Physics 9 — Monday, October 29, 2018

- So far three people plan to come along with me to Acentech's "3D Listening" demo Monday, 4pm, on 17th Street. We'll meet at the main DRL entrance at 3:25pm and walk briskly. We'll walk east on Walnut, then turn left on 17th to reach the Sofitel at 120 S. 17th. My mobile phone number is . . .
- For today, you read Richard Muller's chapter (written in 2009) on Climate Change, as well as his 2012 NYT op-ed.
- There is nothing you need to read for this Wednesday, though if you're interested I have some material from Muller's 2012 book *Energy for Future Presidents* that you could read for XC.

(Digression: connect energy/heat/power physics units to real life.)

A commonly used energy unit in chemistry is the calorie:

$$1\ \mathrm{cal} = 4.18\ \mathrm{J}$$

Even more common is the food Calorie:

$$1\ \mathrm{Cal} = 1000\ \mathrm{cal} = 4180\ \mathrm{J}$$

and remember that power is measured in watts:

$$1\ \mathrm{W} = 1\ \mathrm{J/s}$$

Q: A person typically consumes about 2000 food calories per day. Roughly how many watts of power are required to power a person?

(A) 1 W (B) 10 W (C) 100 W (D) 1000 W (E) 10000 W

We just worked out that a person's daily intake of 2000 dietary Calories implies that it takes on average about 100 watts to power a person.

$$\frac{2000 \times 4180\ \mathrm{J}}{24 \times 60 \times 60\ \mathrm{s}} = 97\ \mathrm{W}$$

Former energy secretary Steven Chu was fond of pointing out that since the USA has an overall energy usage of about 3×10^{12} W (including industry, etc.) and a population of about 300M people, that's an average of 10 kW per person in the USA, or roughly 100 "energy helpers" per person. (I think he used a more pejorative word for "helper.")

In other words, US energy use per person (including industry, etc.) is about 100× a typical person's 100 W metabolic rate.

Heat capacity

As you increase the temperature of a substance, its molecules move faster. So its thermal energy increases. This thermal energy is just an "incoherent" form of kinetic energy: you're adding up a whole bunch of $\frac{1}{2}mv^2$, but the velocities are all pointing in different directions, so the substance as a whole is not moving even though the individual molecules are moving.

The internal thermal energy of a monatomic gas (like helium or argon) is $\frac{3}{2}k_B T$ per molecule, or $\frac{3}{2}RT$ per mole.

You sometimes need to know how much energy is needed to heat an object (like a brick wall, or an old metal baseboard radiator, or a swimming pool) by some number of degrees. Tables usually list **energy per unit mass per degree C**. This is known as **specific heat capacity** (or more commonly just "specific heat").

$$Q = mc\Delta T$$

where c is the "**specific heat**," whose SI units are $\frac{\mathrm{J}}{\mathrm{kg}^\circ\mathrm{C}}$

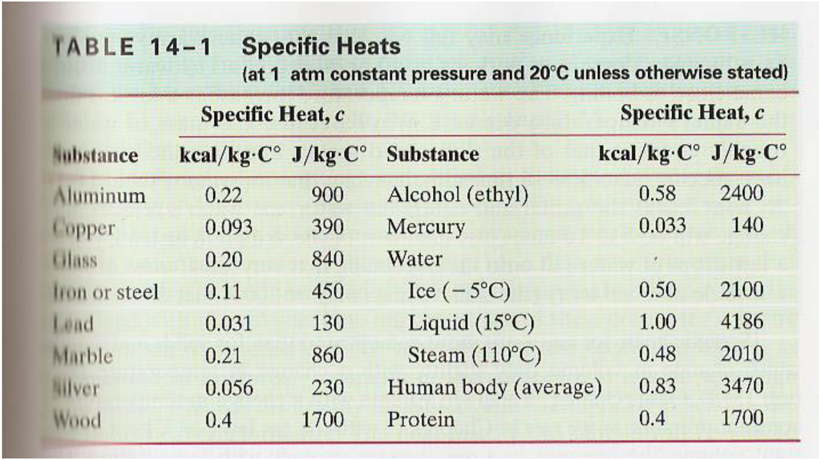

TABLE 14–1 Specific Heats

(at 1 atm constant pressure and 20°C unless otherwise stated)

	Specific Heat, *c*			Specific Heat, *c*	
Substance	**kcal/kg·C°**	**J/kg·C°**	**Substance**	**kcal/kg·C°**	**J/kg·C°**
Aluminum	0.22	900	Alcohol (ethyl)	0.58	2400
Copper	0.093	390	Mercury	0.033	140
Glass	0.20	840	Water		
Iron or steel	0.11	450	Ice (−5°C)	0.50	2100
Lead	0.031	130	Liquid (15°C)	1.00	4186
Marble	0.21	860	Steam (110°C)	0.48	2010
Silver	0.056	230	Human body (average)	0.83	3470
Wood	0.4	1700	Protein	0.4	1700

For example: how many joules of heat are needed to raise 10 kg of water from 20°C to 30°C? How many watts of power are needed to do this in 1000 s (about 17 minutes)?

Latent heat

At the boiling point, the internal energy of the gas phase is higher than that of the liquid phase: need to overcome the forces that keep molecules close together in a liquid.

Similarly, the internal energy of the liquid phase is higher than that of the solid phase at the melting point.

