Physics 9 — Wednesday, October 24, 2018

- For today, you read Giancoli ch15 (thermodynamics).
- HW6 due Friday.
- HW help sessions: Wed 4–6pm DRL $\boxed{\textbf{4C2}}$ (Bill),
  Thu 6:30–8:30pm DRL 2C8 (Grace)
- So far two people plan to come along with me to Acentech's "3D Listening" demo Monday, 4pm, on 17th Street.
- I found a way to run both Odeon and CATT-Acoustic on MacOS without a virtual machine! Stay tuned.

Heat absorbed by Earth per unit time:

$$\frac{\mathrm{d}Q_{\mathrm{absorbed}}}{\mathrm{d}t} = (70\%)(\pi R_{\mathrm{earth}}^2)(1360\ \mathrm{W/m^2}) = 1.2 \times 10^{17}\ \mathrm{W}$$

For Earth to remain at a constant temperature, it must be emitting heat out into space at the same rate (on average) at which it absorbs heat from the Sun.

$$\frac{\mathrm{d}Q_{\mathrm{emitted}}}{\mathrm{d}t} = e\sigma A T^4 = \sigma\ (4\pi R_{\mathrm{earth}}^2)\ (T_{\mathrm{earth}})^4 = 1.2 \times 10^{17}\ \mathrm{W}$$

Heat absorbed by Earth per unit time:

$$\frac{\mathrm{d}Q_{\mathrm{absorbed}}}{\mathrm{d}t} = (70\%)(\pi R_{\mathrm{earth}}^2)(1360\ \mathrm{W/m^2}) = 1.2\times 10^{17}\ \mathrm{W}$$

For Earth to remain at a constant temperature, it must be emitting heat out into space at the same rate (on average) at which it absorbs heat from the Sun.

$$\frac{\mathrm{d}Q_{\mathrm{emitted}}}{\mathrm{d}t} = e\sigma A T^4 = \sigma\ (4\pi R_{\mathrm{earth}}^2)\ (T_{\mathrm{earth}})^4 = 1.2\times 10^{17}\ \mathrm{W}$$

$$\Rightarrow \quad T_{\mathrm{earth}} \approx 255\ \mathrm{K}$$

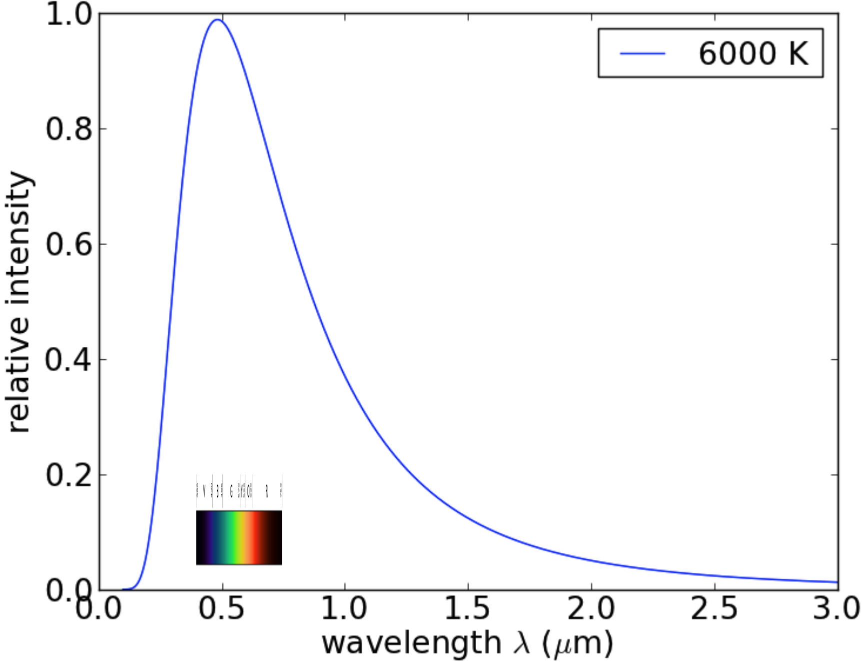
that's $-18^\circ\mathrm{C}$ ($0^\circ\mathrm{F}$). Brrr! That's what you would get if there were no atmosphere to insulate Earth, like a blanket (or a greenhouse). Averaged over seasons & area, $T \approx 288\ \mathrm{K}$ ($+15^\circ\mathrm{C}$, $60^\circ\mathrm{F}$) is the actual temperature.

Suppose I leave my sleek, black iPhone$^{(\mathrm{TM})}$ out in the mid-day sunlight. What happens?

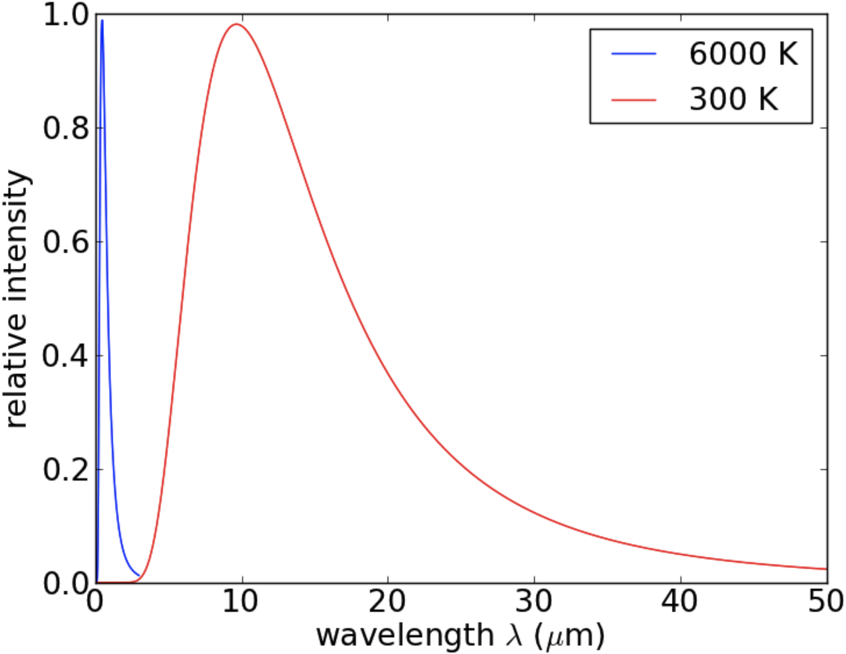
- A Nothing of any consequence happens.
- B The iPhone heats up until it reaches $\pi/2$ times the outdoor temperature (in kelvins), then remains at constant T.
- C The iPhone continues to heat up forever, until finally its plastic case melts and its microchips revert to amorphous silicon.
- D The iPhone's temperature initially rises. As T increases, the rate $\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t$ of heat transferred out of the iPhone increases (because radiation $\propto T^4$, while conduction $\propto T - T_{\mathrm{ambient}}$, and convection also increases as $T - T_{\mathrm{ambient}}$ increases). Eventually, T stabilizes at the point where $\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t = \mathrm{d}Q_{\mathrm{in}}/\mathrm{d}t$.

(Put black object in the "sun." Try varying the distance.)

Wavelength distribution ("spectrum") for 6000 K



Wavelength distribution ("spectrum") for 300 K



After my iPhone's temperature stabilizes in the mid-day sunlight, I then cover it up with an upside-down (transparent) glass bowl. Now what happens?

A The temperature stays the same as before.

B The temperature is reduced, because the transparent glass bowl blocks out a lot of the Sun's light.

C The temperature increases, because the Sun's light can still come in at about the same rate, but it is harder now for the heat to escape. ($\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t$ is lower for a given T because of the insulating layer of glass.) Eventually, T stabilizes at the new, higher point where $\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t = \mathrm{d}Q_{\mathrm{in}}/\mathrm{d}t$.

(Put glass beaker over black object & thermometer.)

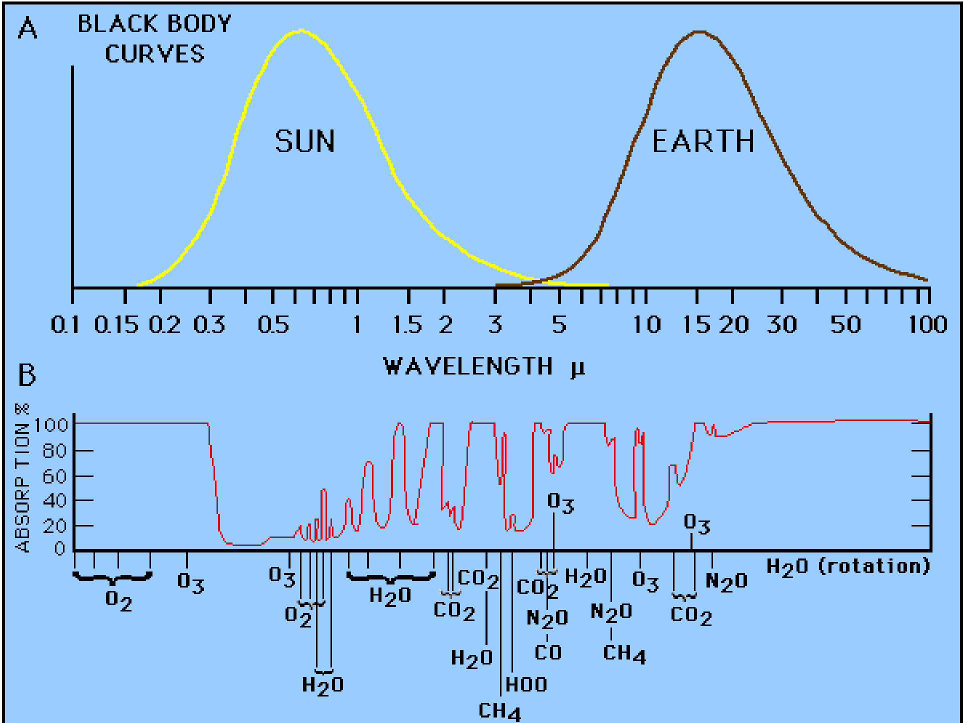
Let's work out what happens if we surround Earth with an insulating layer that (for a given T) cuts $\mathrm{d}Q_{\mathrm{out}}/\mathrm{d}t$ in half.

