\ \mathrm{W/m^2} \times \pi R_{\mathrm{earth}}^2$

(C) $1360\ \mathrm{W/m^2} \times 4\pi R_{\mathrm{earth}}^2$

(D) $0.70 \times 1360\ \mathrm{W/m^2} \times 4\pi R_{\mathrm{earth}}^2$

Heat from Sun absorbed by Earth per unit time:

$$\frac{\mathrm{d}Q_{\mathrm{absorbed}}}{\mathrm{d}t} = (70\%)(\pi R_{\mathrm{earth}}^2)(1360\ \mathrm{W/m^2}) = 1.2 \times 10^{17}\ \mathrm{W}$$

That's an enormous amount of power. It is about 7000× as large as the 18 TW currently consumed by all of human industry, transportation, heating, etc. ($\frac{1}{5}$ of which is used by the USA.)

Earth absorbs 1.2×10^{17} W of energy per unit time from the Sun's rays. Where does that energy go?

(A) The heat from the Sun is stored in the oceans, which (being mostly water!) have a very large heat capacity.

(B) Most sunlight is converted by photosynthesis to grow plants.

(C) Because Earth's temperature is (very nearly) constant over time, we must have $Q_{\mathrm{in}} \approx Q_{\mathrm{out}}$, so Earth must radiate very nearly all of that energy back out into space.

Heat absorbed by Earth per unit time:

$$\frac{\mathrm{d}Q_{\mathrm{absorbed}}}{\mathrm{d}t} = (70\%)(\pi R_{\mathrm{earth}}^2)(1360\ \mathrm{W/m^2}) = 1.2 \times 10^{17}\ \mathrm{W}$$

For Earth to remain at a constant temperature, it must be emitting heat out into space at the same rate (on average) at which it absorbs heat from the Sun.

$$\frac{\mathrm{d}Q_{\mathrm{emitted}}}{\mathrm{d}t} = e\sigma A T^4 = \sigma\ (4\pi R_{\mathrm{earth}}^2)\ (T_{\mathrm{earth}})^4 = 1.2 \times 10^{17}\ \mathrm{W}$$

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$$\Rightarrow \quad T_{\mathrm{earth}} \approx 255\ \mathrm{K}$$

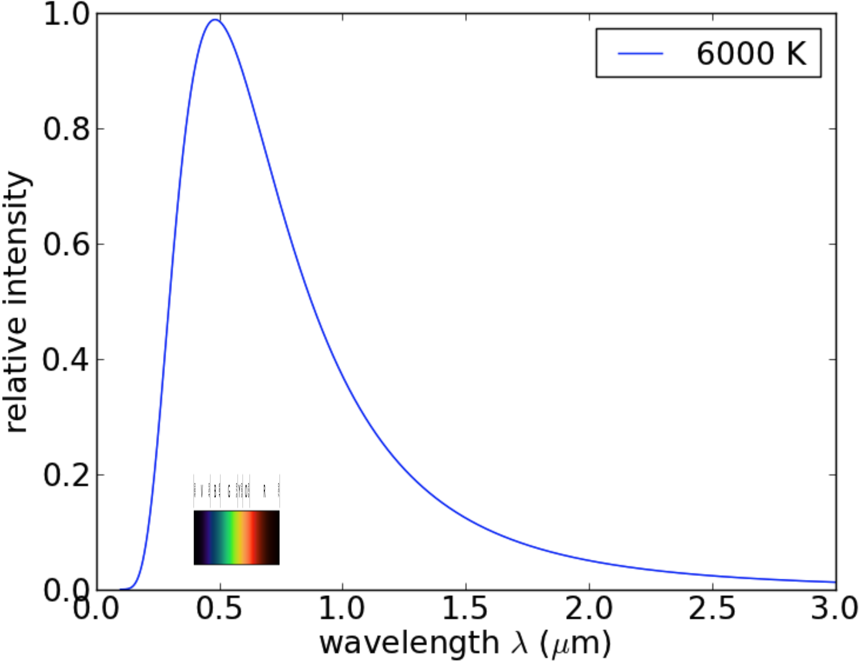
that's $-18^\circ\mathrm{C}$ ($0^\circ\mathrm{F}$). Brrr! That's what you would get if there were no atmosphere to insulate Earth, like a blanket (or a greenhouse). Averaged over seasons & area, $T \approx 288\ \mathrm{K}$ ($+15^\circ\mathrm{C}$, $60^\circ\mathrm{F}$) is the actual temperature.

Suppose I leave my sleek, black iPhone$^{(\mathrm{TM})}$ out in the mid-day sunlight. What happens?