So even at a fixed temperature, you need to add some energy to turn a solid into a liquid, or to turn a liquid into a gas. "Latent heat." Called *heat of fusion* or *heat of vaporization*.

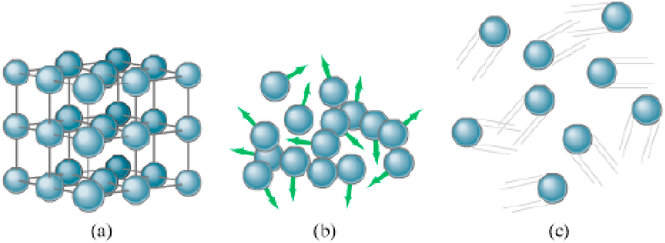

FIGURE 13–2 Atomic arrangements in (a) a crystalline solid, (b) a liquid, and (c) a gas.

Latent heat is extremely useful. For instance, your body takes advantage of the latent heat (heat of vaporization) of water when you sweat.

Suppose that by exercising you double your metabolic rate, from about 100 watts to about 200 watts.

How many milliliters of water ($1 \text{ mL} = 1 \text{ cc} \approx \frac{1}{30}$ ounce) do you need to sweat per minute, in order for your body to remove these extra 100 W by letting the sweat evaporate from your skin?

The latent heat of vaporization for water is 2260 kJ/kg = 2260 J/g. The density of water is 1 g/mL. See if you and your neighbor can work out an answer (in milliliters per minute):

(A) 0.027 mL/min (B) 0.27 mL/min (C) 2.7 mL/min
(D) 27 mL/min (E) 270 mL/min (F) 2700 mL/min

I get

$$\frac{100 \text{ J/s } \times \text{ } 60 \text{ s/min}}{1.0 \text{ g/mL } \times \text{ } 2260 \text{ J/g}} = 2.65 \text{ mL/minute}$$

which is about an ounce every 11 minutes, i.e. a glass of water every hour or two while exercising. Seems like a plausible number.

Conduction of heat (thermal conductivity)

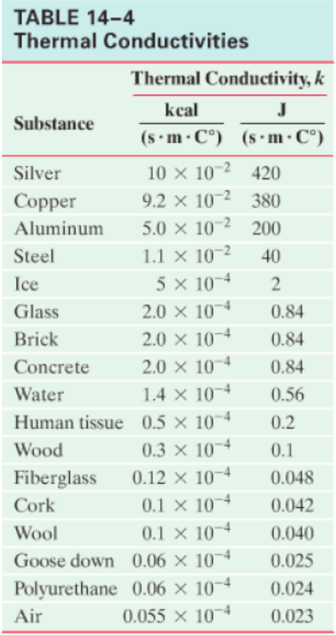

TABLE 14–4
Thermal Conductivities

Substance	Thermal Conductivity, k	
	$\frac{\text{kcal}}{(\text{s}\cdot\text{m}\cdot\text{C}^\circ)}$	$\frac{\text{J}}{(\text{s}\cdot\text{m}\cdot\text{C}^\circ)}$
Silver	10×10^{-2}	420
Copper	9.2×10^{-2}	380
Aluminum	5.0×10^{-2}	200
Steel	1.1×10^{-2}	40
Ice	5×10^{-4}	2
Glass	2.0×10^{-4}	0.84
Brick	2.0×10^{-4}	0.84
Concrete	2.0×10^{-4}	0.84
Water	1.4×10^{-4}	0.56
Human tissue	0.5×10^{-4}	0.2
Wood	0.3×10^{-4}	0.1
Fiberglass	0.12×10^{-4}	0.048
Cork	0.1×10^{-4}	0.042
Wool	0.1×10^{-4}	0.040
Goose down	0.06×10^{-4}	0.025
Polyurethane	0.06×10^{-4}	0.024
Air	0.055×10^{-4}	0.023

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{kA}{\ell}(T_1 - T_2) = \frac{A}{R}(T_1 - T_2)$$

"R value" is

$$\frac{\ell}{k} = \frac{\text{thickness}}{\text{thermal conductivity}}$$

(often given in US customary units, unfortunately)

- more area (in cross-section, perpendicular to heat flow) → faster heat conduction
- bigger temperature difference → faster
- bigger thermal conductivity → faster
- thicker insulating layer → slower
- bigger "R value" → slower

I keep the inside of my house at $20^\circ\mathrm{C}$. If the thermal power $\mathrm{d}Q/\mathrm{d}t$ conducted through the walls of my house is 10 kilowatts when the outdoor temperature is $10^\circ\mathrm{C}$, what will be the thermal power conducted through the walls of my house when the outdoor temperature is $-10^\circ\mathrm{C}$?

(Hint: this is why the gas company often tells you the number of "degree days" for the winter months. Your heating bill should scale like the inside-outside temperature difference, integrated over time.)

(A) 5 kW (B) 10 kW (C) 20 kW (D) 30 kW

By the way: 1 kW = 3400 BTU/hour. 1 BTU = 1055 J.
Anyone know the conventional definition of a BTU?

Concrete has a thermal conductivity k that is about 8.4× that of wood. How thick a layer of concrete would I need to use in order to provide thermal insulation equivalent to that of a 2 cm layer of wood?

