Physics 9 — Wednesday, October 31, 2018

- HW07 due Friday in class.
- HW help sessions: Wed 4–6pm DRL $\boxed{\textbf{4C2}}$ (Bill), Thu 6:30–8:30pm DRL 2C8 (Grace)
- There was no required reading for today, but if you're interested I have a newer Climate Change chapter from Muller's 2012 book *Energy for Future Presidents* that you could read for XC.

We've said that “dry air” is 78% N_2, 21% O_2, 1% Ar, 0.04% CO_2. What important constituent of ordinary air are we missing?

We've said that “dry air” is 78% N_2, 21% O_2, 1% Ar, 0.04% CO_2. What important constituent of ordinary air are we missing?

The fraction of water vapor in the air varies from region to region (climate) and from day to day (weather). The maximum possible fraction of water vapor in the air (before you get rain, snow, fog, etc.) varies with temperature.

We've said that "dry air" is 78% N_2, 21% O_2, 1% Ar, 0.04% CO_2. What important constituent of ordinary air are we missing?

The fraction of water vapor in the air varies from region to region (climate) and from day to day (weather). The maximum possible fraction of water vapor in the air (before you get rain, snow, fog, etc.) varies with temperature.

To discuss the quantity of one gas (e.g. water vapor) that is dissolved in another gas (e.g. air), it helps to introduce the concept of **partial pressure,** which is the pressure due to that constituent alone.

We've said that "dry air" is 78% N_2, 21% O_2, 1% Ar, 0.04% CO_2. What important constituent of ordinary air are we missing?

The fraction of water vapor in the air varies from region to region (climate) and from day to day (weather). The maximum possible fraction of water vapor in the air (before you get rain, snow, fog, etc.) varies with temperature.

To discuss the quantity of one gas (e.g. water vapor) that is dissolved in another gas (e.g. air), it helps to introduce the concept of **partial pressure,** which is the pressure due to that constituent alone.

Example: air at 1 atm is 21% O_2. What is the partial pressure of oxygen (in atmospheres)?

We've said that "dry air" is 78% N_2, 21% O_2, 1% Ar, 0.04% CO_2. What important constituent of ordinary air are we missing?

The fraction of water vapor in the air varies from region to region (climate) and from day to day (weather). The maximum possible fraction of water vapor in the air (before you get rain, snow, fog, etc.) varies with temperature.

To discuss the quantity of one gas (e.g. water vapor) that is dissolved in another gas (e.g. air), it helps to introduce the concept of **partial pressure,** which is the pressure due to that constituent alone.

Example: air at 1 atm is 21% O_2. What is the partial pressure of oxygen (in atmospheres)?

Answer: 0.21 atm, which is about 21300 $\mathrm{N/m^2}$.

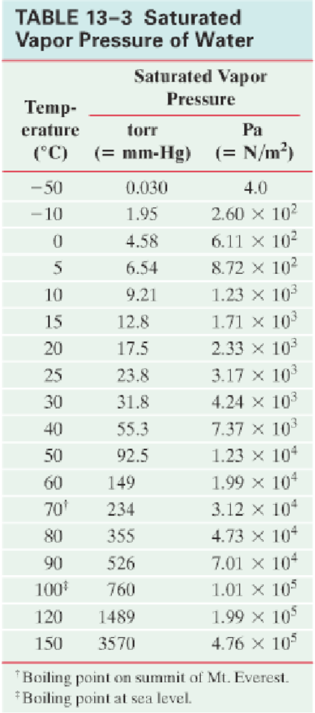


TABLE 13–3 Saturated Vapor Pressure of Water

Temperature (°C)	Saturated Vapor Pressure	
	torr (= mm-Hg)	Pa (= N/m^2)
−50	0.030	4.0
−10	1.95	2.60×10^2
0	4.58	6.11×10^2
5	6.54	8.72×10^2
10	9.21	1.23×10^3
15	12.8	1.71×10^3
20	17.5	2.33×10^3
25	23.8	3.17×10^3
30	31.8	4.24×10^3
40	55.3	7.37×10^3
50	92.5	1.23×10^4
60	149	1.99×10^4
70[†]	234	3.12×10^4
80	355	4.73×10^4
90	526	7.01×10^4
100[‡]	760	1.01×10^5
120	1489	1.99×10^5
150	3570	4.76×10^5

[†] Boiling point on summit of Mt. Everest.

[‡] Boiling point at sea level.

At a given temperature, the maximum possible partial pressure of water vapor is called the "**(saturated) vapor pressure of water.**"

When partial pressure of water vapor equals saturated vapor pressure, a pool of standing water is in equilibrium with water vapor: rate of evaporation (liquid to gas) equals rate of condensation (gas to liquid).

If partial pressure of water vapor exceeds the saturated vapor pressure (e.g. if you take saturated air and cool it), precipitation occurs.