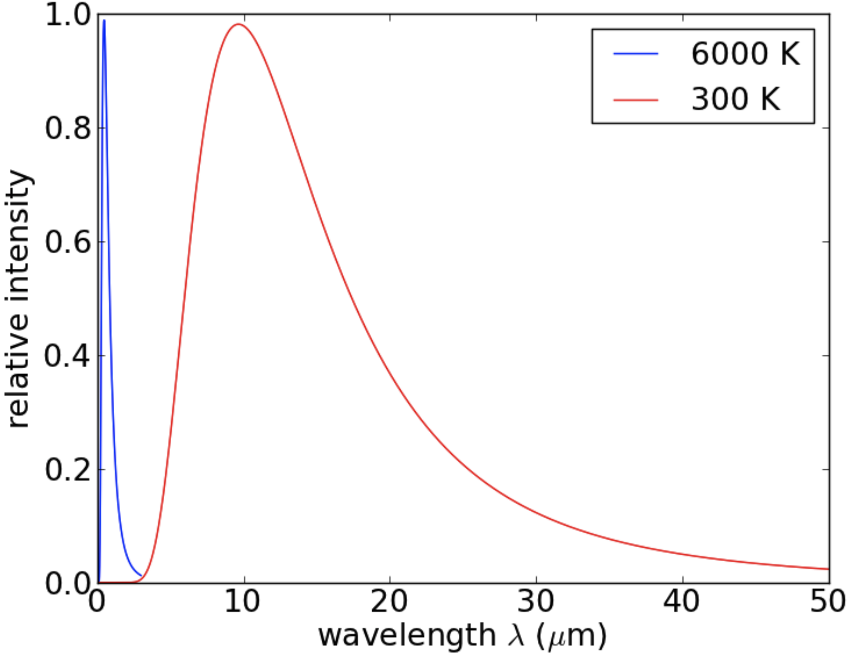
- A Nothing of any consequence happens.
- B The iPhone heats up until it reaches $\pi/2$ times the outdoor temperature (in kelvins), then remains at constant T.
- C The iPhone continues to heat up forever, until finally its plastic case melts and its microchips revert to amorphous silicon.
- D The iPhone's temperature initially rises. As T increases, the rate $\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t$ of heat transferred out of the iPhone increases (because radiation $\propto T^4$, while conduction $\propto T - T_{\mathrm{ambient}}$, and convection also increases as $T - T_{\mathrm{ambient}}$ increases). Eventually, T stabilizes at the point where $\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t = \mathrm{d}Q_{\mathrm{in}}/\mathrm{d}t$.

(Put black object in the "sun." Try varying the distance.)

Wavelength distribution (“spectrum”) for 6000 K



Wavelength distribution (“spectrum”) for 300 K



After my iPhone's temperature stabilizes in the mid-day sunlight, I then cover it up with an upside-down (transparent) glass bowl. Now what happens?

- A The temperature stays the same as before.
- B The temperature is reduced, because the transparent glass bowl blocks out a lot of the Sun's light.
- C The temperature increases, because the Sun's light can still come in at about the same rate, but it is harder now for the heat to escape. ($\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t$ is lower for a given T because of the insulating layer of glass.) Eventually, T stabilizes at the new, higher point where $\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t = \mathrm{d}Q_{\mathrm{in}}/\mathrm{d}t$.

(Put glass beaker over black object & thermometer.)

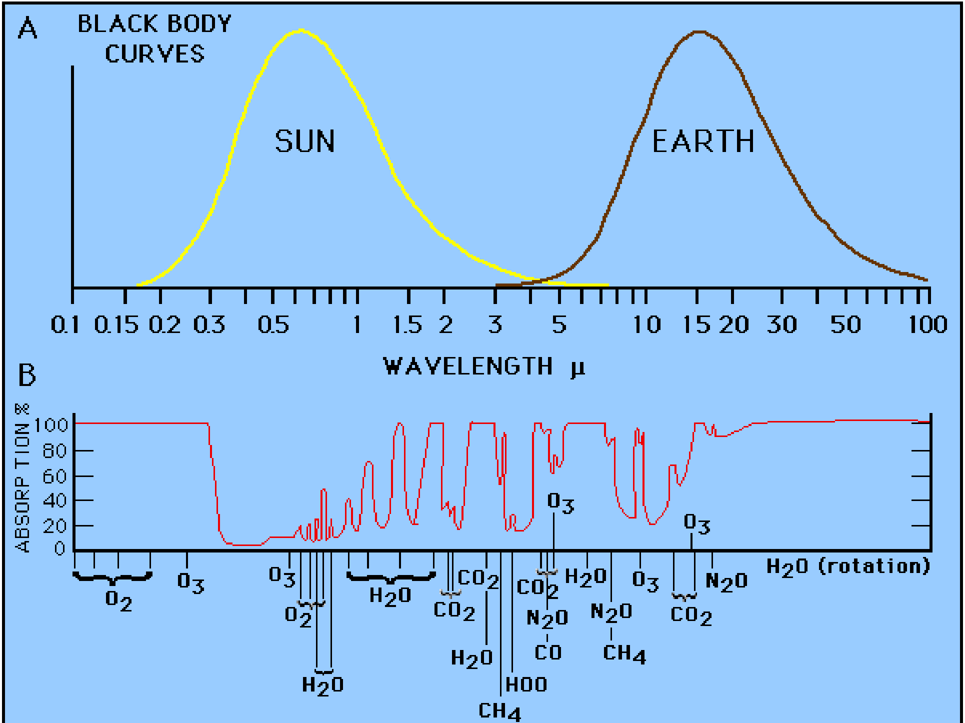
Let's work out what happens if we surround Earth with an insulating layer that (for a given T) cuts $\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t$ in half.

(You know that water vapor is one such insulator: it is cooler in the morning after a clear night, and warmer after a cloudy night. The weather report sometimes mentions "radiational cooling" after a clear night. Water vapor acts something like the glass roof of a greenhouse.)

It turns out that the more jiggly molecules in the atmosphere (e.g. $\mathrm{H_2O}$ vapor, $\mathrm{CO_2}$, $\mathrm{CH_4}$, which have many "degrees of freedom") are transparent to incoming visible light but absorb and re-emit outgoing infrared radiation. Dense clouds can, of course, block both incoming and outgoing radiation.

3-minute video (made here at Penn Physics!)