(A) 0.12 cm (B) 2 cm (C) 8.4 cm (D) 16.8 cm

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The "R value" goes like (thickness) / (thermal conductivity). So if the thermal conductivity is multiplied $\times 8.4$, then the thickness also needs to be multiplied $\times 8.4$. So I need 16.8 cm of concrete to get the same thermal insulation as 2 cm of wood.

If 10 kilowatts of power (heat per unit time) is conducted through the 10 $\mathrm{m} \times 10\ \mathrm{m}$ roof of my house on a given winter day, how much power would be conducted through a similar roof (same kind of insulation, same thickness, etc.) that is 20 $\mathrm{m} \times 20\ \mathrm{m}$ on the same winter day?

(A) 2.5 kW (B) 10 kW (C) 20 kW (D) 40 kW

If 10 kilowatts of power (heat per unit time) is conducted through the $10\ \mathrm{m} \times 10\ \mathrm{m}$ roof of my house on a given winter day, how much power would be conducted through a similar roof (same kind of insulation, same thickness, etc.) that is $20\ \mathrm{m} \times 20\ \mathrm{m}$ on the same winter day?

(A) 2.5 kW (B) 10 kW (C) 20 kW (D) 40 kW

The area is quadrupled, so the conducted heat per unit time is quadrupled. I get 40 kW.

If you buy fiberglass insulation at Home Depot, the “R value” is written on the paper backing.

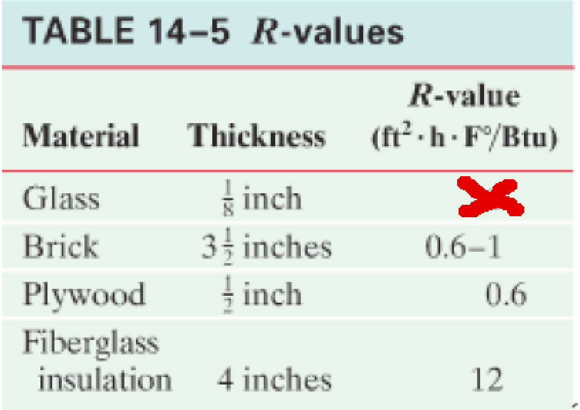


TABLE 14–5 *R*-values

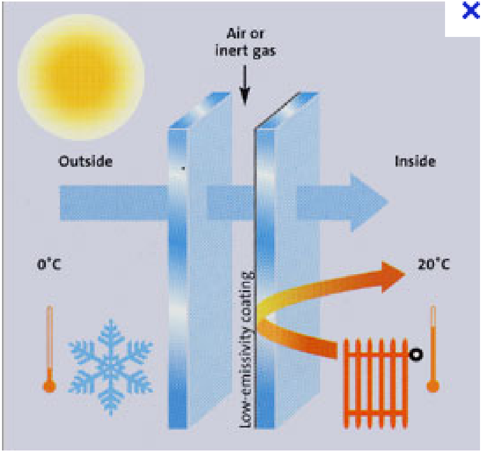
Material	Thickness	*R*-value ($ft^2 \cdot h \cdot F^\circ/Btu$)
Glass	$\frac{1}{8}$ inch	[redacted]
Brick	$3\frac{1}{2}$ inches	0.6–1
Plywood	$\frac{1}{2}$ inch	0.6
Fiberglass insulation	4 inches	12

$$R = \frac{\ell}{k}$$

A U.S. customary R value of 1 is (in metric units) $0.176\ \frac{\mathrm{m^2\,^\circ C}}{W}$.

R values add: twice the thickness means twice the R value, which means half as much heat conducted per unit time.

R values add: double-glazed window



$$R = \frac{\ell}{k}$$

$$k_{\mathrm{glass}} = 0.84\ \frac{\mathrm{W}}{\mathrm{m}^\circ\mathrm{C}}$$

$$k_{\mathrm{air}} = 0.026\ \frac{\mathrm{W}}{\mathrm{m}^\circ\mathrm{C}}$$

$$k_{\mathrm{argon}} = 0.018\ \frac{\mathrm{W}}{\mathrm{m}^\circ\mathrm{C}}$$

What is ℓ/k for 6 mm of glass? How about 3 mm of glass, then 10 mm of air, then 3 mm of glass?

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$$\frac{0.006\ \mathrm{m}}{0.84\ \frac{\mathrm{W}}{\mathrm{m}^\circ\mathrm{C}}} = 0.0071\ \frac{\mathrm{m}^{2\circ}\mathrm{C}}{\mathrm{W}} \qquad \frac{0.010\ \mathrm{m}}{0.026\ \frac{\mathrm{W}}{\mathrm{m}^\circ\mathrm{C}}} = 0.38\ \frac{\mathrm{m}^{2\circ}\mathrm{C}}{\mathrm{W}}$$

So in theory, two 3 mm panes of glass separated by 10 mm of air will *conduct* about 1/50 as much heat per unit time as a single 6 mm pane of glass. (But we ignored convection and radiation. And in real life, even a single layer of glass will build up an insulating layer of cool air next to it, increasing its effectiveness.)

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To discuss the quantity of one gas (e.g. water vapor) that is dissolved in another gas (e.g. air), it helps to introduce the concept of **partial pressure,** which is the pressure due to that constituent alone.

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Answer: 0.21 atm, which is about 21300 $\mathrm{N/m^2}$.