(You know that water vapor is one such insulator: it is cooler in the morning after a clear night, and warmer after a cloudy night. The weather report sometimes mentions "radiational cooling" after a clear night. Water vapor acts something like the glass roof of a greenhouse.)

It turns out that the more jiggly molecules in the atmosphere (e.g. $\mathrm{H_2O}$ vapor, $\mathrm{CO_2}$, $\mathrm{CH_4}$, which have many "degrees of freedom") are transparent to incoming visible light but absorb and re-emit outgoing infrared radiation. Dense clouds can, of course, block both incoming and outgoing radiation.

3-minute video (made here at Penn Physics!)

`https://www.youtube.com/watch?v=0eI9zxZoipA`


A
BLACK BODY CURVES
SUN
EARTH
0.1
0.15
0.2
0.3
0.5
1
1.5
2
3
5
10
15
20
30
50
100
WAVELENGTH μ
B
ABSORPTION %
100
80
60
40
20
0
O3
O3
O_2
O_3
O_3
O_2
H_2O
CO_2
CO_2
H_2O
CO_2
N_2O
CO
H_2O
N_2O
CH_4
O_3
CO_2
N_2O
H_2O (rotation)
H_2O
H_2O
HOO
CH_4

The incoming power is still

$$\frac{\mathrm{d}Q_{\mathrm{absorbed}}}{\mathrm{d}t} = (70\%)(\pi R_{\mathrm{earth}}^2)(1360\ \mathrm{W/m^2}) = 1.2 \times 10^{17}\ \mathrm{W}$$

But if the atmosphere absorbs the IR and re-scatters it half upward and half downward, then the power radiated by Earth's surface must double, so that the net power leaving the top of the atmosphere equals the power coming in from the Sun:

$$\frac{\mathrm{d}Q_{\mathrm{emitted}}}{\mathrm{d}t} \;=\; e\sigma A T^4 \;=\; 2 \;\times\; 1.2 \times 10^{17}\ \mathrm{W}$$

$$\Rightarrow \quad T_{\mathrm{earth}} \;=\; (2)^{1/4} \;\times\; 255\ \mathrm{K} \;=\; 303\ \mathrm{K}$$

That's closer to the right answer (which is $15^\circ\mathrm{C} = 288\ \mathrm{K}$), but too hot. If instead of trapping 50%, the atmosphere instead traps 39% of Q_{out}, we get 288 K at the surface.

More effective insulating blanket → warmer surface.

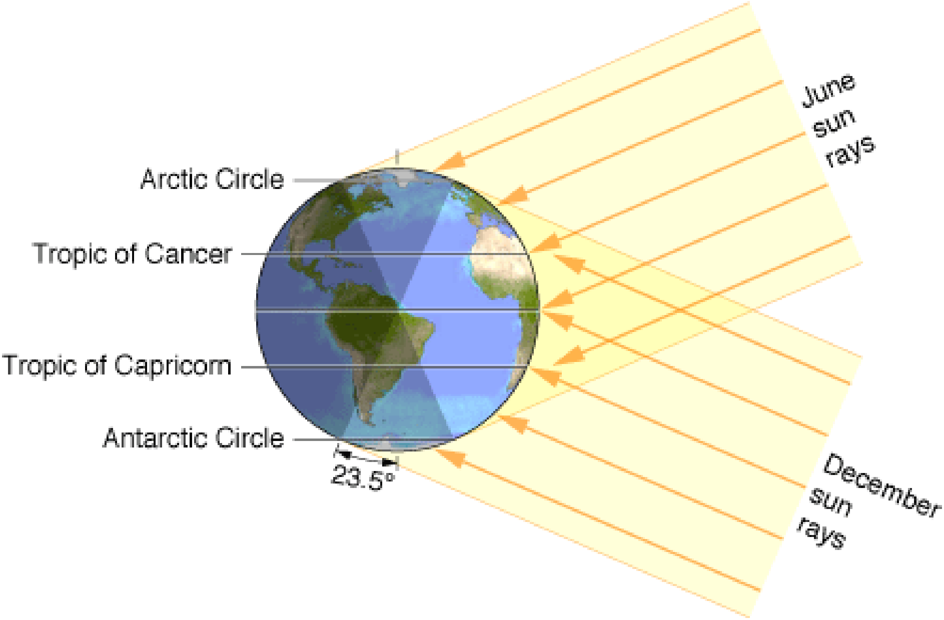
One more thing about the Sun's rays . . .

Remember that Sun's intensity scales like

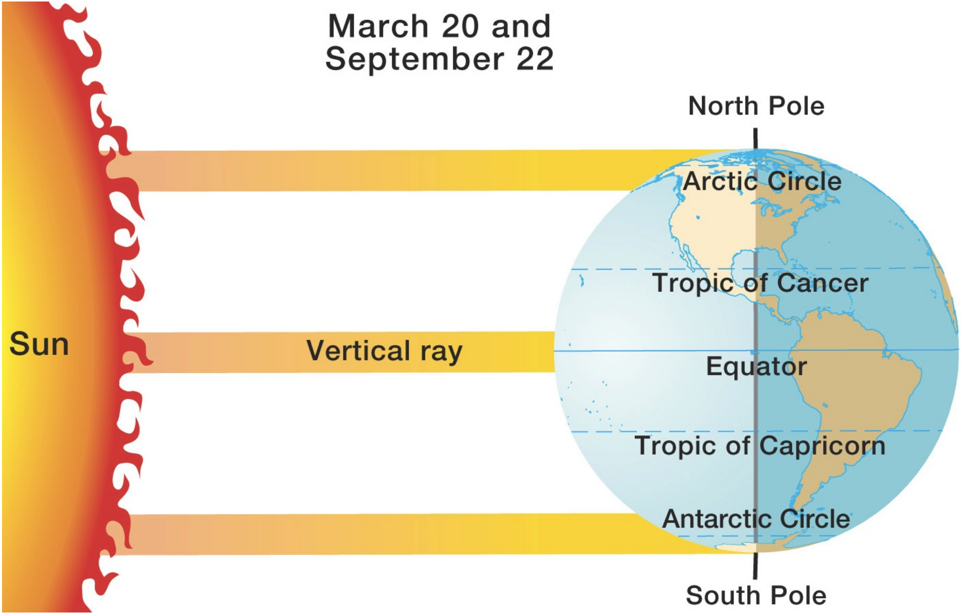
$$\text{intensity} \;\propto\; \cos\theta$$

where θ is the angle between the sun's rays and the surface normal.

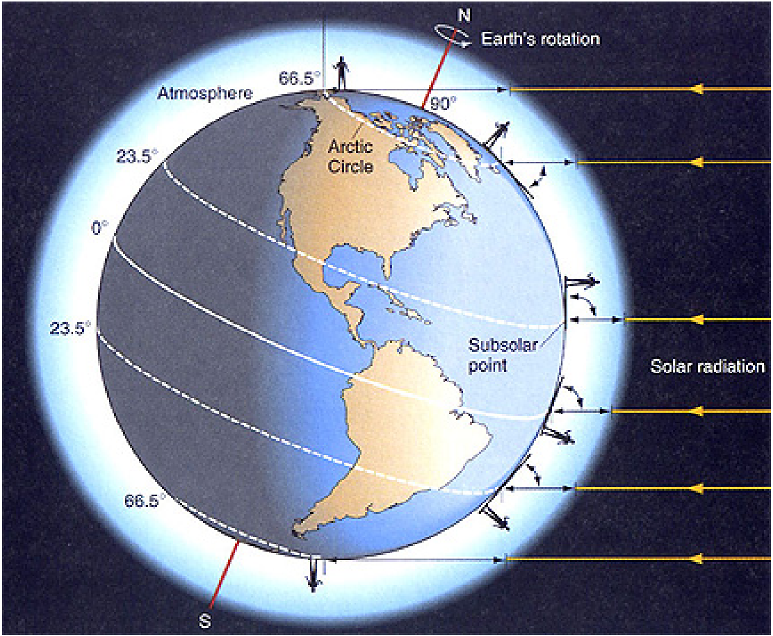
In your senior studio work, you may see this in various solar-modeling software (several years ago, it was Autodesk Ecotect).