Below this partial pressure, standing water will evaporate.

When saturated vapor pressure equals external atmospheric pressure, boiling occurs!

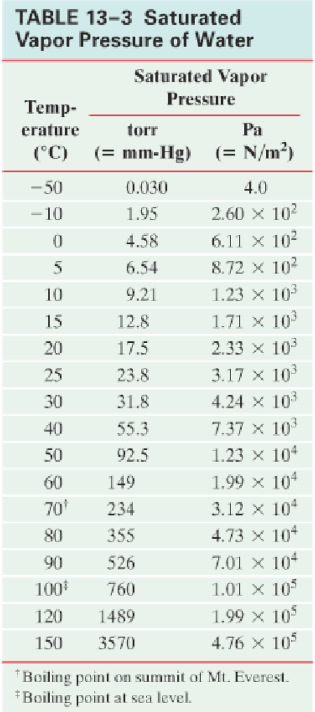


TABLE 13–3 Saturated Vapor Pressure of Water

Temperature ($^\circ$C)	Saturated Vapor Pressure	
	torr (= mm-Hg)	Pa (= N/m^2)
−50	0.030	4.0
−10	1.95	2.60×10^2
0	4.58	6.11×10^2
5	6.54	8.72×10^2
10	9.21	1.23×10^3
15	12.8	1.71×10^3
20	17.5	2.33×10^3
25	23.8	3.17×10^3
30	31.8	4.24×10^3
40	55.3	7.37×10^3
50	92.5	1.23×10^4
60	149	1.99×10^4
70[†]	234	3.12×10^4
80	355	4.73×10^4
90	526	7.01×10^4
100[‡]	760	1.01×10^5
120	1489	1.99×10^5
150	3570	4.76×10^5

[†] Boiling point on summit of Mt. Everest.

[‡] Boiling point at sea level.

In humid air, the rate at which sweat can evaporate from your skin (hence cooling your body, via latent heat) is lower than in dry air. So a hot, humid day feels more uncomfortable than a hot, dry day.

HVAC climate-control systems monitor and regulate “relative humidity” =

$$\frac{\text{partial pressure of water vapor}}{\text{saturated water vapor pressure}} \times 100\%$$

R.H. $\sim$ 40%-50% is most comfortable.

Q: if $T = 30^\circ\mathrm{C}$ and R.H.=50%, what is partial pressure of water vapor (in $\mathrm{N/m^2}$)?

In that case, what fraction of molecules in the air are H_2O molecules?

We learned last fall to associate *irreversible* processes (for which a movie played backwards looks impossible) with the dissipation of (coherent) mechanical energy into (incoherent) thermal energy.

- So you're not surprised that after a few bounces, the initial mechanical energy of a ball dissipates into thermal energy.
- But a movie of this process played backwards would look like an impossible situation.

Similarly, you're accustomed to seeing heat flow spontaneously from a warm object to a cooler object. That's why thermal insulation is needed to slow this process down.

- So you're not surprised when your glass of ice-water reaches room temperature after sitting out for an hour.
- But again a movie of this process played backwards would show something that looks impossible.

Both of these situations involve an **increase in entropy**; and hence are **irreversible.** Let's ponder the meaning of entropy and its connection to probability.

Entropy

You might have learned in a math class at some point that if I toss N coins, the probability of seeing m of those N coins land heads-up is given by the *binomial distribution,*

$$P(m,N) = \frac{N!}{m!\,(N-m)!}\,\left(\frac{1}{2}\right)^N$$

For $N = 2$, the probabilities for $m = 0, 1, 2$ are

$$\frac{1}{4},\quad \frac{2}{4},\quad \frac{1}{4}$$

For $N = 4$, the probabilities for $m = 0, 1, 2, 3, 4$ are

$$\frac{1}{16},\quad \frac{4}{16},\quad \frac{6}{16},\quad \frac{4}{16},\quad \frac{1}{16}$$

For $N = 8$, the probabilities for $m = 0, 1, 2, 3, 4, 5, 6, 7, 8$ are

$$\frac{1}{256},\quad \frac{8}{256},\quad \frac{28}{256},\quad \frac{56}{256},\quad \frac{70}{256},\quad \frac{56}{256},\quad \frac{28}{256},\quad \frac{8}{256},\quad \frac{1}{256}$$

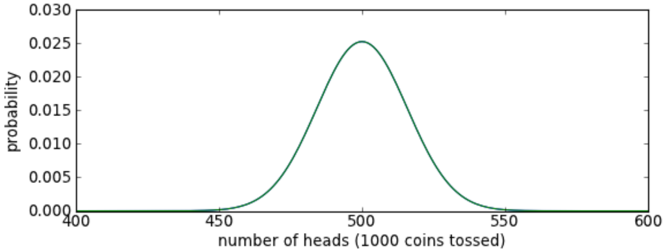
$$P(m,N) = \frac{N!}{m!\,(N-m)!}\left(\frac{1}{2}\right)^N$$