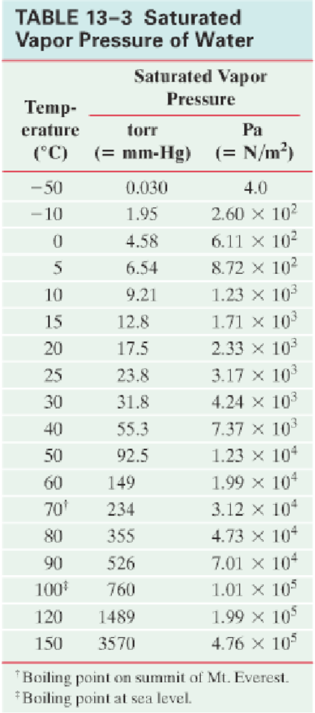


TABLE 13–3 Saturated Vapor Pressure of Water

Temperature (°C)	Saturated Vapor Pressure torr (= mm-Hg)	Saturated Vapor Pressure Pa (= N/m²)
−50	0.030	4.0
−10	1.95	2.60×10^2
0	4.58	6.11×10^2
5	6.54	8.72×10^2
10	9.21	1.23×10^3
15	12.8	1.71×10^3
20	17.5	2.33×10^3
25	23.8	3.17×10^3
30	31.8	4.24×10^3
40	55.3	7.37×10^3
50	92.5	1.23×10^4
60	149	1.99×10^4
70[†]	234	3.12×10^4
80	355	4.73×10^4
90	526	7.01×10^4
100[‡]	760	1.01×10^5
120	1489	1.99×10^5
150	3570	4.76×10^5

[†] Boiling point on summit of Mt. Everest.
[‡] Boiling point at sea level.

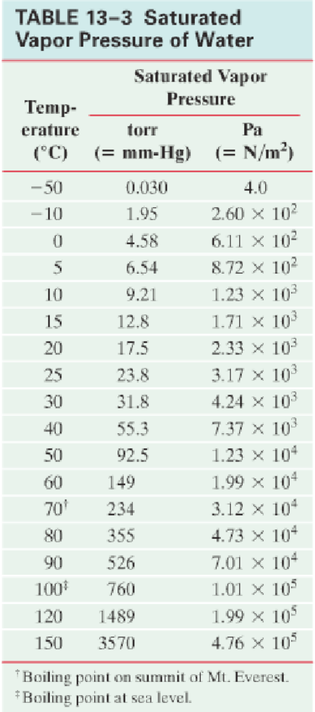
At a given temperature, the maximum possible partial pressure of water vapor is called the "**(saturated) vapor pressure of water.**"

When partial pressure of water vapor equals saturated vapor pressure, a pool of standing water is in equilibrium with water vapor: rate of evaporation (liquid to gas) equals rate of condensation (gas to liquid).

If partial pressure of water vapor exceeds the saturated vapor pressure (e.g. if you take saturated air and cool it), precipitation occurs.

Below this partial pressure, standing water will evaporate.

When saturated vapor pressure equals external atmospheric pressure, boiling occurs!



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In humid air, the rate at which sweat can evaporate from your skin (hence cooling your body, via latent heat) is lower than in dry air. So a hot, humid day feels more uncomfortable than a hot, dry day.

HVAC climate-control systems monitor and regulate "relative humidity" =

$$\frac{\text{partial pressure of water vapor}}{\text{saturated water vapor pressure}} \times 100\%$$

R.H. $\sim$ 40%-50% is most comfortable.

Q: if $T = 30^\circ\mathrm{C}$ and R.H.=50%, what is partial pressure of water vapor (in $\mathrm{N/m^2}$)?

In that case, what fraction of molecules in the air are H_2O molecules?

We learned last fall to associate *irreversible* processes (for which a movie played backwards looks impossible) with the dissipation of (coherent) mechanical energy into (incoherent) thermal energy.

- So you're not surprised that after a few bounces, the initial mechanical energy of a ball dissipates into thermal energy.
- But a movie of this process played backwards would look like an impossible situation.

Similarly, you're accustomed to seeing heat flow spontaneously from a warm object to a cooler object. That's why thermal insulation is needed to slow this process down.

- So you're not surprised when your glass of ice-water reaches room temperature after sitting out for an hour.
- But again a movie of this process played backwards would show something that looks impossible.

Both of these situations involve an **increase in entropy**; and hence are **irreversible.** Let's ponder the meaning of entropy and its connection to probability.

Entropy

You might have learned in a math class at some point that if I toss N coins, the probability of seeing m of those N coins land heads-up is given by the *binomial distribution*,

$$P(m, N) = \frac{N!}{m!\,(N-m)!}\,\left(\frac{1}{2}\right)^N$$

For $N = 2$, the probabilities for $m = 0, 1, 2$ are

$$\frac{1}{4}, \quad \frac{2}{4}, \quad \frac{1}{4}$$

For $N = 4$, the probabilities for $m = 0, 1, 2, 3, 4$ are

$$\frac{1}{16}, \quad \frac{4}{16}, \quad \frac{6}{16}, \quad \frac{4}{16}, \quad \frac{1}{16}$$