June sun rays
Arctic Circle
Tropic of Cancer
Tropic of Capricorn
Antarctic Circle
23.5°
December sun rays

March 20 and September 22



What month is this?



About how much solar power is incident on one square meter of flat, level ground near Philadelphia, at noon on March 21? (Note that $70\% \times 1350\ \mathrm{W/m^2} \approx 1000\ \mathrm{W}$.)

(A) 1000 watts $\times \sin(40^\circ) \approx$ 640 watts

(B) 1000 watts $\times \cos(40^\circ) \approx$ 770 watts

(C) 1000 watts $\times \tan(40^\circ) \approx$ 840 watts

(D) 1000 watts

(E) 1350 watts

About how much solar power is incident on one square meter of a vertical, south-facing window near Philadelphia, at noon on March 21? (Note that 70% $\times$ 1350 $\mathrm{W/m^2} \approx$ 1000 W.)

(A) 1000 watts $\times$ sin$(40^\circ) \approx$ 640 watts

(B) 1000 watts $\times$ cos$(40^\circ) \approx$ 770 watts

(C) 1000 watts $\times$ tan$(40^\circ) \approx$ 840 watts

(D) 1000 watts

(E) 1350 watts

What is the purpose of an “expansion joint” on a bridge?

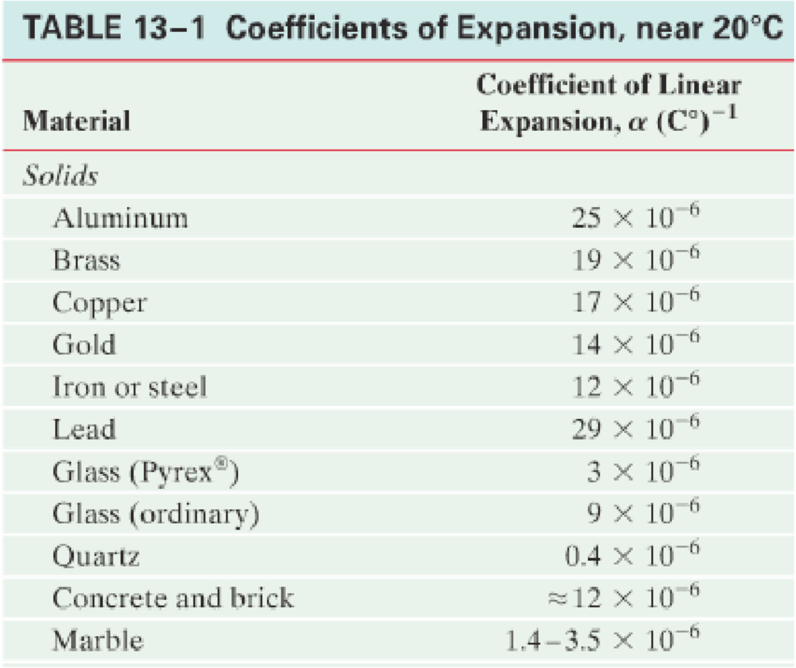


TABLE 13–1 Coefficients of Expansion, near 20°C

Material	Coefficient of Linear Expansion, α $(\mathrm{C}^\circ)^{-1}$
Solids	
Aluminum	25×10^{-6}
Brass	19×10^{-6}
Copper	17×10^{-6}
Gold	14×10^{-6}
Iron or steel	12×10^{-6}
Lead	29×10^{-6}
Glass (Pyrex®)	3×10^{-6}
Glass (ordinary)	9×10^{-6}
Quartz	0.4×10^{-6}
Concrete and brick	$\approx 12 \times 10^{-6}$
Marble	$1.4–3.5 \times 10^{-6}$

A bridge deck is made of concrete slabs that are 10.0 meters long at $20^\circ\mathrm{C}$. How wide should the expansion cracks between the slabs be (at $20^\circ\mathrm{C}$) to prevent buckling if the range of temperatures is $-20^\circ\mathrm{C}$ to $+40^\circ\mathrm{C}$? Use $\alpha = 12 \times 10^{-6}/^\circ\mathrm{C}$ for the coefficient of linear expansion for concrete.

(a) 2.4 mm (b) 4.8 mm (c) 7.2 mm

A bridge deck is made of concrete slabs that are 10.0 meters long at $20^\circ\mathrm{C}$. If the expansion cracks between the slabs are 2.4 mm wide at $20^\circ\mathrm{C}$, how wide will they be at $-20^\circ\mathrm{C}$? Use $\alpha = 12 \times 10^{-6}/^\circ\mathrm{C}$ for the coefficient of linear expansion for concrete.

(a) 1.2 mm (b) 2.4 mm (c) 4.8 mm (d) 7.2 mm

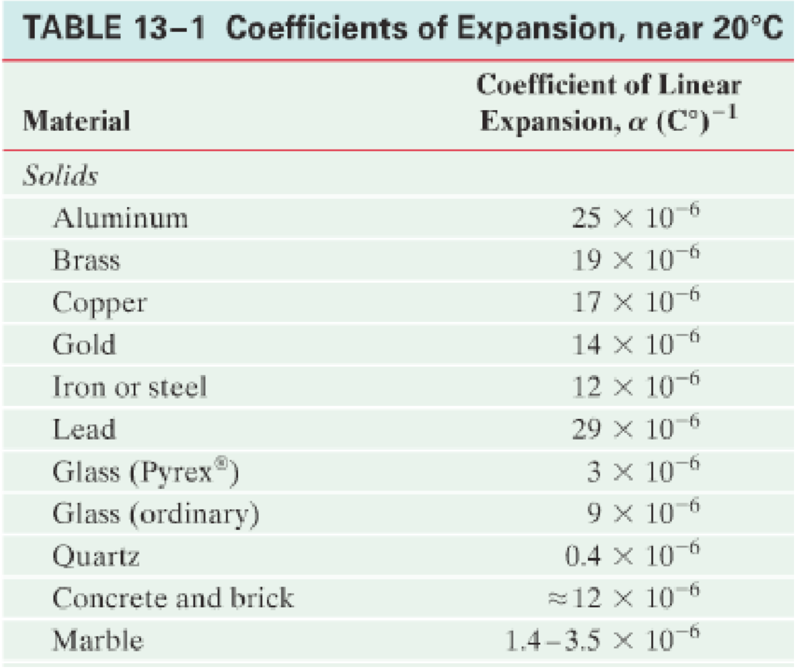
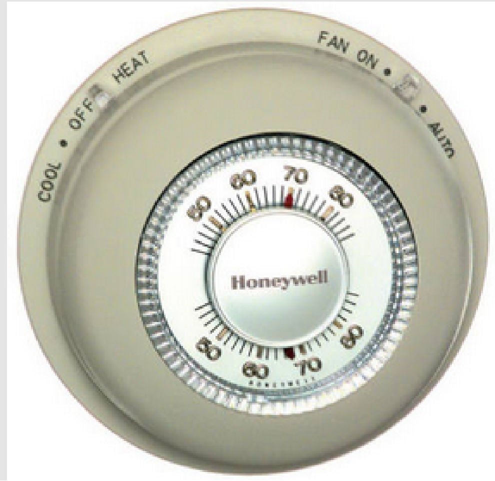


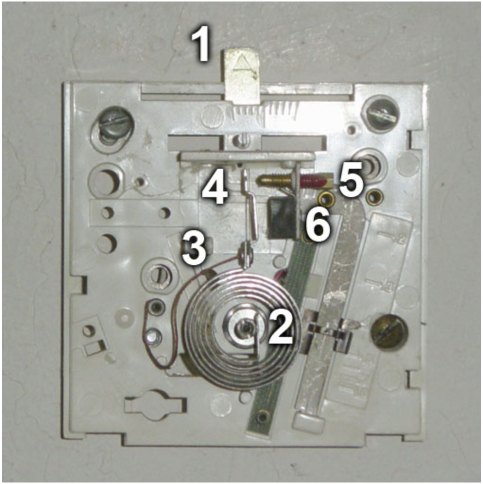
TABLE 13–1 Coefficients of Expansion, near 20°C

Material	Coefficient of Linear Expansion, α (C°)$^{-1}$
Solids	
Aluminum	25×10^{-6}
Brass	19×10^{-6}
Copper	17×10^{-6}
Gold	14×10^{-6}
Iron or steel	12×10^{-6}
Lead	29×10^{-6}
Glass (Pyrex®)	3×10^{-6}
Glass (ordinary)	9×10^{-6}
Quartz	0.4×10^{-6}
Concrete and brick	$\approx 12 \times 10^{-6}$
Marble	$1.4\text{–}3.5 \times 10^{-6}$

Do you remember Richard Muller's story about Pyrex vs. ordinary glass?

Have you ever seen an old thermostat that looks like this?





How do you suppose this works?

Related question: suppose I take a long strip of steel and a long strip of aluminum, and I solder them together, like a submarine sandwich, with the steel on top. What happens when I heat it up?