If I toss $N = 1000$ coins, the probability of finding m heads-up is

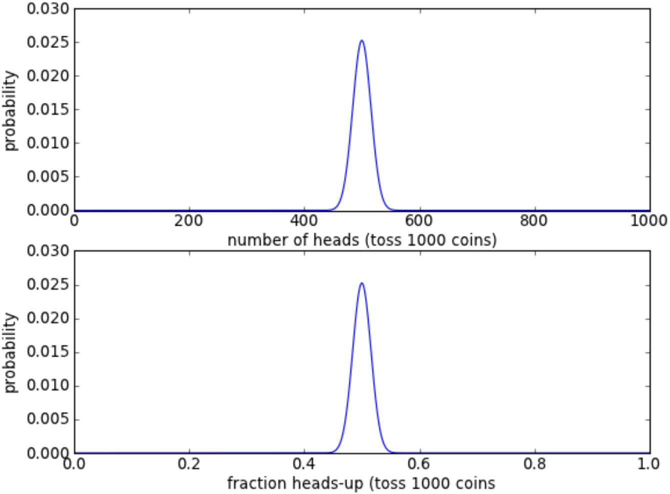
heads	probability
0	0.9×10^{-301}
1	0.9×10^{-298}
450	0.02%
480	1.1%
499	2.5%
500	2.5%

. . .

heads	probability
1000	0.9×10^{-301}
999	0.9×10^{-298}
550	0.02%
520	1.1%
501	2.5%

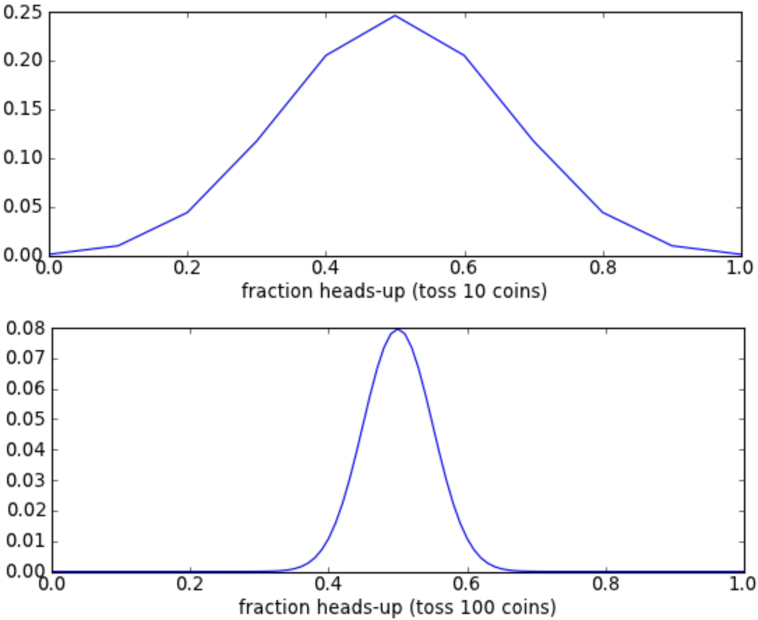


Let's take that same graph (for tossing 1000 coins) and change the x-axis so that it tells us **what fraction** of the coins lands heads-up.

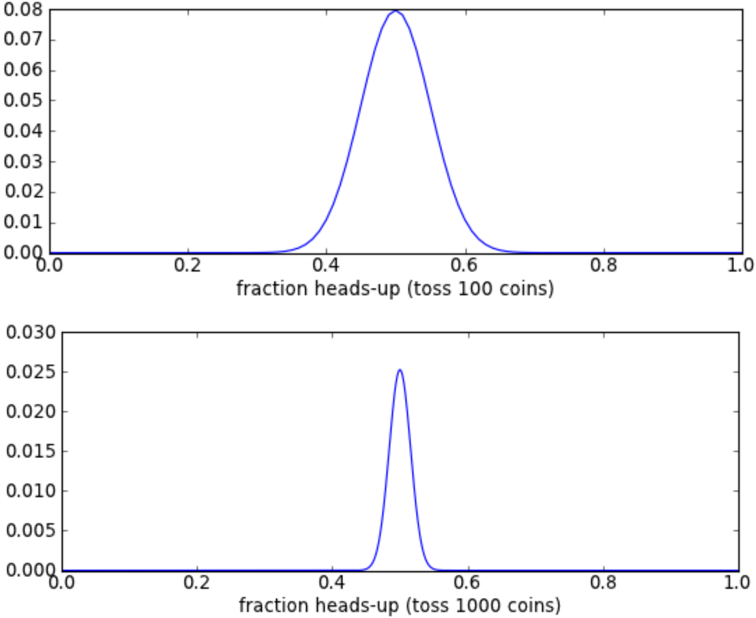


Now let's see how the bottom graph changes as we vary the number of coins that we toss.