`https://www.youtube.com/watch?v=0eI9zxZoipA`


A
BLACK BODY CURVES
SUN
EARTH
0.1 0.15 0.2 0.3 0.5 1 1.5 2 3 5 10 15 20 30 50 100
WAVELENGTH μ
B
ABSORPTION %
100
80
60
40
20
0
O_2
O_3
O_3
O_2
H_2O
H_2O
CO_2
CO_2
H_2O
CH_4
HDO
O_3
CO_2
N_2O
CO
H_2O
N_2O
CH_4
O_3
O_3
CO_2
N_2O
H_2O (rotation)

The incoming power is still

$$\frac{\mathrm{d}Q_{\mathrm{absorbed}}}{\mathrm{d}t} = (70\%)(\pi R_{\mathrm{earth}}^2)(1360\ \mathrm{W/m^2}) = 1.2 \times 10^{17}\ \mathrm{W}$$

But if the atmosphere absorbs the IR and re-scatters it half upward and half downward, then the power radiated by Earth's surface must double, so that the net power leaving the top of the atmosphere equals the power coming in from the Sun:

$$\frac{\mathrm{d}Q_{\mathrm{emitted}}}{\mathrm{d}t} = e\sigma A T^4 = 2 \times 1.2 \times 10^{17}\ \mathrm{W}$$

$$\Rightarrow \quad T_{\mathrm{earth}} = (2)^{1/4} \times 255\ \mathrm{K} = 303\ \mathrm{K}$$

That's closer to the right answer (which is $15^\circ\mathrm{C} = 288\ \mathrm{K}$), but too hot. If instead of trapping 50%, the atmosphere instead traps 39% of Q_{out}, we get 288 K at the surface.

More effective insulating blanket → warmer surface.

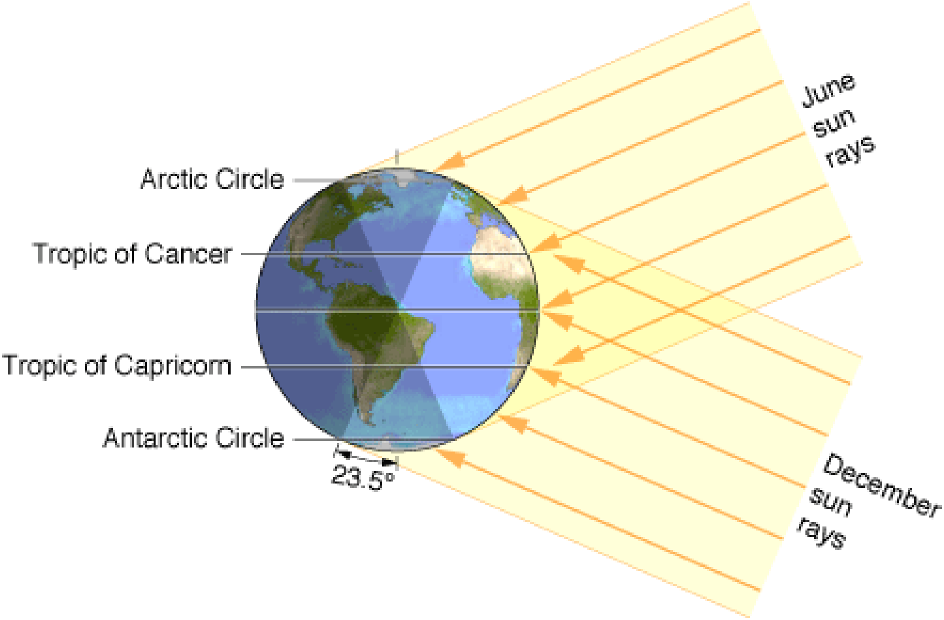
One more thing about the Sun's rays . . .

Remember that Sun's intensity scales like

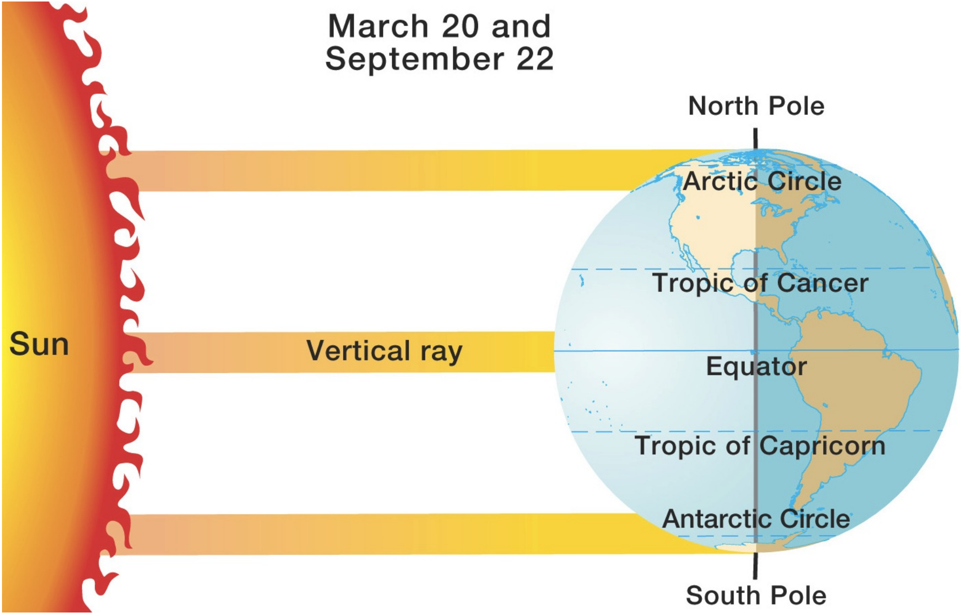
$$\text{intensity} \quad \propto \quad \cos\theta$$

where θ is the angle between the sun's rays and the surface normal.