Thermal expansion

It turns out that a simple coefficient of linear (or volume) expansion works pretty well for most materials. But these things are usually tabulated as a function of temperature.

$$\Delta L = \alpha L_0 \Delta T \quad \rightarrow \quad \frac{\mathrm{d}L}{\mathrm{d}T} = \alpha L_0$$

$$\Delta V = \beta V_0 \quad \rightarrow \quad \frac{\mathrm{d}V}{\mathrm{d}T} = \beta V_0$$

If $V = L^3$ and material is isotropic, then

$$\frac{\mathrm{d}V}{\mathrm{d}T} = \frac{\mathrm{d}}{\mathrm{d}T}(L^3) = 3L_0^2(\alpha L_0) = 3\alpha L_0^3 = (3\alpha)V_0$$

so $\beta \approx 3\alpha$ for most materials.

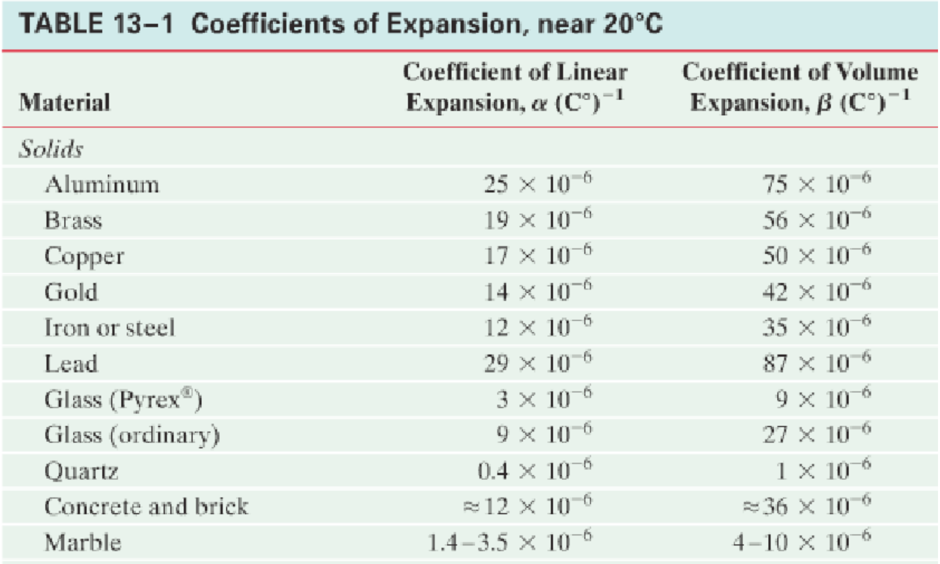


TABLE 13–1 Coefficients of Expansion, near 20°C

Material	Coefficient of Linear Expansion, α $(\mathrm{C}^\circ)^{-1}$	Coefficient of Volume Expansion, β $(\mathrm{C}^\circ)^{-1}$
Solids		
Aluminum	25×10^{-6}	75×10^{-6}
Brass	19×10^{-6}	56×10^{-6}
Copper	17×10^{-6}	50×10^{-6}
Gold	14×10^{-6}	42×10^{-6}
Iron or steel	12×10^{-6}	35×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Glass (Pyrex®)	3×10^{-6}	9×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Quartz	0.4×10^{-6}	1×10^{-6}
Concrete and brick	$\approx 12 \times 10^{-6}$	$\approx 36 \times 10^{-6}$
Marble	$1.4\text{–}3.5 \times 10^{-6}$	$4\text{–}10 \times 10^{-6}$

Thermal stress

Remember from last year that (E = Young's modulus)

$$\frac{\Delta L}{L_0} = \frac{1}{E}\ \frac{F}{A}$$

If I heat something up so that it tries to expand thermally:

$$\frac{\Delta L}{L_0} = \alpha \Delta T$$

but I don't let it expand (because it is held in place rigidly), then it is effectively being squished by a factor $\alpha \Delta T$

So holidng it at its original length when it heats up induces a stress (force per unit area):

$$\frac{F}{A} = E\alpha \Delta T$$

which the material may or may not be able to tolerate (depending on its compressive strength).

A bridge deck is made of concrete slabs that are 10.0 meters long at $20^\circ\mathrm{C}$. They are laid end-to-end with no expansion cracks!! So the length of each slab cannot increase. (Oops!)

If the temperature rises to $30^\circ\mathrm{C}$, how large is the compressive stress that develops in the concrete in order to maintain its original length?

Use $\alpha = 12 \times 10^{-6}/^\circ\mathrm{C}$ for the coefficient of linear expansion for concrete. Use $E = 20 \times 10^{9}\ \mathrm{N/m^2}$ for the Young's modulus of concrete.

A bridge deck is made of concrete slabs that are 10.0 meters long at $20^\circ\mathrm{C}$. They are laid end-to-end with no expansion cracks!! So the length of each slab cannot increase. (Oops!)

If the temperature rises to $30^\circ\mathrm{C}$, how large is the compressive stress that develops in the concrete in order to maintain its original length?

Use $\alpha = 12 \times 10^{-6}/^\circ\mathrm{C}$ for the coefficient of linear expansion for concrete. Use $E = 20 \times 10^9\ \mathrm{N/m^2}$ for the Young's modulus of concrete.

(I get stress $= 2.4 \times 10^6\ \mathrm{N/m^2} \ = \ \alpha E \Delta T$. This is about ×8 below the compressive strength of concrete, so the slabs won't crush, but they may buckle, as a slender column would.)

Giancoli's illustration of solid / liquid / gas

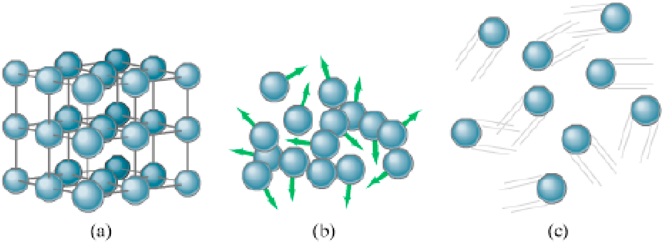


FIGURE 13–2 Atomic arrangements in (a) a crystalline solid, (b) a liquid, and (c) a gas.

Atomic mass unit (u):

$$1\ \mathrm{u} \;=\; 1\ \frac{\mathrm{gram}}{\mathrm{mole}} \;=\; 1.66 \times 10^{-27}\ \mathrm{kg}$$

This is very close (within 1%) to the mass of a proton:

$$m_{\mathrm{proton}} = 1.67 \times 10^{-27}\ \mathrm{kg}$$

A dozen eggs is 12 eggs. A mole of protons is $N_A = 6.022 \times 10^{23}$ protons.

A $^{12}\mathrm{C}$ nucleus contains 6 protons + 6 neutrons. So a carbon atom has a mass of 12 u, or 12 g/mol.

A mole of protons has a mass (within 1%) of 1.0 gram. A mole of $^{12}\mathrm{C}$ atoms has a mass (by definition of mole) of exactly 12 grams.

An oxygen nucleus contains

$$8 \text{ protons} + 8 \text{ neutrons} = 16 \text{ nucleons}$$

A hydrogen nucleus contains just 1 proton (and no neutrons). What is the mass of a mole of water $(\mathrm{H_2O})$ molecules?

(A) 0.016 kg
(B) 0.017 kg
(C) 0.018 kg
(D) 0.020 kg
(E) 0.034 kg

Ideal gas law

Anybody remember this from high school chemistry?

$$PV = nRT$$

$$R = 8.315 \ \frac{\mathrm{J}}{\mathrm{mol} \cdot \mathrm{K}}$$

$$R = 0.0821 \ \frac{\mathrm{L} \cdot \mathrm{atm}}{\mathrm{mol} \cdot \mathrm{K}}$$

Notice that temperature is measured in K (kelvin). To get temperature in K, take temperature in °C and add 273.15.

By the way, how does a temperature difference $\Delta T = 1$ K compare with a temperature difference $\Delta T = 1^\circ$C?

(A) A temperature difference of 1 kelvin is bigger than a temperature difference of 1 degree Celsius.

(B) A temperature difference of 1 kelvin is smaller than a temperature difference of 1 degree Celsius.

(C) A temperature difference of 1 kelvin is the same as a temperature difference of 1 degree Celsius.

By the way, how does a temperature difference $\Delta T = 1^\circ$F compare with a temperature difference $\Delta T = 1^\circ$C?

(A) A temperature difference of 1 degree Farenheit is 1.8× as large as temperature difference of 1 degree Celsius.

(B) A temperature difference of 1 degree Celsius is 1.8× as large as temperature difference of 1 degree Farenheit.

(C) A temperature difference of 1 degree Farenheit is the same as a temperature difference of 1 degree Celsius.

What is the final pressure P_f of a sealed bottle of air after you raise its temperature from 27°C to 57°C?