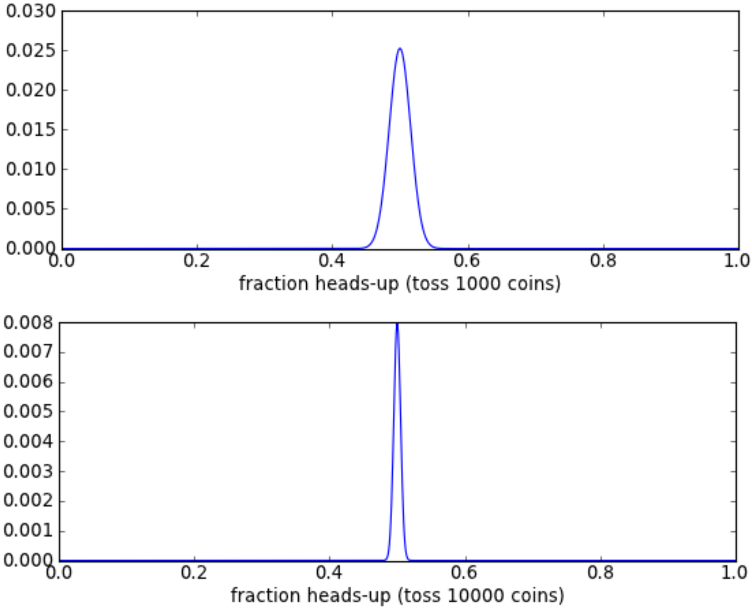
Toss 10 coins (top) or 100 coins (bottom)



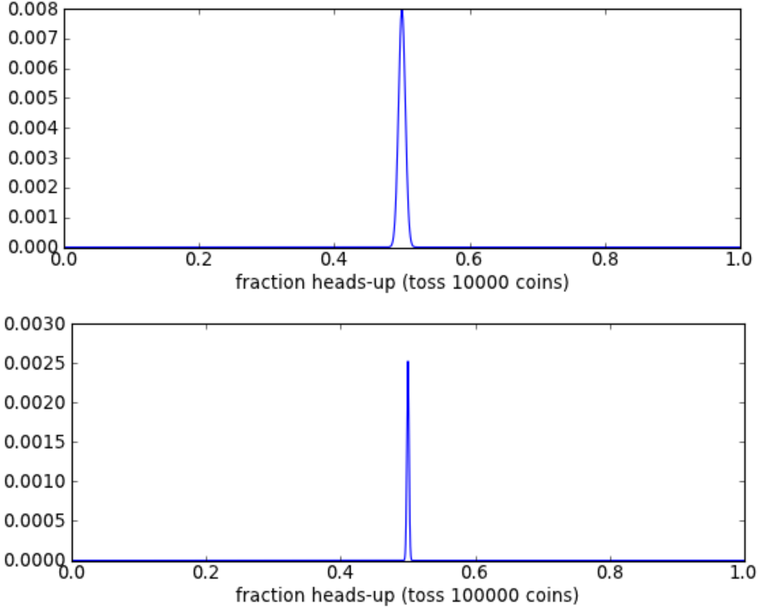
Toss 100 coins (top) or 1000 coins (bottom)



Toss 1000 coins (top) or 10000 coins (bottom)



Toss 10000 coins (top) or 100000 coins (bottom)



No matter how many coins you toss, the most probable outcome is that half of the coins will land heads-up. But for a small number of coins, it's not so unlikely to see a fraction that is quite different from 50%. As the number of coins increases, it becomes less and less probable that the fraction of heads is very different from 50%.

When you toss N coins, the number of values of m that have some reasonable probability of occurring is proportional to $\sqrt{N}$

So the number of heads that you expect to see goes roughly like

$$m \pm \delta m \;=\; \frac{N}{2} \times \left(1 \pm \frac{1}{\sqrt{N}}\right)$$

i.e. you expect to be within about 10% of 50/50 for 100 coins, to be within about 1% of 5000/5000 for 10000 coins, to be within about 0.1% of 500000/500000 for a million coins, etc.

$$m \pm \delta m \;=\; \frac{N}{2} \;\times\; \left(1 \pm \frac{1}{\sqrt{N}}\right)$$

If I toss $N = 100$ coins, I am not surprised to find any value $40 \leq m \leq 60$, but I would be very surprised to find $m = 30$.