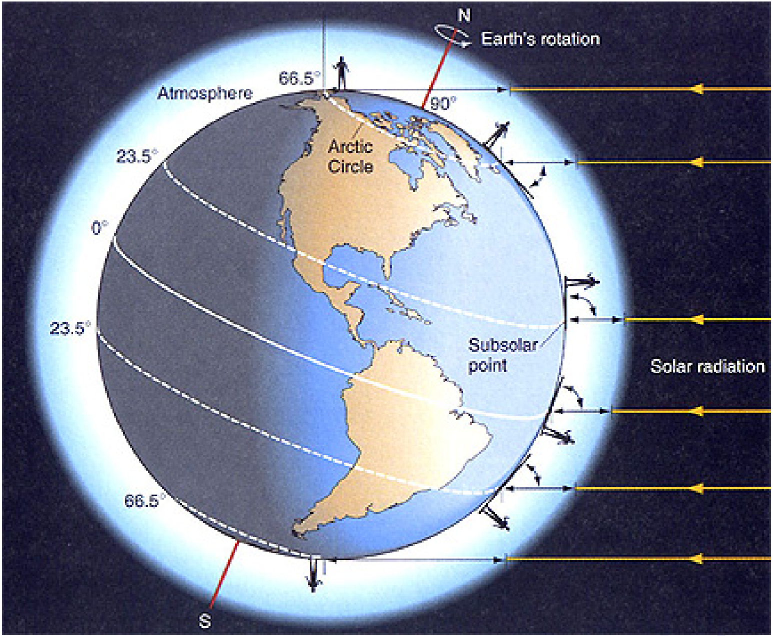
In your senior studio work, you may see this in various solar-modeling software (several years ago, it was Autodesk Ecotect).


Arctic Circle
Tropic of Cancer
Tropic of Capricorn
Antarctic Circle
23.5°
June sun rays
December sun rays

March 20 and September 22



What month is this?



About how much solar power is incident on one square meter of flat, level ground near Philadelphia, at noon on March 21? (Note that 70% $\times$ 1350 $\mathrm{W/m^2} \approx$ 1000 W.)

(A) 1000 watts $\times$ sin(40°) $\approx$ 640 watts

(B) 1000 watts $\times$ cos(40°) $\approx$ 770 watts

(C) 1000 watts $\times$ tan(40°) $\approx$ 840 watts

(D) 1000 watts

(E) 1350 watts

About how much solar power is incident on one square meter of a vertical, south-facing window near Philadelphia, at noon on March 21? (Note that 70% $\times$ 1350 $\mathrm{W/m^2} \approx$ 1000 W.)

(A) 1000 watts $\times$ sin(40°) $\approx$ 640 watts

(B) 1000 watts $\times$ cos(40°) $\approx$ 770 watts

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What is the purpose of an “expansion joint” on a bridge?