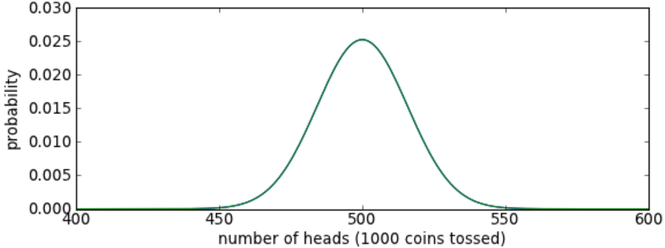
For $N = 8$, the probabilities for $m = 0, 1, 2, 3, 4, 5, 6, 7, 8$ are

$$\frac{1}{256}, \quad \frac{8}{256}, \quad \frac{28}{256}, \quad \frac{56}{256}, \quad \frac{70}{256}, \quad \frac{56}{256}, \quad \frac{28}{256}, \quad \frac{8}{256}, \quad \frac{1}{256}$$

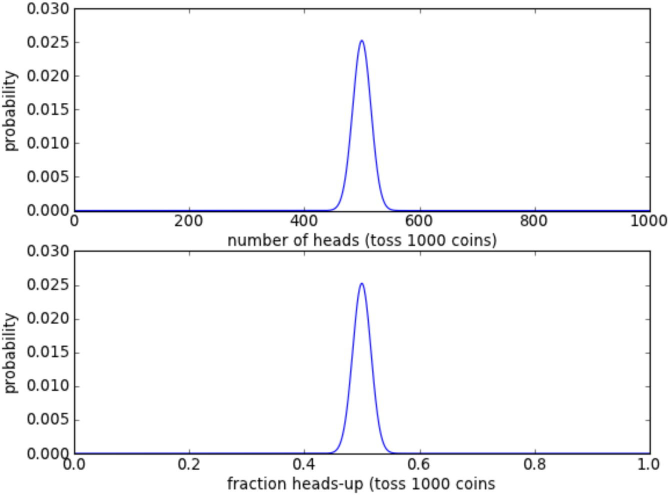
$$P(m, N) = \frac{N!}{m!\ (N-m)!}\ \left(\frac{1}{2}\right)^N$$

If I toss $N = 1000$ coins, the probability of finding m heads-up is

heads	probability	. . .	heads	probability
0	0.9×10^{-301}		1000	0.9×10^{-301}
1	0.9×10^{-298}		999	0.9×10^{-298}
450	0.02%		550	0.02%
480	1.1%		520	1.1%
499	2.5%		501	2.5%
500	2.5%			

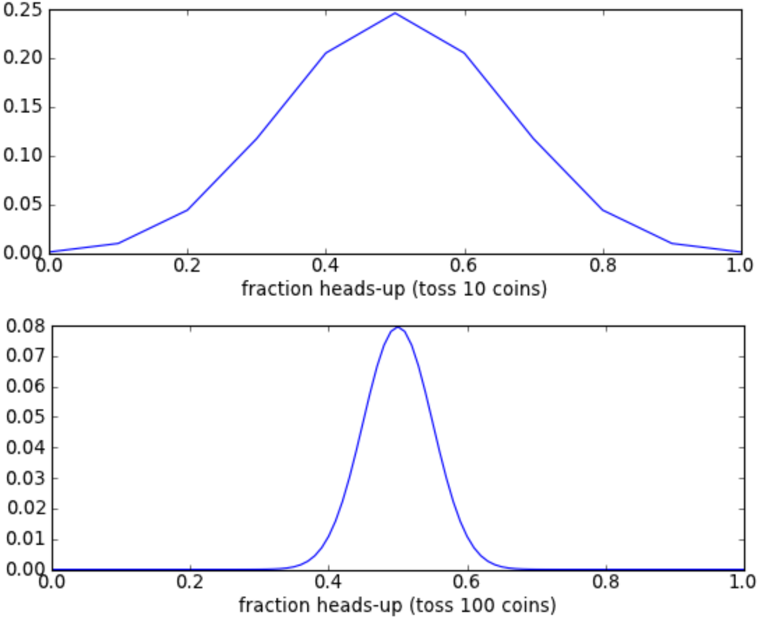


Let's take that same graph (for tossing 1000 coins) and change the x-axis so that it tells us **what fraction** of the coins lands heads-up.



Now let's see how the bottom graph changes as we vary the number of coins that we toss.

Toss 10 coins (top) or 100 coins (bottom)