If I toss 10^{20} coins, the fraction of heads I expect to find is, on average, 0.5. How far from 0.5 would be surprising?

$$\frac{1}{\sqrt{N}} = \frac{1}{\sqrt{10^{20}}} = 10^{-10}$$

So I would be surprised to find a fraction of heads that is less than about 0.4999999999 or larger than about 0.5000000001

If I poll 10000 architects and find that 50% agree and 50% disagree with the assertion that the spire atop the new One World Trade Center legitimately makes 1 WTC taller than Chicago's Willis (Sears) Tower, what is the "margin of error" of my survey?

(A) roughly ±10%
(B) roughly ±1%
(C) roughly ±0.1%
(D) roughly ±0.01%

Answer: roughly ±1%, because $1/\sqrt{10000} = 0.01$

Note for statistics buffs: I think if you calculate more carefully, the r.m.s. fluctuation is more like $\pm\frac{1}{2}\%$, so my rule-of-thumb ±1% roughly corresponds to the 95% confidence interval in this case.

Just to reiterate:

Suppose I repeatedly toss 1000 coins such that **on average** the number of heads-up coins is 500.

I don't always get exactly 500 heads. The range of heads-up coins that would be "not surprising" (or not suspicious) is approximately

$$(500 - \sqrt{500}) \quad \leq \quad N_{\text{heads}} \quad \leq \quad (500 + \sqrt{500})$$

So anything in the 475–525 range would be unsurprising. Even 460 or 540 heads would not be too surprising. Finding 400 heads (or 600 heads) would be extremely surprising.

In general, when you count things that are randomly sampled, and you expect on average to count N of them, the "unsurprising" range of fluctuations from sample to sample is roughly $\pm\sqrt{N}$.

Suppose I survey 20000 people and find that 9970 of them prefer chocolate ice cream, while 10030 of them prefer vanilla ice cream.

If I repeat the survey (with different people, randomly chosen) again and again each week, roughly how much do I expect that 9970 number to vary, just because of the finite size of the number of people surveyed?

I don't expect to get exactly 9970 votes for chocolate each week. Instead, I expect the number to vary from survey to survey by about . . .

(A) varies by about ±1

(B) varies by about ±10

(C) varies by about ±100

(D) varies by about ±1000

Imagine that you have 10^{23} gas molecules in a box. Then you mark off half of the volume with a ruler and ask “What fraction of the molecules are in the left half of the box?”

This is like tossing 10^{23} coins and asking what fraction f is heads-up. You expect f to be almost exactly 0.5.

If you do the math, you expect something like

$$0.499999999997 < f < 0.500000000003$$

So for all practical purposes, exactly half of the molecules are in each half of the volume. That’s called “equipartition of space.”

The number of heads m (or equivalently the fraction of heads m/N) could be called the “macro state,” or the observable state of the system. If I tossed 10^{20} coins that had one side painted black and one side painted white, I could measure the “macro state” without looking at every coin.

Similarly, the observable state (“macro state”) of 10^{23} gas molecules is given by P, V, and N. (The “basic state,” a.k.a. “micro state,” is described by writing down **every particle’s** position and velocity.)

The number of particles is so huge that the chance that all of the molecules are going to huddle up on one side of the room is beyond infinitesimal.

The most probable macrostate is the one with the largest number of basic states. Once a closed system has reached thermal equilibrium (e.g. once it has a well-defined temperature), it will stay in the most probable state or states.

Entropy

The logarithm of [the number, Ω, of different basic states that contribute to a given macrostate] is called the *entropy* of that macrostate:

$$S = \ln(\Omega)$$

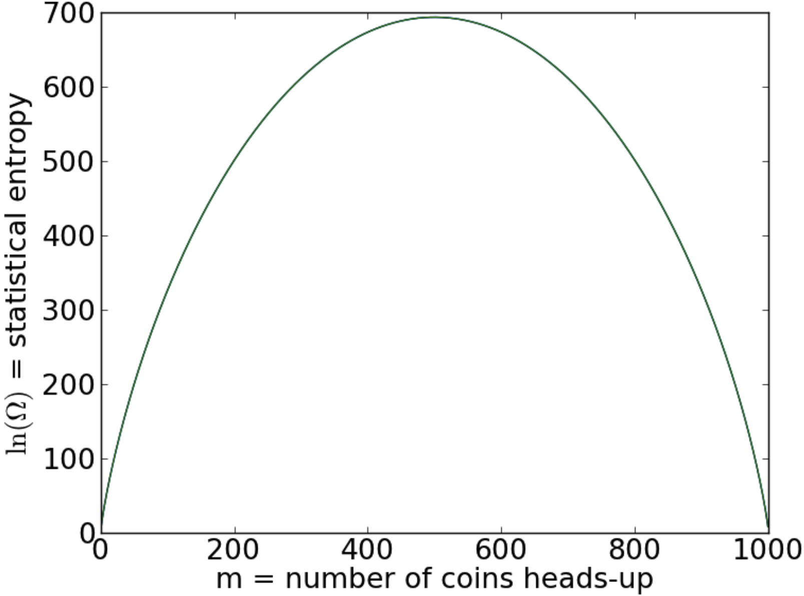
for the coin toss, the macrostate corresponds to knowing N and m, and Ω corresponds to $\frac{N!}{m!\ (N-m)!}$