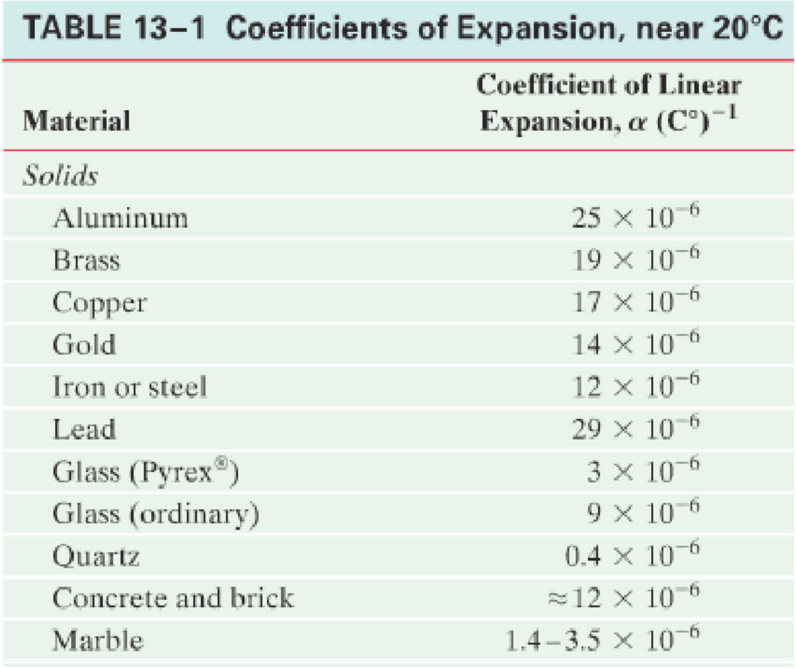


TABLE 13–1 Coefficients of Expansion, near 20°C

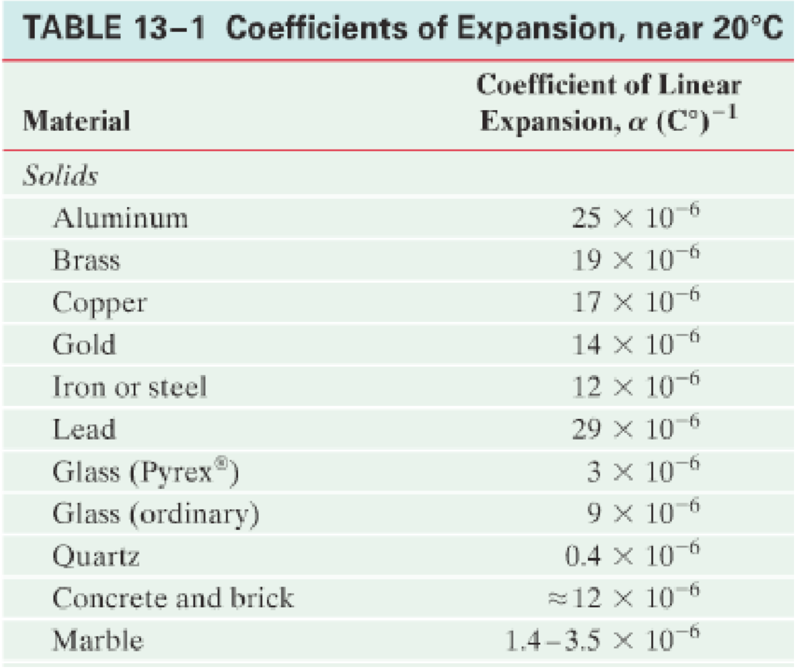
Material	Coefficient of Linear Expansion, α $(\mathrm{C}^\circ)^{-1}$
Solids	
Aluminum	25×10^{-6}
Brass	19×10^{-6}
Copper	17×10^{-6}
Gold	14×10^{-6}
Iron or steel	12×10^{-6}
Lead	29×10^{-6}
Glass (Pyrex®)	3×10^{-6}
Glass (ordinary)	9×10^{-6}
Quartz	0.4×10^{-6}
Concrete and brick	$\approx 12 \times 10^{-6}$
Marble	$1.4 - 3.5 \times 10^{-6}$

A bridge deck is made of concrete slabs that are 10.0 meters long at $20^\circ\mathrm{C}$. How wide should the expansion cracks between the slabs be (at $20^\circ\mathrm{C}$) to prevent buckling if the range of temperatures is $-20^\circ\mathrm{C}$ to $+40^\circ\mathrm{C}$? Use $\alpha = 12 \times 10^{-6}/^\circ\mathrm{C}$ for the coefficient of linear expansion for concrete.

(a) 2.4 mm (b) 4.8 mm (c) 7.2 mm

A bridge deck is made of concrete slabs that are 10.0 meters long at $20^\circ\mathrm{C}$. If the expansion cracks between the slabs are 2.4 mm wide at $20^\circ\mathrm{C}$, how wide will they be at $-20^\circ\mathrm{C}$? Use $\alpha = 12 \times 10^{-6}/^\circ\mathrm{C}$ for the coefficient of linear expansion for concrete.

(a) 1.2 mm (b) 2.4 mm (c) 4.8 mm (d) 7.2 mm



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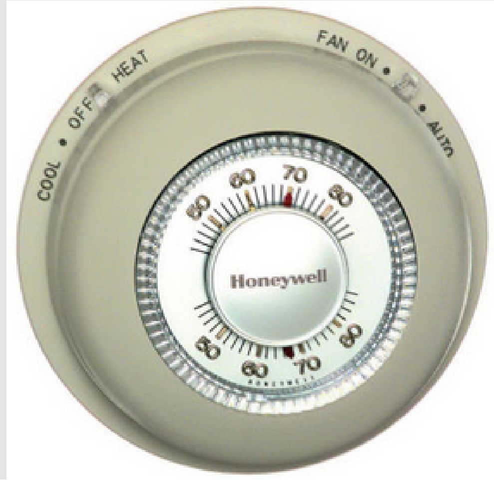
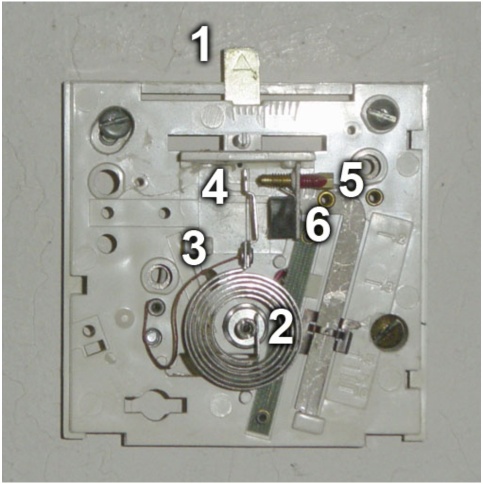
Glass (ordinary)	9×10^{-6}
Quartz	0.4×10^{-6}
Concrete and brick	$\approx 12 \times 10^{-6}$
Marble	$1.4\text{–}3.5 \times 10^{-6}$

Do you remember Richard Muller's story about Pyrex vs. ordinary glass?

Have you ever seen an old thermostat that looks like this?





How do you suppose this works?

Related question: suppose I take a long strip of steel and a long strip of aluminum, and I solder them together, like a submarine sandwich, with the steel on top. What happens when I heat it up?

Thermal expansion

It turns out that a simple coefficient of linear (or volume) expansion works pretty well for most materials. But these things are usually tabulated as a function of temperature.

$$\Delta L = \alpha L_0 \Delta T \quad \rightarrow \quad \frac{\mathrm{d}L}{\mathrm{d}T} = \alpha L_0$$

$$\Delta V = \beta V_0 \quad \rightarrow \quad \frac{\mathrm{d}V}{\mathrm{d}T} = \beta V_0$$

If $V = L^3$ and material is isotropic, then

$$\frac{\mathrm{d}V}{\mathrm{d}T} = \frac{\mathrm{d}}{\mathrm{d}T}(L^3) = 3L_0^2(\alpha L_0) = 3\alpha L_0^3 = (3\alpha)V_0$$

so $\beta \approx 3\alpha$ for most materials.