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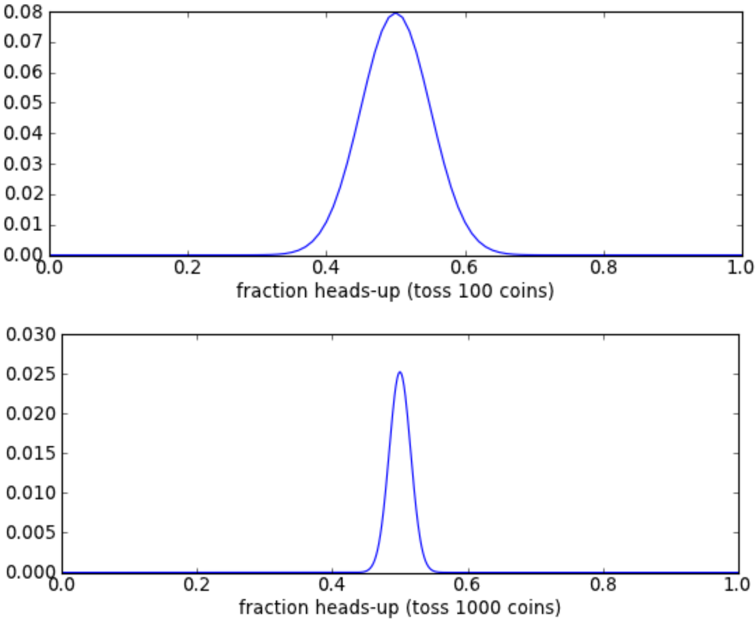
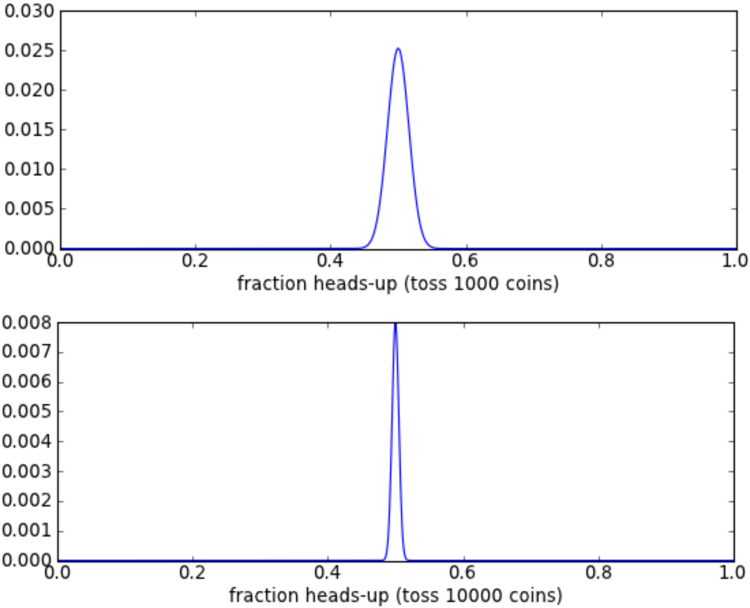
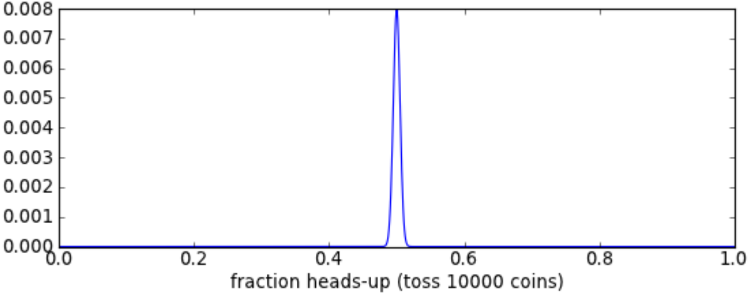
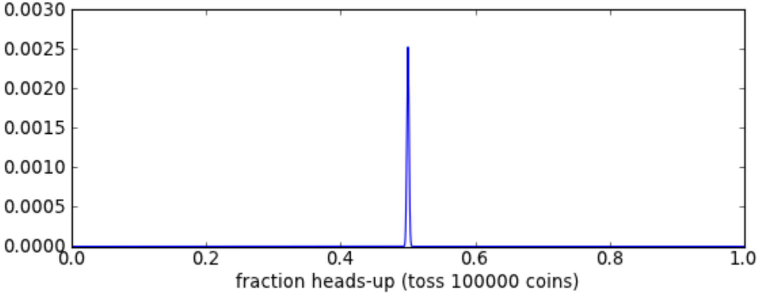


Toss 1000 coins (top) or 10000 coins (bottom)



Toss 10000 coins (top) or 100000 coins (bottom)





No matter how many coins you toss, the most probable outcome is that half of the coins will land heads-up. But for a small number of coins, it's not so unlikely to see a fraction that is quite different from 50%. As the number of coins increases, it becomes less and less probable that the fraction of heads is very different from 50%.

When you toss N coins, the number of values of m that have some reasonable probability of occurring is proportional to $\sqrt{N}$

So the number of heads that you expect to see goes roughly like

$$m \pm \delta m \quad = \quad \frac{N}{2} \; \times \; \left(1 \pm \frac{1}{\sqrt{N}}\right)$$

i.e. you expect to be within about 10% of 50/50 for 100 coins, to be within about 1% of 5000/5000 for 10000 coins, to be within about 0.1% of 500000/500000 for a million coins, etc.

$$m \pm \delta m \;=\; \frac{N}{2} \;\times\; \left(1 \pm \frac{1}{\sqrt{N}}\right)$$

If I toss $N = 100$ coins, I am not surprised to find any value $40 \leq m \leq 60$, but I would be very surprised to find $m = 30$.

If I toss 10^{20} coins, the fraction of heads I expect to find is, on average, 0.5. How far from 0.5 would be surprising?

$$\frac{1}{\sqrt{N}} = \frac{1}{\sqrt{10^{20}}} = 10^{-10}$$

So I would be surprised to find a fraction of heads that is less than 0.4999999999 or larger than 0.5000000001

If I poll 10000 architects and find that 50% agree and 50% disagree with the assertion that the spire atop the new One World Trade Center legitimately makes 1 WTC taller than Chicago's Willis (Sears) Tower, what is the "margin of error" of my survey?