Interesting nerd fact ("Stirling's formula"): as $N \to \infty$,

$$\ln(N!) \quad \to \quad N\ \left(\ln(N) - 1\right) \quad \approx \quad N\ \ln(N)$$

So the "statistical entropy," S, for m coins heads-up is

$$S = \ln(\Omega) = \ln\left(\frac{N!}{m!\ (N-m)!}\right) \approx N \ln N - m \ln m - (N-m) \ln(N-m)$$



$$S = \ln(\Omega) = \ln\left(\frac{N!}{m!\,(N-m)!}\right) \approx N \ln N - m \ln m - (N-m) \ln(N-m)$$

Suppose I start out with a jar of 1000 coins that are carefully arranged to be entirely heads-up, and I shake the jar for a long time. What will I find?

Suppose I start out with a jar of 1000 coins that are carefully arranged to be entirely heads-up, and I shake the jar for a long time. What will I find?

Shaking will tend toward more probable (higher entropy) states. After shaking, I will find that the entropy has increased to the largest possible value (plus or minus very small fluctuations).

Suppose I start out with a jar of 1000 coins that are carefully arranged to be entirely heads-up, and I shake the jar for a long time. What will I find?

Shaking will tend toward more probable (higher entropy) states. After shaking, I will find that the entropy has increased to the largest possible value (plus or minus very small fluctuations).

Once you reach "equilibrium," i.e. the $m_{\text{heads}} \approx 500$ state, you will never spontaneously go back to the $m_{\text{heads}} \approx 0$ state. It's just too improbable. Once you shake for long enough to get close to 500, you'll stay pretty close to 500 (roughly within $500 \pm \sqrt{500}$ or so).

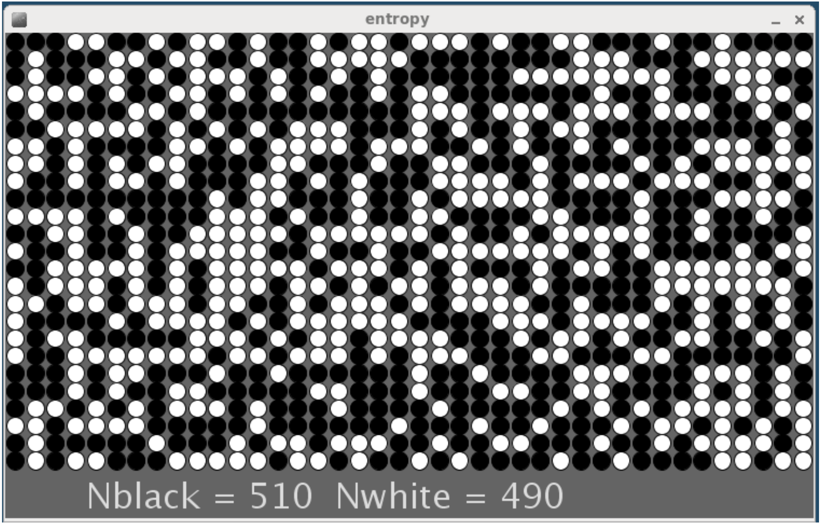
Suppose I start out with a jar of 1000 coins that are carefully arranged to be entirely heads-up, and I shake the jar for a long time. What will I find?

Shaking will tend toward more probable (higher entropy) states. After shaking, I will find that the entropy has increased to the largest possible value (plus or minus very small fluctuations).

Once you reach "equilibrium," i.e. the $m_{\mathrm{heads}} \approx 500$ state, you will never spontaneously go back to the $m_{\mathrm{heads}} \approx 0$ state. It's just too improbable. Once you shake for long enough to get close to 500, you'll stay pretty close to 500 (roughly within $500 \pm \sqrt{500}$ or so).

Similarly, the *second law of thermodynamics* states that the entropy of a closed system will never decrease with time. (It can stay the same or can increase.) The 2nd law is just a statement that a closed system evolves toward the most probable macrostate.

Entropy



Nblack = 510 Nwhite = 490

Entropy

Again, *second law of thermodynamics* ("the entropy law") states that the entropy of a closed system will never decrease with time. (It can stay the same or can increase.)

For a system that is not closed (i.e. it can exchange energy with its environment), the entropy law states that the combined entropy of the system + its environment will never decrease with time. (It can stay the same or can increase.) This is equivalent to treating "system + environment" as a (much larger) closed system.

Increases in entropy are associated with irreversible processes, like the dissipation of mechanical energy into heat when I drop a tennis ball and let it bounce until it comes to rest.