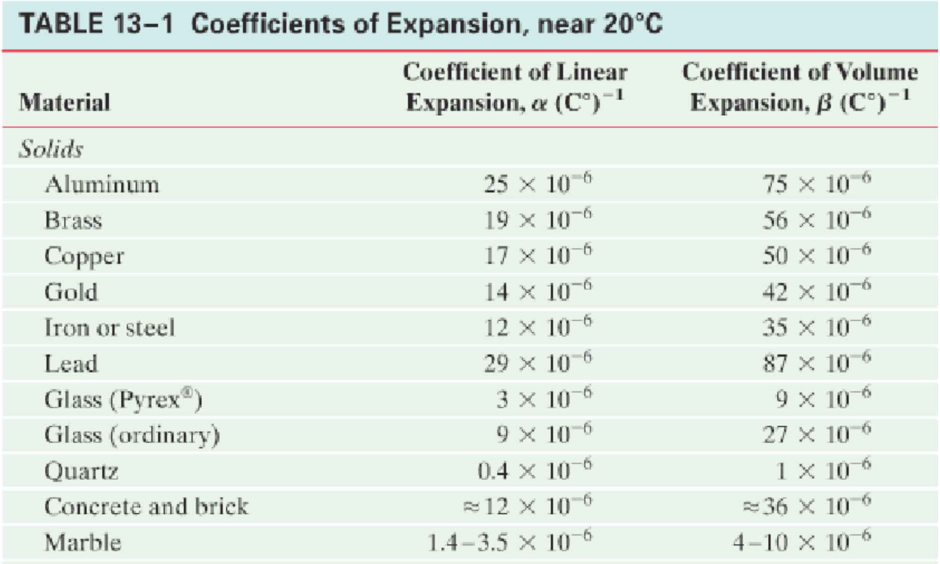


TABLE 13–1 Coefficients of Expansion, near 20°C

Material	Coefficient of Linear Expansion, α $(\mathrm{C}^\circ)^{-1}$	Coefficient of Volume Expansion, β $(\mathrm{C}^\circ)^{-1}$
Solids		
Aluminum	25×10^{-6}	75×10^{-6}
Brass	19×10^{-6}	56×10^{-6}
Copper	17×10^{-6}	50×10^{-6}
Gold	14×10^{-6}	42×10^{-6}
Iron or steel	12×10^{-6}	35×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Glass (Pyrex®)	3×10^{-6}	9×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Quartz	0.4×10^{-6}	1×10^{-6}
Concrete and brick	$\approx 12 \times 10^{-6}$	$\approx 36 \times 10^{-6}$
Marble	$1.4\text{–}3.5 \times 10^{-6}$	$4\text{–}10 \times 10^{-6}$

Thermal stress

Remember from the the fall term that ($E =$ Young's modulus)

$$\frac{\Delta L}{L_0} = \frac{1}{E}\ \frac{F}{A}$$

If I heat something up so that it tries to expand thermally:

$$\frac{\Delta L}{L_0} = \alpha \Delta T$$

but I don't let it expand (because it is held in place rigidly), then it is effectively being squished by a factor $\alpha \Delta T$

So holidng it at its original length when it heats up induces a stress (force per unit area):

$$\frac{F}{A} = E\alpha \Delta T$$

which the material may or may not be able to tolerate (depending on its compressive strength).

A bridge deck is made of concrete slabs that are 10.0 meters long at $20^\circ\mathrm{C}$. They are laid end-to-end with no expansion cracks!! So the length of each slab cannot increase. (Oops!)

If the temperature rises to $30^\circ\mathrm{C}$, how large is the compressive stress that develops in the concrete in order to maintain its original length?

Use $\alpha = 12 \times 10^{-6}/^\circ\mathrm{C}$ for the coefficient of linear expansion for concrete. Use $E = 20 \times 10^{9}\ \mathrm{N/m^2}$ for the Young's modulus of concrete.

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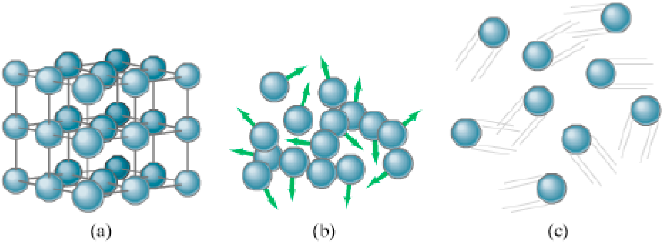
Use $\alpha = 12 \times 10^{-6}/^\circ\mathrm{C}$ for the coefficient of linear expansion for concrete. Use $E = 20 \times 10^9\ \mathrm{N/m^2}$ for the Young's modulus of concrete.

(I get stress $= 2.4 \times 10^6\ \mathrm{N/m^2}\ =\ \alpha E \Delta T$. This is about ×8 below the compressive strength of concrete, so the slabs won't crush, but they may buckle, as a slender column would.)

Giancoli's illustration of solid / liquid / gas



(a) (b) (c)

FIGURE 13–2 Atomic arrangements in (a) a crystalline solid, (b) a liquid, and (c) a gas.

Atomic mass unit (u):

$$1\ \mathrm{u} \quad = \quad 1\ \frac{\mathrm{gram}}{\mathrm{mole}} \quad = \quad 1.66 \times 10^{-27}\ \mathrm{kg}$$

This is very close (within 1%) to the mass of a proton:

$$m_{\mathrm{proton}} = 1.67 \times 10^{-27}\ \mathrm{kg}$$

A dozen eggs is 12 eggs. A mole of protons is $N_A = 6.022 \times 10^{23}$ protons.