(A) roughly ±10%

(B) roughly ±1%

(C) roughly ±0.1%

(D) roughly ±0.01%

Answer: roughly ±1%, because $1/\sqrt{10000} = 0.01$

Note for statistics buffs: I think if you calculate more carefully, the r.m.s. fluctuation is more like $\pm\frac{1}{2}\%$, so my rule-of-thumb ±1% roughly corresponds to the 95% confidence interval in this case.

Just to reiterate:

Suppose I repeatedly toss 1000 coins such that **on average** the number of heads-up coins is 500.

I don't always get exactly 500 heads. The range of heads-up coins that would be "not surprising" (or not suspicious) is approximately

$$(500 - \sqrt{500}) \quad \le \quad N_{\mathrm{heads}} \quad \le \quad (500 + \sqrt{500})$$

So anything in the 475–525 range would be unsurprising. Even 460 or 540 heads would not be too surprising. Finding 400 heads (or 600 heads) would be extremely surprising.

In general, when you count things that are randomly sampled, and you expect on average to count N of them, the "unsurprising" range of fluctuations from sample to sample is roughly $\pm\sqrt{N}$.

Suppose I survey 20000 people and find that 9970 of them prefer chocolate ice cream, while 10030 of them prefer vanilla ice cream.

If I repeat the survey (with different people, randomly chosen) again and again each week, roughly how much do I expect that 9970 number to vary, just because of the finite size of the number of people surveyed?

I don't expect to get exactly 9970 votes for chocolate each week. Instead, I expect the number to vary from survey to survey by about . . .

(A) varies by about ± 1

(B) varies by about ± 10

(C) varies by about ± 100

(D) varies by about ± 1000

Imagine that you have 10^{23} gas molecules in a box. Then you mark off half of the volume with a ruler and ask "What fraction of the molecules are in the left half of the box?"

This is like tossing 10^{23} coins and asking what fraction f is heads-up. You expect f to be almost exactly 0.5.

If you do the math, you expect something like

$$0.499999999997 < f < 0.500000000003$$

So for all practical purposes, exactly half of the molecules are in each half of the volume. That's called "equipartition of space."

The number of heads m (or equivalently the fraction of heads m/N) could be called the "macro state," or the observable state of the system. If I tossed 10^{20} coins that had one side painted black and one side painted white, I could measure the "macro state" without looking at every coin.

Similarly, the observable state ("macro state") of 10^{23} gas molecules is given by P, V, and N. (The "basic state," a.k.a. "micro state," is described by writing down **every particle's** position and velocity.)

The number of particles is so huge that the chance that all of the molecules are going to huddle up on one side of the room is beyond infinitesimal.

The most probable macrostate is the one with the largest number of basic states. Once a closed system has reached thermal equilibrium (e.g. once it has a well-defined temperature), it will stay in the most probable state or states.

Entropy

The logarithm of [the number, Ω, of different basic states that contribute to a given macrostate] is called the *entropy* of that macrostate:

$$S = \ln(\Omega)$$

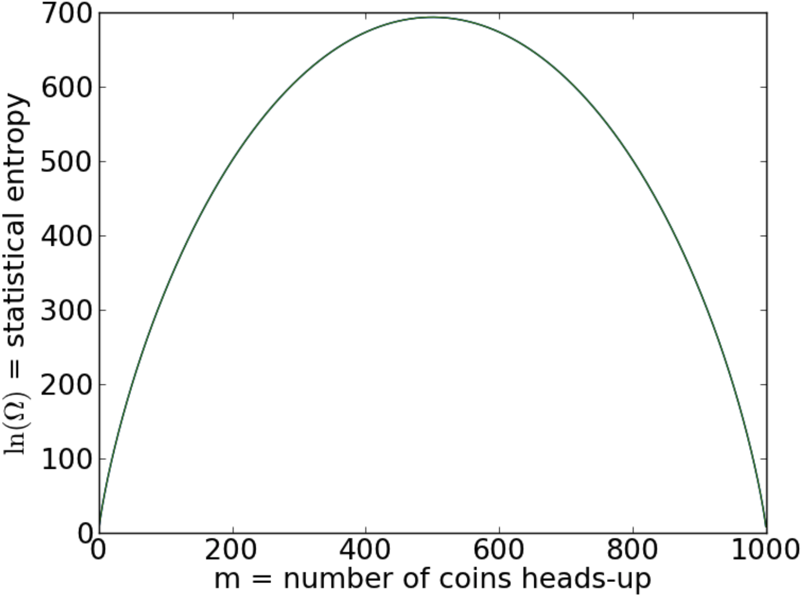
for the coin toss, the macrostate corresponds to knowing N and m, and Ω corresponds to $\frac{N!}{m!\ (N-m)!}$

Interesting nerd fact ("Stirling's formula"): as $N \to \infty$,

$$\ln(N!) \quad \to \quad N\ (\ln(N) - 1) \quad \approx \quad N\ \ln(N)$$

So the "statistical entropy," S, for m coins heads-up is

$$S = \ln(\Omega) = \ln\left(\frac{N!}{m!\ (N-m)!}\right) \approx N \ln N - m \ln m - (N-m) \ln(N-m)$$



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Once you reach “equilibrium,” i.e. the $m_{\rm heads} \approx 500$ state, you will never spontaneously go back to the $m_{\rm heads} \approx 0$ state. It’s just too improbable. Once you shake for long enough to get close to 500, you’ll stay pretty close to 500 (roughly within $500 \pm \sqrt{500}$ or so).