(A) $P_f = 0.47\ P_i$

(B) $P_f = 0.9\ P_i$

(C) $P_f = 1.1\ P_i$

(D) $P_f = 2.1\ P_i$

(E) not enough information to decide

A cylinder initially contains one liter of air at atmospheric pressure. I then compress the gas *isothermally* (i.e. at constant temperature) to half its initial volume. What is the final pressure of the gas (in atmospheres)?

(A) $P_f = 0.50$ atm
(B) $P_f = 0.71$ atm
(C) $P_f = 1.00$ atm
(D) $P_f = 1.41$ atm
(E) $P_f = 2.00$ atm
(F) not enough information to decide

Dry air is $78\%\ \mathrm{N_2}$, $21\%\ \mathrm{O_2}$, $1\%\ \mathrm{Ar}$, $0.04\%\ \mathrm{CO_2}$, $\ldots$.
For simplicity, let's call it $80\%\ \mathrm{N_2}$ and $20\%\ \mathrm{O_2}$.

What is the density of $\mathrm{N_2}$ gas at $T = 22^\circ\mathrm{C}$? (Each nitrogen **atom** contains 7 protons and 7 neutrons.)

How do you find the mass of one mole of $\mathrm{N_2}$ molecules?

How do you find the volume of one mole of $\mathrm{N_2}$ molecules?

Dry air is $78\%\ \mathrm{N_2}$, $21\%\ \mathrm{O_2}$, $1\%\ \mathrm{Ar}$, $0.04\%\ \mathrm{CO_2}$, $\ldots$.
For simplicity, let's call it 80% $\mathrm{N_2}$ and 20% $\mathrm{O_2}$.

What is the density of $\mathrm{N_2}$ gas at $T = 22^\circ\mathrm{C}$? (Each nitrogen **atom** contains 7 protons and 7 neutrons.)

How do you find the mass of one mole of $\mathrm{N_2}$ molecules?

How do you find the volume of one mole of $\mathrm{N_2}$ molecules?

I get $m = 28\ \mathrm{g}$, $V = \frac{nRT}{P} = (.0821)(295) = 24.22\ \mathrm{L}$, so density $m/V = 1.16\ \mathrm{g/L} = 1.16\ \mathrm{kg/m^3}$.

Same thing for $\mathrm{O_2}$: one oxygen **atom** contains 8 protons and 8 neutrons.

Dry air is 78% $\mathrm{N_2}$, 21% $\mathrm{O_2}$, 1% Ar, 0.04% $\mathrm{CO_2}$, $\ldots$.
For simplicity, let's call it 80% $\mathrm{N_2}$ and 20% $\mathrm{O_2}$.

What is the density of $\mathrm{N_2}$ gas at $T = 22^\circ\mathrm{C}$? (Each nitrogen **atom** contains 7 protons and 7 neutrons.)

How do you find the mass of one mole of $\mathrm{N_2}$ molecules?

How do you find the volume of one mole of $\mathrm{N_2}$ molecules?

I get $m = 28\ \mathrm{g}$, $V = \frac{nRT}{P} = (.0821)(295) = 24.22\ \mathrm{L}$, so density $m/V = 1.16\ \mathrm{g/L} = 1.16\ \mathrm{kg/m^3}$.

Same thing for $\mathrm{O_2}$: one oxygen **atom** contains 8 protons and 8 neutrons. I get $1.32\ \mathrm{kg/m^3}$.

So for dry room-temperature air:
$(0.8)(1.16) + (0.2)(1.32) = 1.19\ \mathrm{kg/m^3}$.

The ideal gas law you learned in high school chemistry was:

$$PV = nRT$$

$$R = 8.315\ \frac{\mathrm{J}}{\mathrm{mol}\cdot\mathrm{K}}$$

$$R = 0.0821\ \frac{\mathrm{L}\cdot\mathrm{atm}}{\mathrm{mol}\cdot\mathrm{K}}$$

The analogous ideal gas law in terms of number of molecules (N) rather than number of moles (n) is:

$$PV = Nk_BT$$

where k_B is *Boltzmann's constant*

$$k_B = 1.38\times10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}$$

Q: what do you get when you divide 8.315 $\frac{\mathrm{J}}{\mathrm{mol}\cdot\mathrm{K}}$ by $6.022\times10^{23}/\mathrm{mol}$?

Boltzmann's constant

$$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$$

$\frac{1}{2} k_B T$ is the average thermal energy per *degree of freedom.* A single atom of monotomic gas can move from place to place in three dimensions, but can't rotate or vibrate, so it has 3 d.o.f.

$$\frac{1}{2} m v_{\mathrm{rms}}^2 = \frac{3}{2} k_B T$$

The average energy of an atom or molecule is directly proportional to temperature. v_{rms} means the "root mean squared" speed.

$$v_{\mathrm{rms}} = \sqrt{\frac{3 k_B T}{m}}$$

for an ideal gas. The lighter molecules tend to move faster!

Boltzmann's constant

$$v_{\mathrm{rms}} = \sqrt{\frac{3k_B T}{m}}$$

for an ideal gas. The lighter molecules tend to move faster! For helium gas (4 g/mol) at room temperature,

$$v_{\mathrm{rms}} \approx 1360 \mathrm{~m/s}$$

For nitrogen gas ($\mathrm{N_2}$, 28 g/mol) at 298 K,

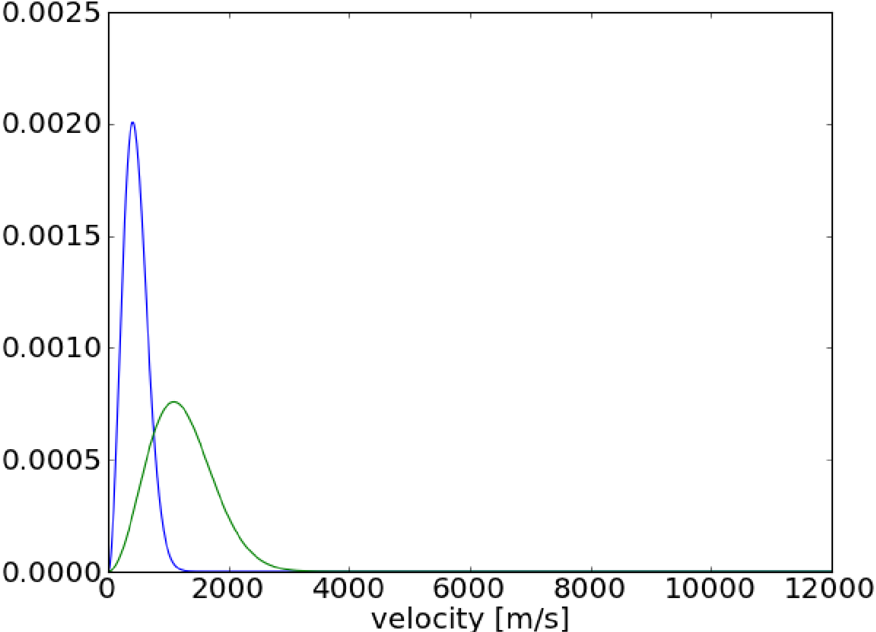
$$v_{\mathrm{rms}} \approx 520 \mathrm{~m/s}$$

For gasoline vapor ($\mathrm{C_8H_{18}}$, 114 g/mol) at 298 K,

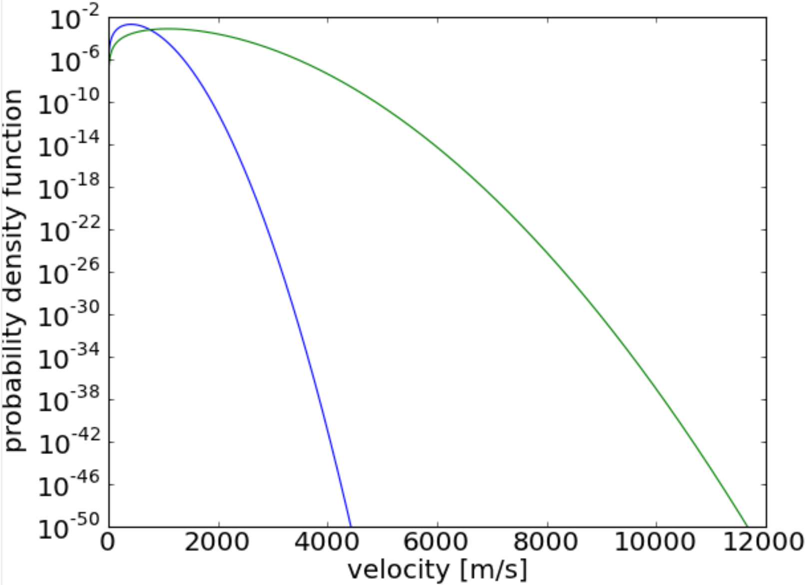
$$v_{\mathrm{rms}} \approx 260 \mathrm{~m/s}$$

Richard Muller pointed out that this is why the helium escaped from Earth's atmosphere: some small fraction of the helium atoms move fast enough (11200 m/s) to escape Earth's gravity.

One of these two curves is the velocity distribution (at $T = 288\ \mathrm{K}$) for $\mathrm{N_2}$ (nitrogen) and one is for He (helium). Which is which?