A $^{12}\mathrm{C}$ nucleus contains 6 protons + 6 neutrons. So a carbon atom has a mass of 12 u, or 12 g/mol.

A mole of protons has a mass (within 1%) of 1.0 gram. A mole of $^{12}\mathrm{C}$ atoms has a mass (by definition of mole) of exactly 12 grams.

An oxygen nucleus contains

$$8 \text{ protons} + 8 \text{ neutrons} = 16 \text{ nucleons}$$

A hydrogen nucleus contains just 1 proton (and no neutrons). What is the mass of a mole of water (H_2O) molecules?

(A) 0.016 kg
(B) 0.017 kg
(C) 0.018 kg
(D) 0.020 kg
(E) 0.034 kg

Ideal gas law

Anybody remember this from high school chemistry?

$$PV = nRT$$

$$R = 8.315 \ \frac{\mathrm{J}}{\mathrm{mol} \cdot \mathrm{K}}$$

$$R = 0.0821 \ \frac{\mathrm{L} \cdot \mathrm{atm}}{\mathrm{mol} \cdot \mathrm{K}}$$

Notice that temperature is measured in K (kelvin). To get temperature in K, take temperature in °C and add 273.15.

By the way, how does a temperature difference $\Delta T = 1$ K compare with a temperature difference $\Delta T = 1^\circ$C?

(A) A temperature difference of 1 kelvin is bigger than a temperature difference of 1 degree Celsius.

(B) A temperature difference of 1 kelvin is smaller than a temperature difference of 1 degree Celsius.

(C) A temperature difference of 1 kelvin is the same as a temperature difference of 1 degree Celsius.

By the way, how does a temperature difference $\Delta T = 1^\circ\mathrm{F}$ compare with a temperature difference $\Delta T = 1^\circ\mathrm{C}$?

(A) A temperature difference of 1 degree Farenheit is $1.8\times$ as large as temperature difference of 1 degree Celsius.

(B) A temperature difference of 1 degree Celsius is $1.8\times$ as large as temperature difference of 1 degree Farenheit.

(C) A temperature difference of 1 degree Farenheit is the same as a temperature difference of 1 degree Celsius.

What is the final pressure P_f of a sealed bottle of air after you raise its temperature from 27°C to 57°C?

(A) $P_f = 0.47\ P_i$

(B) $P_f = 0.9\ P_i$

(C) $P_f = 1.1\ P_i$

(D) $P_f = 2.1\ P_i$

(E) not enough information to decide

A cylinder initially contains one liter of air at atmospheric pressure. I then compress the gas *isothermally* (i.e. at constant temperature) to half its initial volume. What is the final pressure of the gas (in atmospheres)?

(A) $P_f = 0.50$ atm

(B) $P_f = 0.71$ atm

(C) $P_f = 1.00$ atm

(D) $P_f = 1.41$ atm

(E) $P_f = 2.00$ atm

(F) not enough information to decide

The ideal gas law you learned in high school chemistry was:

$$PV = nRT$$

$$R = 8.315\ \frac{\mathrm{J}}{\mathrm{mol}\cdot\mathrm{K}}$$

$$R = 0.0821\ \frac{\mathrm{L}\cdot\mathrm{atm}}{\mathrm{mol}\cdot\mathrm{K}}$$

The analogous ideal gas law in terms of number of molecules (N) rather than number of moles (n) is:

$$PV = Nk_B T$$

where k_B is *Boltzmann's constant*

$$k_B = 1.38 \times 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}$$

Q: what do you get when you divide 8.315 $\frac{\mathrm{J}}{\mathrm{mol}\cdot\mathrm{K}}$ by $6.022 \times 10^{23}/\mathrm{mol}$?

Boltzmann's constant

$$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$$

$\frac{1}{2}k_B T$ is the average thermal energy per *degree of freedom.* A single atom of monotomic gas can move from place to place in three dimensions, but can't rotate or vibrate, so it has 3 d.o.f.

$$\frac{1}{2}mv_{\mathrm{rms}}^2 = \frac{3}{2}k_B T$$

The average energy of an atom or molecule is directly proportional to temperature. v_{rms} means the "root mean squared" speed.

$$v_{\mathrm{rms}} = \sqrt{\frac{3k_B T}{m}}$$

for an ideal gas. The lighter molecules tend to move faster!

Boltzmann's constant

$$v_{\mathrm{rms}} = \sqrt{\frac{3k_B T}{m}}$$

for an ideal gas. The lighter molecules tend to move faster! For helium gas (4 g/mol) at room temperature,

$$v_{\mathrm{rms}} \approx 1360 \mathrm{~m/s}$$

For nitrogen gas ($\mathrm{N_2}$, 28 g/mol) at 298 K,

$$v_{\mathrm{rms}} \approx 520 \mathrm{~m/s}$$

For gasoline vapor ($\mathrm{C_8H_{18}}$, 114 g/mol) at 298 K,

$$v_{\mathrm{rms}} \approx 260 \mathrm{~m/s}$$

Richard Muller pointed out that this is why the helium escaped from Earth's atmosphere: some small fraction of the helium atoms move fast enough (11200 m/s) to escape Earth's gravity.

Nitrogen (N_2) makes up about 78% of the air we breathe, while oxygen (O_2) accounts for approximately 21%. On average, are the nitrogen or the oxygen molecules moving faster? (The mass of an N_2 molecule is 28 amu, while the mass of an O_2 molecule is 32 amu.)