Reversible processes (i.e. processes for which a movie played either forward or backward looks like physics that is possible) correspond to zero change in entropy.

For a system that is in equilibrium with a thermal reservoir at temperature T, you can relate the system's change in entropy to the "energy transferred thermally" (a.k.a. heat) into the system:

$$\Delta S = \frac{Q}{k_B T}$$

or in other words,

$$Q = k_B T \Delta S$$

A net heat (Q) flow into a system increases the system's entropy. (Giancoli and Mazur have different conventions about the factor of k_B. Mazur uses "statistical entropy," which no units; Giancoli uses "thermodynamic entropy," which is in J/K (joules per kelvin).)

For an ideal gas, we learned

$$PV = Nk_B T$$

The thermal energy of the gas (if monatomic like He, Ar, etc.) is

$$E_{\mathrm{thermal}} \;=\; \frac{3}{2} Nk_B T \;=\; \frac{3}{2} PV$$

(Complication that we'll ignore: the $\frac{3}{2}$ becomes $\frac{d}{2}$ for the non-monatomic case.)

If I compress the gas without letting any heat escape, I will increase its thermal energy. You know from experience (e.g. bicycle pump) that work (W) is required to compress a gas: as I push down on a piston, I exert a force that opposes the gas pressure.

So we can increase the energy of a gas by doing mechanical work (W) on the gas, which decreases the gas's volume:

$$W_{(\mathrm{ON\ gas})} = -\int P\,\mathrm{d}V$$

$W > 0$ (work done ON the gas) increases the gas's energy and decreases the gas's volume.

You also know from experience that you can increase the temperature (and therefore the energy) of the gas by **heating** it, i.e. by putting it in contact with an object of higher temperature.

The transfer of incoherent thermal energy into a system (usually because of a temperature difference) is called **heat** (Q).

FYI: Don't confuse "heat" (Q) with "thermal energy" (E_{thermal}, a.k.a. U). Because the word "heat" is used ambiguously, Mazur's book refers to Q as "energy transferred thermally."

So we have two ways to add energy to a gas: W and Q.

$$\Delta E_{\text{thermal}} = W + Q$$

W is due to (coherent) mechanical interactions, like a piston moving up and down or an electric motor pumping on the gas.

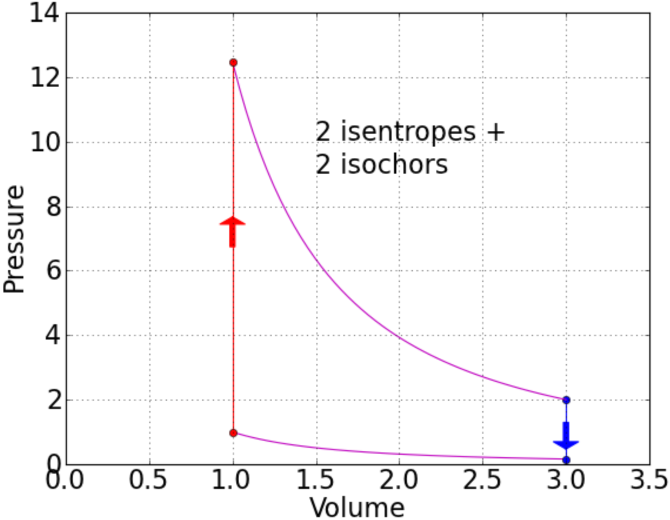
Q is due to (incoherent) thermal interactions, like the jiggling molecules of an adjacent heat bath (thermal reservoir) causing the molecules of the gas to reach the temperature of the heat bath.

- If $Q = 0$ the entropy of the gas does not change.
- If $Q > 0$ the entropy of the gas increases.
- If $Q < 0$ the entropy of the gas decreases.
- For heat transfer Q into the gas at constant temperature T,

$$\Delta S_{\text{gas}} = \frac{Q}{k_B T}$$

$Q < 0$ is only possible if there is some compensating change in entropy somewhere else, such that the overall $\Delta S \geq 0$.

Heating ($Q > 0$) or cooling ($Q < 0$) the gas while $W = 0$ looks like $\Delta V = 0$ (isochore, "constant volume"). Doing work on the gas ($W > 0$) or letting the gas do work ($W < 0$) while $Q = 0$ looks like $P_f V_f^{(5/3)} = P_i V_i^{(5/3)}$ (isentrope, "constant entropy"). Giancoli says "adiabatic" where Mazur says "isentropic."