If you make the vertical axis logarithmic, you can see that indeed some tiny fraction of the helium atoms move faster than the 11 km/s “escape velocity” of Earth’s gravity.



Nitrogen (N_2) makes up about 78% of the air we breathe, while oxygen (O_2) accounts for approximately 21%. On average, are the nitrogen or the oxygen molecules moving faster? (The mass of an N_2 molecule is 28 amu, while the mass of an O_2 molecule is 32 amu.)

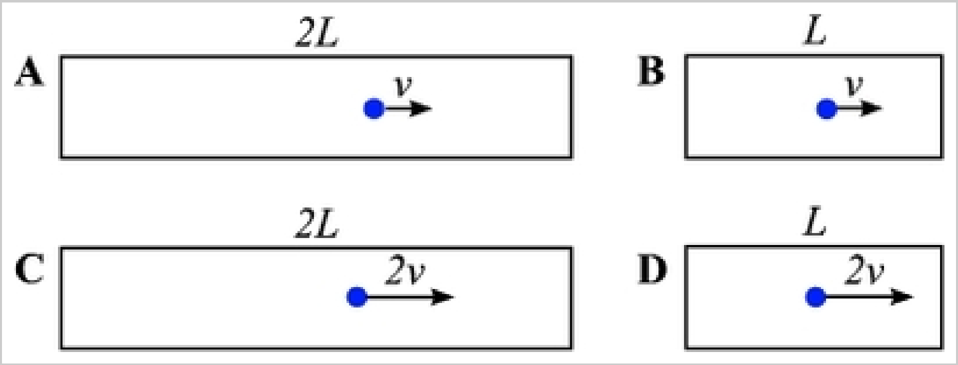
(A) The rms speed of the N_2 molecules is $\frac{32}{28}$ as large as the rms speed of the O_2 molecules, so the N_2 molecules move slightly faster, on average.

(B) The rms speed of the N_2 molecules is $\sqrt{\frac{32}{28}}$ as large as the rms speed of the O_2 molecules, so the N_2 molecules move slightly faster, on average.

(C) The rms speed of the O_2 molecules is $\frac{32}{28}$ as large as the rms speed of the N_2 molecules, so the O_2 molecules move slightly faster, on average.

(D) The rms speed of the O_2 molecules is $\sqrt{\frac{32}{28}}$ as large as the rms speed of the N_2 molecules, so the O_2 molecules move slightly faster, on average.

A gas atom of mass m moves in a straight line with speed v or $2v$ in a box with length L or $2L$, as shown below. The atom collides elastically with the right wall, travels to the left wall, collides elastically, returns to the right wall and repeats this process. In which case does the atom exert the largest average force on the walls?



Bigger thermal energy → higher rms speed → larger force per unit area (pressure)

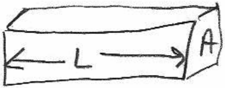
Smaller volume → particles collide more often with walls → larger force per unit area (pressure)

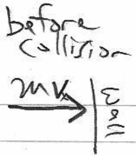
More particles in container → collisions with walls occur more often → larger force per unit area (pressure)

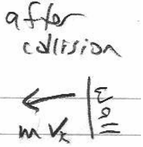
$$P = \frac{N k_B T}{V}$$

$$\frac{1}{2} m v_{\mathrm{rms}}^2 = \frac{3}{2} k_B T$$

"Deriving" the ideal-gas law is not as difficult as you may think







$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$\text{force}_x = \frac{\Delta\,\text{momentum}_x}{\Delta\,\text{time}} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$$

$$\rightarrow \underset{\text{(due to one molecule)}}{\text{pressure}} = \frac{mv_x^2}{L \cdot A} = \frac{m\overline{v_x^2}}{\text{volume}} = \frac{k_BT}{\text{volume}}$$

$$\frac{1}{2}m\overline{v_x^2} = \frac{1}{2}m\overline{v_y^2} = \frac{1}{2}m\overline{v_z^2} = \frac{1}{2}k_BT \Leftrightarrow \frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT$$

$$\rightarrow \boxed{P = \frac{Nk_BT}{V}}$$

(Digression: connect energy/heat/power physics units to real life.)

A commonly used energy unit in chemistry is the calorie:

$$1\ \mathrm{cal} = 4.18\ \mathrm{J}$$

Even more common is the food Calorie:

$$1\ \mathrm{Cal} = 1000\ \mathrm{cal} = 4180\ \mathrm{J}$$

and remember that power is measured in watts:

$$1\ \mathrm{W} = 1\ \mathrm{J/s}$$

Q: A person typically consumes about 2000 food calories per day. Roughly how many watts of power are required to power a person?

(A) 1 W (B) 10 W (C) 100 W (D) 1000 W (E) 10000 W

We just worked out that a person's daily intake of 2000 dietary Calories implies that it takes on average about 100 watts to power a person.

$$\frac{2000 \times 4180 \text{ J}}{24 \times 60 \times 60 \text{ s}} = 97 \text{ W}$$

Former energy secretary Steven Chu was fond of pointing out that since the USA has an overall energy usage of about 3×10^{12} W (including industry, etc.) and a population of about 300M people, that's an average of 10 kW per person in the USA, or roughly 100 "energy helpers" per person. (I think he used a more pejorative word for "helper.")

In other words, US energy use per person (including industry, etc.) is about 100× a typical person's 100 W metabolic rate.

Heat capacity

As you increase the temperature of a substance, its molecules move faster. So its thermal energy increases. This thermal energy is just an "incoherent" form of kinetic energy: you're adding up a whole bunch of $\frac{1}{2}mv^2$, but the velocities are all pointing in different directions, so the substance as a whole is not moving even though the individual molecules are moving.

The internal thermal energy of a monatomic gas (like helium or argon) is $\frac{3}{2}k_B T$ per molecule, or $\frac{3}{2}RT$ per mole.

You sometimes need to know how much energy is needed to heat an object (like a brick wall, or an old metal baseboard radiator, or a swimming pool) by some number of degrees. Tables usually list **energy per unit mass per degree C**. This is known as **specific heat capacity** (or more commonly just "specific heat").

$$Q = m c \Delta T$$

where c is the "**specific heat**," whose SI units are $\frac{\mathrm{J}}{\mathrm{kg}^\circ\mathrm{C}}$

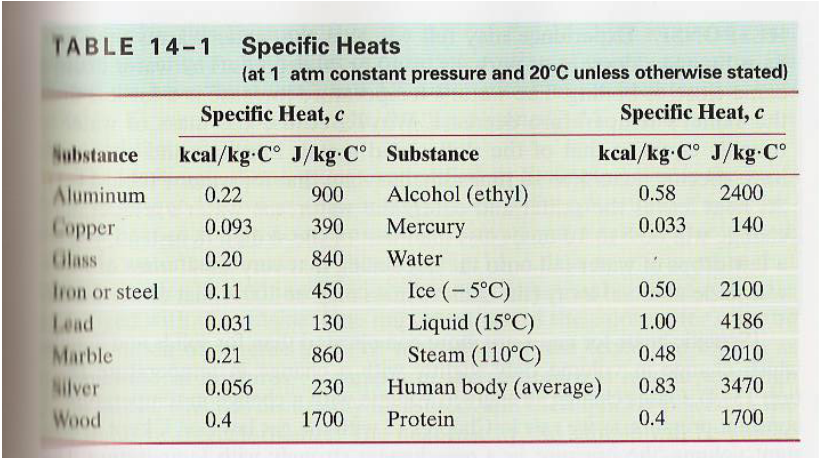


TABLE 14–1 Specific Heats
(at 1 atm constant pressure and 20°C unless otherwise stated)

	Specific Heat, *c*			Specific Heat, *c*	
Substance	**kcal/kg·C°**	**J/kg·C°**	**Substance**	**kcal/kg·C°**	**J/kg·C°**
Aluminum	0.22	900	Alcohol (ethyl)	0.58	2400
Copper	0.093	390	Mercury	0.033	140
Glass	0.20	840	Water		
Iron or steel	0.11	450	Ice (−5°C)	0.50	2100
Lead	0.031	130	Liquid (15°C)	1.00	4186
Marble	0.21	860	Steam (110°C)	0.48	2010
Silver	0.056	230	Human body (average)	0.83	3470
Wood	0.4	1700	Protein	0.4	1700

For example: how many joules of heat are needed to raise 10 kg of water from 20°C to 30°C? How many watts of power are needed to do this in 1000 s (about 17 minutes)?

Latent heat

At the boiling point, the internal energy of the gas phase is higher than that of the liquid phase: need to overcome the forces that keep molecules close together in a liquid.

Similarly, the internal energy of the liquid phase is higher than that of the solid phase at the melting point.

So even at a fixed temperature, you need to add some energy to turn a solid into a liquid, or to turn a liquid into a gas. "Latent heat." Called *heat of fusion* or *heat of vaporization*.