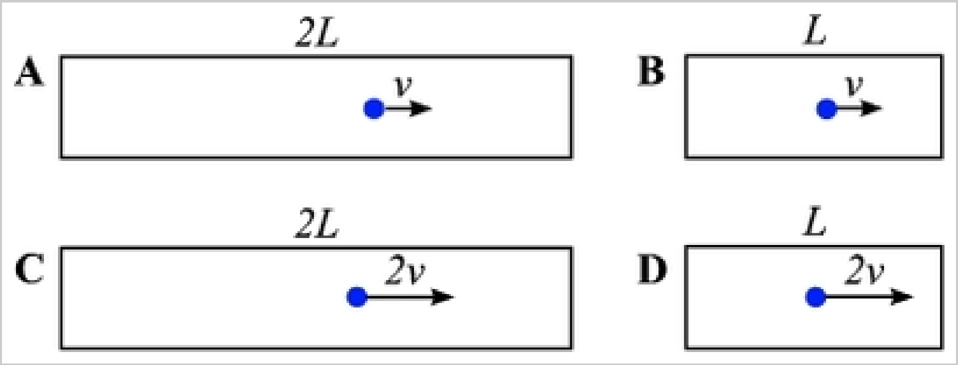
(A) The rms speed of the N_2 molecules is $\frac{32}{28}$ as large as the rms speed of the O_2 molecules, so the N_2 molecules move slightly faster, on average.

(B) The rms speed of the N_2 molecules is $\sqrt{\frac{32}{28}}$ as large as the rms speed of the O_2 molecules, so the N_2 molecules move slightly faster, on average.

(C) The rms speed of the O_2 molecules is $\frac{32}{28}$ as large as the rms speed of the N_2 molecules, so the O_2 molecules move slightly faster, on average.

(D) The rms speed of the O_2 molecules is $\sqrt{\frac{32}{28}}$ as large as the rms speed of the N_2 molecules, so the O_2 molecules move slightly faster, on average.

A gas atom of mass m moves in a straight line with speed v or $2v$ in a box with length L or $2L$, as shown below. The atom collides elastically with the right wall, travels to the left wall, collides elastically, returns to the right wall and repeats this process. In which case does the atom exert the largest average force on the walls?



Bigger thermal energy → higher rms speed → larger force per unit area (pressure)

Smaller volume → particles collide more often with walls → larger force per unit area (pressure)

More particles in container → collisions with walls occur more often → larger force per unit area (pressure)

$$P = \frac{N k_B T}{V}$$

$$\frac{1}{2} m v_{\mathrm{rms}}^2 = \frac{3}{2} k_B T$$

(Digression: connect energy/heat/power physics units to real life.)

A commonly used energy unit in chemistry is the calorie:

$$1 \text{ cal} = 4.18 \text{ J}$$

Even more common is the food Calorie:

$$1 \text{ Cal} = 1000 \text{ cal} = 4180 \text{ J}$$

and remember that power is measured in watts:

$$1 \text{ W} = 1 \text{ J/s}$$

Q: A person typically consumes about 2000 food calories per day. Roughly how many watts of power are required to power a person?

(A) 1 W (B) 10 W (C) 100 W (D) 1000 W (E) 10000 W

Latent heat

At the boiling point, the internal energy of the gas phase is higher than that of the liquid phase: need to overcome the forces that keep molecules close together in a liquid.

Similarly, the internal energy of the liquid phase is higher than that of the solid phase at the melting point.

So even at a fixed temperature, you need to add some energy to turn a solid into a liquid, or to turn a liquid into a gas. "Latent heat." Called *heat of fusion* or *heat of vaporization*.

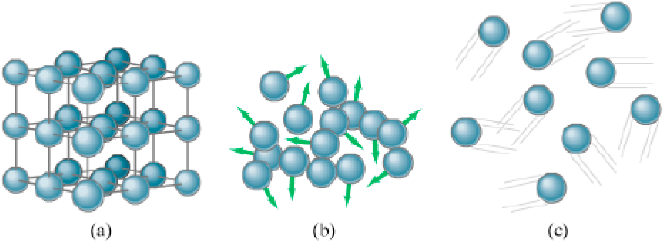


FIGURE 13–2 Atomic arrangements in (a) a crystalline solid, (b) a liquid, and (c) a gas.

Latent heat is extremely useful. For instance, your body takes advantage of the latent heat (heat of vaporization) of water when you sweat.

Suppose that by exercising you double your metabolic rate, from about 100 watts to about 200 watts.

How many milliliters of water ($1\ \text{mL} = 1\ \text{cc} \approx \frac{1}{30}$ ounce) do you need to sweat per minute, in order for your body to remove these extra 100 W by letting the sweat evaporate from your skin?

The latent heat of vaporization for water is $2260\ \text{kJ/kg} = 2260\ \text{J/g}$. The density of water is $1\ \text{g/mL}$.

I get

$$\frac{100 \text{ J/s} \ \times \ 60 \text{ s/min}}{1.0 \text{ g/mL} \ \times \ 2260 \text{ J/g}} = 2.65 \text{ mL/minute}$$

which is about an ounce every 11 minutes, i.e. a glass of water every hour or two while exercising. Seems like a plausible number.

Physics 9 — Monday, October 22, 2018

- For today, you read Giancoli ch14 (heat).
- For Wednesday, read Giancoli ch15 (thermodynamics).
- HW6 due Friday.
- Are there a few people interested in coming along with me to Acentech's "3D Listening" demo next Monday, 4pm, on 17th Street?
- I found a way to run both Odeon and CATT-Acoustic on MacOS without a virtual machine! Stay tuned.