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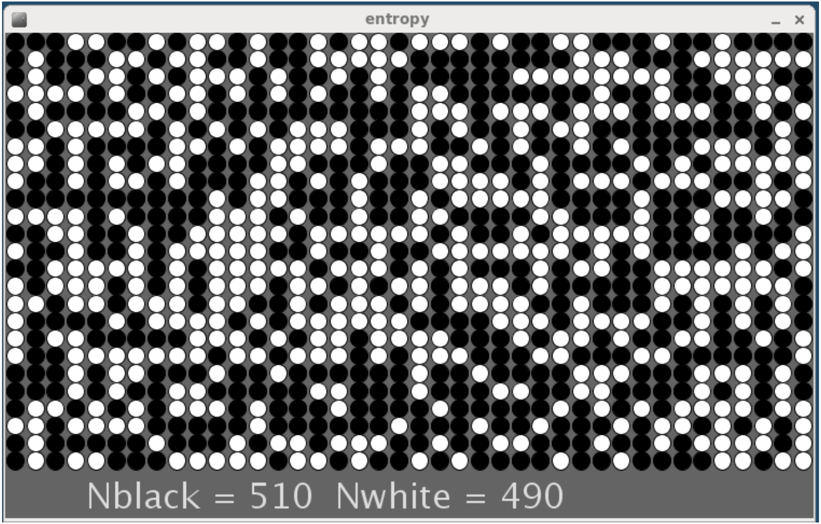
Similarly, the *second law of thermodynamics* states that the entropy of a closed system will never decrease with time. (It can stay the same or can increase.) The 2nd law is just a statement that a closed system evolves toward the most probable macrostate.

Entropy



Nblack = 510 Nwhite = 490

Entropy

Again, *second law of thermodynamics* ("the entropy law") states that the entropy of a closed system will never decrease with time. (It can stay the same or can increase.)

For a system that is not closed (i.e. it can exchange energy with its environment), the entropy law states that the combined entropy of the system + its environment will never decrease with time. (It can stay the same or can increase.) This is equivalent to treating "system + environment" as a (much larger) closed system.

Increases in entropy are associated with irreversible processes, like the dissipation of mechanical energy into heat when I drop a tennis ball and let it bounce until it comes to rest.

Reversible processes (i.e. processes for which a movie played either forward or backward looks like physics that is possible) correspond to zero change in entropy.

For a system that is in equilibrium with a thermal reservoir at temperature T, you can relate the system's change in entropy to the "energy transferred thermally" (a.k.a. heat) into the system:

$$\Delta S = \frac{Q}{k_B T}$$

or in other words,

$$Q = k_B T \Delta S$$

A net heat (Q) flow into a system increases the system's entropy. (Giancoli and Mazur have different conventions about the factor of k_B. Mazur uses "statistical entropy," which no units; Giancoli uses "thermodynamic entropy," which is in J/K (joules per kelvin).)

About HW problem 2 for today (Reynolds number):

If **the same flow (volume per unit time)** of a fluid passes through both wide and narrow sections of the pipe/duct/river/etc., the narrow section is more likely to be turbulent, hence more likely to be noisy. If you only partially close off an HVAC duct, so that (approximately) the same air flow must pass (at higher speed) through a reduced area, the air flow makes more noise.

Reynolds number:

$$Re \;=\; \frac{2r\overline{v}\rho}{\eta} \;\propto\; r\overline{v}$$

Flow rate (volume/time):

$$Q \;=\; A\overline{v} \;=\; \pi r^2\,\overline{v} \;\propto\; r^2\,\overline{v}$$

Suppose $r_1 = R$, $\overline{v}_1 = V$, and $r_2 = 2R$. To get same flow, $Q_2 = Q_1$, you need $\overline{v}_2 = (V/4)$. Then

$$\frac{Q_2}{Q_1} \;=\; \frac{r_2^2\,\overline{v}_2}{r_1^2\,\overline{v}_1} \;=\; \frac{(2R)^2(V/4)}{(R)^2(V)} \;=\; 1$$

But

$$\frac{Re_2}{Re_1} \;=\; \frac{r_2\,\overline{v}_2}{r_1\,\overline{v}_1} \;=\; \frac{(2R)(V/4)}{(R)(V)} \;=\; \frac{1}{2}$$

Same flow through wider duct has smaller Reynolds number, so is less turbulent. Consistent with pictures of river. (This argument depends on same flow rate: if making the pipe bigger just allows a bigger flow, then you're not reducing turbulence.)

Physics 9 — Monday, October 29, 2018

- So far three people plan to come along with me to Acentech's "3D Listening" demo Monday, 4pm, on 17th Street. We'll meet at the main DRL entrance at 3:25pm and walk briskly. We'll walk east on Walnut, then turn left on 17th to reach the Sofitel at 120 S. 17th. My mobile phone number is . . .
- For today, you read Richard Muller's chapter (written in 2009) on Climate Change, as well as his 2012 NYT op-ed.
- There is nothing you need to read for this Wednesday, though if you're interested I have some material from Muller's 2012 book *Energy for Future Presidents* that you could read for XC.