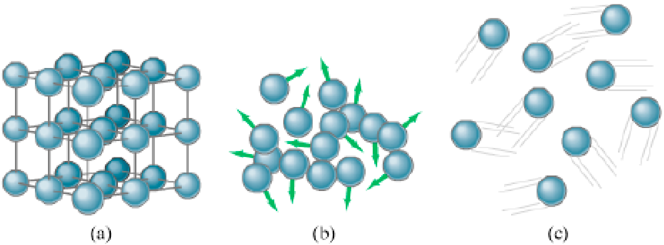

(a) (b) (c)

FIGURE 13–2 Atomic arrangements in (a) a crystalline solid, (b) a liquid, and (c) a gas.

Latent heat is extremely useful. For instance, your body takes advantage of the latent heat (heat of vaporization) of water when you sweat.

Suppose that by exercising you double your metabolic rate, from about 100 watts to about 200 watts.

How many milliliters of water ($1\ \mathrm{mL} = 1\ \mathrm{cc} \approx \frac{1}{30}$ ounce) do you need to sweat per minute, in order for your body to remove these extra 100 W by letting the sweat evaporate from your skin?

The latent heat of vaporization for water is $2260\ \mathrm{kJ/kg} = 2260\ \mathrm{J/g}$. The density of water is $1\ \mathrm{g/mL}$. See if you and your neighbor can work out an answer (in milliliters per minute):

(A) 0.027 mL/min (B) 0.27 mL/min (C) 2.7 mL/min

(D) 27 mL/min (E) 270 mL/min (F) 2700 mL/min

I get

$$\frac{100 \text{ J/s} \ \times \ 60 \text{ s/min}}{1.0 \text{ g/mL} \ \times \ 2260 \text{ J/g}} = 2.65 \text{ mL/minute}$$

which is about an ounce every 11 minutes, i.e. a glass of water every hour or two while exercising. Seems like a plausible number.

Conduction of heat (thermal conductivity)

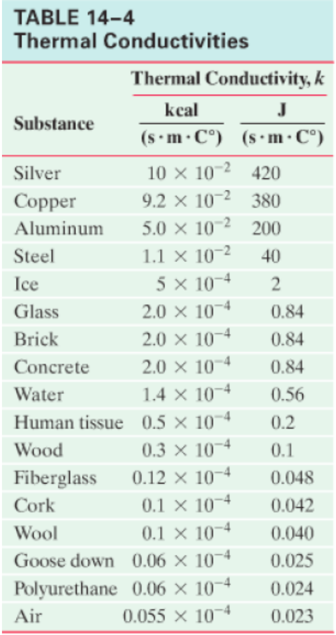


TABLE 14–4
Thermal Conductivities

Substance	Thermal Conductivity, k $\frac{\text{kcal}}{(\text{s}\cdot\text{m}\cdot\text{C}^\circ)}$	Thermal Conductivity, k $\frac{\text{J}}{(\text{s}\cdot\text{m}\cdot\text{C}^\circ)}$
Silver	10×10^{-2}	420
Copper	9.2×10^{-2}	380
Aluminum	5.0×10^{-2}	200
Steel	1.1×10^{-2}	40
Ice	5×10^{-4}	2
Glass	2.0×10^{-4}	0.84
Brick	2.0×10^{-4}	0.84
Concrete	2.0×10^{-4}	0.84
Water	1.4×10^{-4}	0.56
Human tissue	0.5×10^{-4}	0.2
Wood	0.3×10^{-4}	0.1
Fiberglass	0.12×10^{-4}	0.048
Cork	0.1×10^{-4}	0.042
Wool	0.1×10^{-4}	0.040
Goose down	0.06×10^{-4}	0.025
Polyurethane	0.06×10^{-4}	0.024
Air	0.055×10^{-4}	0.023

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{kA}{\ell}(T_1 - T_2) = \frac{A}{R}(T_1 - T_2)$$

"R value" is

$$\frac{\ell}{k} = \frac{\text{thickness}}{\text{thermal conductivity}}$$

(often given in US customary units, unfortunately)

- more area (in cross-section, perpendicular to heat flow) → faster heat conduction
- bigger temperature difference → faster
- bigger thermal conductivity → faster
- thicker insulating layer → slower
- bigger "R value" → slower

I keep the inside of my house at $20^\circ\mathrm{C}$. If the thermal power $\mathrm{d}Q/\mathrm{d}t$ conducted through the walls of my house is 10 kilowatts when the outdoor temperature is $10^\circ\mathrm{C}$, what will be the thermal power conducted through the walls of my house when the outdoor temperature is $-10^\circ\mathrm{C}$?

(Hint: this is why the gas company often tells you the number of "degree days" for the winter months. Your heating bill should scale like the inside-outside temperature difference, integrated over time.)

(A) 5 kW (B) 10 kW (C) 20 kW (D) 30 kW

By the way: 1 kW = 3400 BTU/hour. 1 BTU = 1055 J. Anyone know the conventional definition of a BTU?

Concrete has a thermal conductivity k that is about $8.4\times$ that of wood. How thick a layer of concrete would I need to use in order to provide thermal insulation equivalent to that of a 2 cm layer of wood?

(A) 0.12 cm (B) 2 cm (C) 8.4 cm (D) 16.8 cm

Concrete has a thermal conductivity k that is about $8.4\times$ that of wood. How thick a layer of concrete would I need to use in order to provide thermal insulation equivalent to that of a 2 cm layer of wood?

(A) 0.12 cm (B) 2 cm (C) 8.4 cm (D) 16.8 cm

The "R value" goes like (thickness) / (thermal conductivity). So if the thermal conductivity is multiplied $\times 8.4$, then the thickness also needs to be multiplied $\times 8.4$. So I need 16.8 cm of concrete to get the same thermal insulation as 2 cm of wood.

If 10 kilowatts of power (heat per unit time) is conducted through the $10\ \mathrm{m} \times 10\ \mathrm{m}$ roof of my house on a given winter day, how much power would be conducted through a similar roof (same kind of insulation, same thickness, etc.) that is $20\ \mathrm{m} \times 20\ \mathrm{m}$ on the same winter day?

(A) 2.5 kW (B) 10 kW (C) 20 kW (D) 40 kW

If 10 kilowatts of power (heat per unit time) is conducted through the 10 $\mathrm{m} \times 10\ \mathrm{m}$ roof of my house on a given winter day, how much power would be conducted through a similar roof (same kind of insulation, same thickness, etc.) that is 20 $\mathrm{m} \times 20\ \mathrm{m}$ on the same winter day?

(A) 2.5 kW (B) 10 kW (C) 20 kW (D) 40 kW

The area is quadrupled, so the conducted heat per unit time is quadrupled. I get 40 kW.

If you buy fiberglass insulation at Home Depot, the “R value” is written on the paper backing.

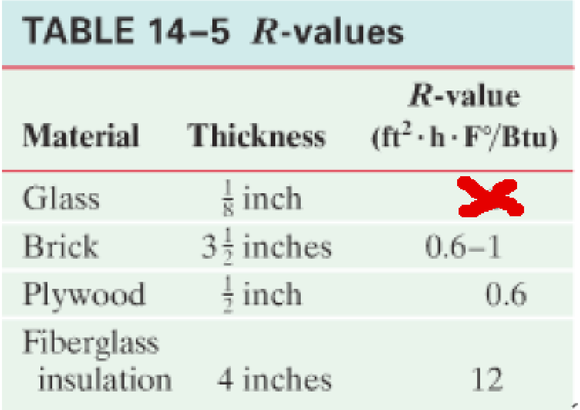


TABLE 14–5 ***R*-values**

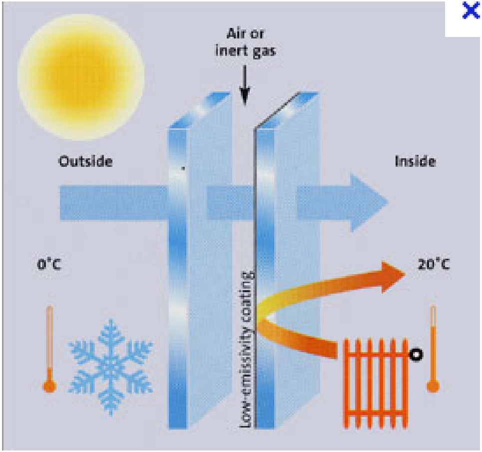
Material	Thickness	*R*-value ($\mathrm{ft^2 \cdot h \cdot F^\circ/Btu}$)
Glass	$\frac{1}{8}$ inch	[redacted]
Brick	$3\frac{1}{2}$ inches	0.6–1
Plywood	$\frac{1}{2}$ inch	0.6
Fiberglass insulation	4 inches	12

$$R = \frac{\ell}{k}$$

A U.S. customary R value of 1 is (in metric units) $0.176\ \frac{\mathrm{m^2{}^\circ C}}{W}$.

R values add: twice the thickness means twice the R value, which means half as much heat conducted per unit time.

R values add: double-glazed window



$$R = \frac{\ell}{k}$$

$$k_{\mathrm{glass}} = 0.84 \ \frac{\mathrm{W}}{\mathrm{m}^{\circ}\mathrm{C}}$$

$$k_{\mathrm{air}} = 0.026 \ \frac{\mathrm{W}}{\mathrm{m}^{\circ}\mathrm{C}}$$

$$k_{\mathrm{argon}} = 0.018 \ \frac{\mathrm{W}}{\mathrm{m}^{\circ}\mathrm{C}}$$

What is ℓ/k for 6 mm of glass? How about 3 mm of glass, then 10 mm of air, then 3 mm of glass?

$$R = \frac{\ell}{k}$$

$$k_{\mathrm{glass}} = 0.84\ \frac{\mathrm{W}}{\mathrm{m}^\circ\mathrm{C}} \qquad k_{\mathrm{air}} = 0.026\ \frac{\mathrm{W}}{\mathrm{m}^\circ\mathrm{C}} \qquad k_{\mathrm{argon}} = 0.018\ \frac{\mathrm{W}}{\mathrm{m}^\circ\mathrm{C}}$$

What is ℓ/k for 6 mm of glass? How about 3 mm of glass, then 10 mm of air, then 3 mm of glass?

$$\frac{0.006\ \mathrm{m}}{0.84\ \frac{\mathrm{W}}{\mathrm{m}^\circ\mathrm{C}}} = 0.0071\ \frac{\mathrm{m}^{2\circ}\mathrm{C}}{\mathrm{W}} \qquad \frac{0.010\ \mathrm{m}}{0.026\ \frac{\mathrm{W}}{\mathrm{m}^\circ\mathrm{C}}} = 0.38\ \frac{\mathrm{m}^{2\circ}\mathrm{C}}{\mathrm{W}}$$

So in theory, two 3 mm panes of glass separated by 10 mm of air will *conduct* about 1/50 as much heat per unit time as a single 6 mm pane of glass. (But we ignored convection and radiation. And in real life, even a single layer of glass will build up an insulating layer of cool air next to it, increasing its effectiveness.)

We've said that "dry air" is 78% N_2, 21% O_2, 1% Ar, 0.04% CO_2.
What important constituent of ordinary air are we missing?

We've said that "dry air" is 78% N_2, 21% O_2, 1% Ar, 0.04% CO_2. What important constituent of ordinary air are we missing?

The fraction of water vapor in the air varies from region to region (climate) and from day to day (weather). The maximum possible fraction of water vapor in the air (before you get rain, snow, fog, etc.) varies with temperature.

We've said that "dry air" is 78% N_2, 21% O_2, 1% Ar, 0.04% CO_2. What important constituent of ordinary air are we missing?

The fraction of water vapor in the air varies from region to region (climate) and from day to day (weather). The maximum possible fraction of water vapor in the air (before you get rain, snow, fog, etc.) varies with temperature.

To discuss the quantity of one gas (e.g. water vapor) that is dissolved in another gas (e.g. air), it helps to introduce the concept of **partial pressure,** which is the pressure due to that constituent alone.

We've said that "dry air" is 78% N_2, 21% O_2, 1% Ar, 0.04% CO_2. What important constituent of ordinary air are we missing?

The fraction of water vapor in the air varies from region to region (climate) and from day to day (weather). The maximum possible fraction of water vapor in the air (before you get rain, snow, fog, etc.) varies with temperature.

To discuss the quantity of one gas (e.g. water vapor) that is dissolved in another gas (e.g. air), it helps to introduce the concept of **partial pressure,** which is the pressure due to that constituent alone.

Example: air at 1 atm is 21% O_2. What is the partial pressure of oxygen (in atmospheres)?

We've said that "dry air" is 78% N_2, 21% O_2, 1% Ar, 0.04% CO_2. What important constituent of ordinary air are we missing?

The fraction of water vapor in the air varies from region to region (climate) and from day to day (weather). The maximum possible fraction of water vapor in the air (before you get rain, snow, fog, etc.) varies with temperature.

To discuss the quantity of one gas (e.g. water vapor) that is dissolved in another gas (e.g. air), it helps to introduce the concept of **partial pressure,** which is the pressure due to that constituent alone.

Example: air at 1 atm is 21% O_2. What is the partial pressure of oxygen (in atmospheres)?

Answer: 0.21 atm, which is about 21300 $\mathrm{N/m^2}$.

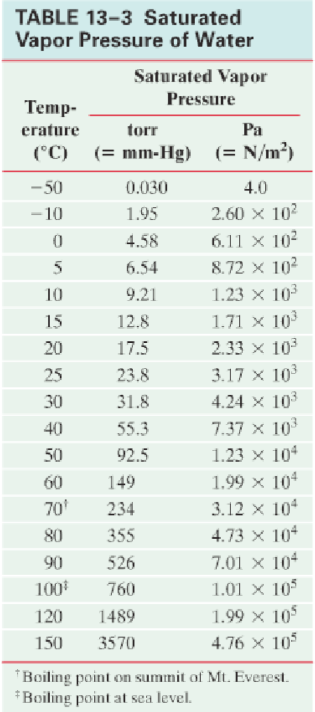


TABLE 13–3 Saturated Vapor Pressure of Water

Temp-erature (°C)	Saturated Vapor Pressure torr (= mm-Hg)	Saturated Vapor Pressure Pa (= N/m^2)
−50	0.030	4.0
−10	1.95	2.60×10^2
0	4.58	6.11×10^2
5	6.54	8.72×10^2
10	9.21	1.23×10^3
15	12.8	1.71×10^3
20	17.5	2.33×10^3
25	23.8	3.17×10^3
30	31.8	4.24×10^3
40	55.3	7.37×10^3
50	92.5	1.23×10^4
60	149	1.99×10^4
70[†]	234	3.12×10^4
80	355	4.73×10^4
90	526	7.01×10^4
100[‡]	760	1.01×10^5
120	1489	1.99×10^5
150	3570	4.76×10^5

[†] Boiling point on summit of Mt. Everest.

[‡] Boiling point at sea level.

At a given temperature, the maximum possible partial pressure of water vapor is called the "**(saturated) vapor pressure of water.**"

When partial pressure of water vapor equals saturated vapor pressure, a pool of standing water is in equilibrium with water vapor: rate of evaporation (liquid to gas) equals rate of condensation (gas to liquid).

If partial pressure of water vapor exceeds the saturated vapor pressure (e.g. if you take saturated air and cool it), precipitation occurs.

Below this partial pressure, standing water will evaporate.

When saturated vapor pressure equals external atmospheric pressure, boiling occurs!

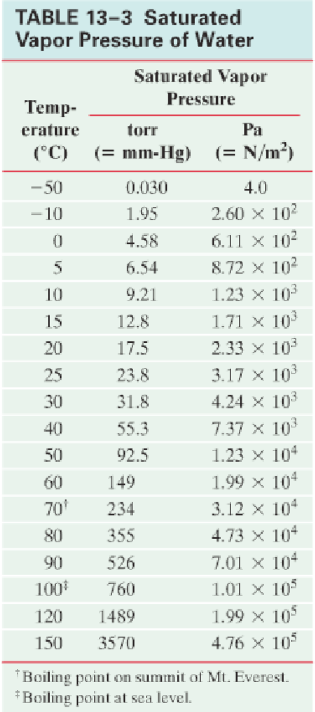


TABLE 13–3 Saturated Vapor Pressure of Water

Temperature (°C)	Saturated Vapor Pressure: torr (= mm-Hg)	Saturated Vapor Pressure: Pa (= N/m^2)
−50	0.030	4.0
−10	1.95	2.60×10^2
0	4.58	6.11×10^2
5	6.54	8.72×10^2
10	9.21	1.23×10^3
15	12.8	1.71×10^3
20	17.5	2.33×10^3
25	23.8	3.17×10^3
30	31.8	4.24×10^3
40	55.3	7.37×10^3
50	92.5	1.23×10^4
60	149	1.99×10^4
70[†]	234	3.12×10^4
80	355	4.73×10^4
90	526	7.01×10^4
100[‡]	760	1.01×10^5
120	1489	1.99×10^5
150	3570	4.76×10^5

[†] Boiling point on summit of Mt. Everest.

[‡] Boiling point at sea level.

In humid air, the rate at which sweat can evaporate from your skin (hence cooling your body, via latent heat) is lower than in dry air. So a hot, humid day feels more uncomfortable than a hot, dry day.

HVAC climate-control systems monitor and regulate "relative humidity" =

$$\frac{\text{partial pressure of water vapor}}{\text{saturated water vapor pressure}} \times 100\%$$

R.H. $\sim$ 40%-50% is most comfortable.

Q: if $T = 30^\circ\mathrm{C}$ and R.H.=50%, what is partial pressure of water vapor (in $\mathrm{N/m^2}$)?

In that case, what fraction of molecules in the air are H_2O molecules?

Physics 9 — Wednesday, October 24, 2018

- For today, you read Giancoli ch15 (thermodynamics).
- HW6 due Friday.
- HW help sessions: Wed 4–6pm DRL $\boxed{\textbf{4C2}}$ (Bill), Thu 6:30–8:30pm DRL 2C8 (Grace)
- So far two people plan to come along with me to Acentech's "3D Listening" demo Monday, 4pm, on 17th Street.
- I found a way to run both Odeon and CATT-Acoustic on MacOS without a virtual machine! Stay tuned.