In the **unlikely event** that anyone wants to know where the $\frac{5}{3}$ in $PV^{(5/3)}$ comes from, it's because (for a monatomic gas, $d = 3$)

$$0 = \Delta S = N \ln\left(\frac{V_f}{V_i}\right) + \frac{3}{2} N \ln\left(\frac{T_f}{T_i}\right)$$

then using the ideal gas law, $PV \propto T$,

$$0 = \Delta S = N \ln\left(\frac{V_f}{V_i}\right) + \frac{3}{2} N \ln\left(\frac{P_f V_f}{P_i V_i}\right)$$

$$0 = \Delta S = \frac{5}{2} N \ln\left(\frac{V_f}{V_i}\right) + \frac{3}{2} N \ln\left(\frac{P_f}{P_i}\right)$$

$$1 = \left(\frac{V_f}{V_i}\right)^5 \times \left(\frac{P_f}{P_i}\right)^3$$

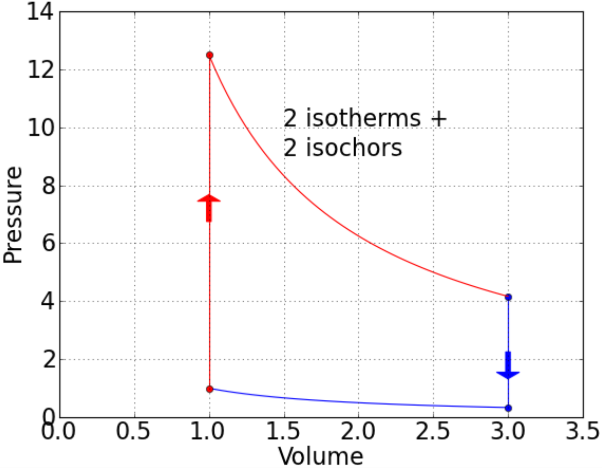
so finally (for an isentropic process on a monatomic gas)

$$P_i V_i^{(5/3)} = P_f V_f^{(5/3)}$$

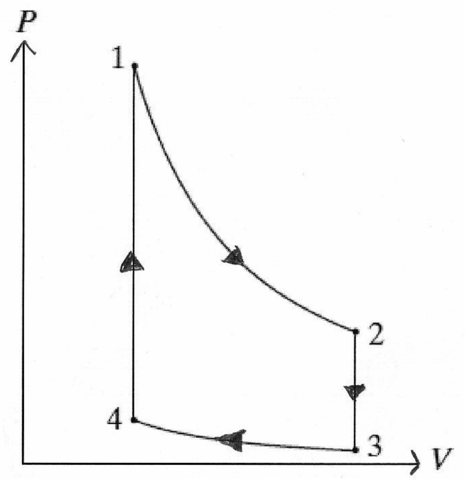
The starting point was Mazur eq 19.61. But you don't need to know this!

Heating ($Q > 0$) or cooling ($Q < 0$) the gas while $W = 0$ looks like $\Delta V = 0$ (isochore).

Doing work on the gas ($W > 0$) or letting the gas do work ($W < 0$) at constant temperature T looks like $P_f V_f = P_i V_i$ (isotherm). (For an engine, $W > 0$ happens at low T.)



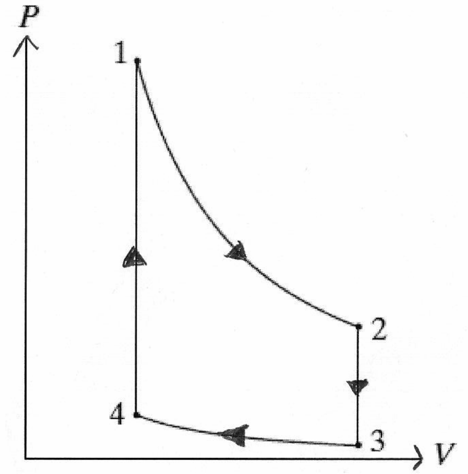
This cycle consists of two isotherms ($\Delta T = 0$) and two isochores ($\Delta V = 0$). The cycle is $4 \to 1 \to 2 \to 3 \to 4 \to \ldots$ On which parts of the cycle is positive work done ON the gas ($W_{\text{in}} > 0$)?



(A) $4 \to 1$ (B) $1 \to 2$ (C) $2 \to 3$ (D) $3 \to 4$

(E) $1 \to 2$ and $3 \to 4$ (F) $4 \to 1$ and $2 \to 3$

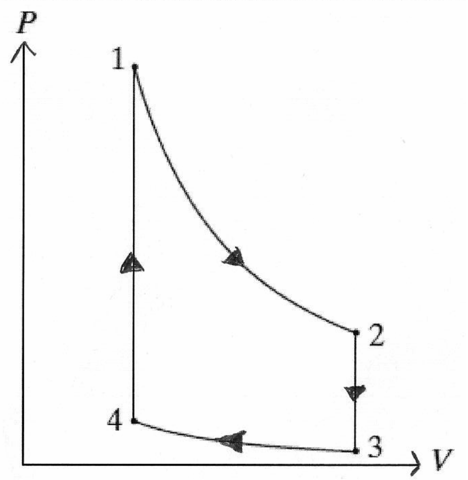
This cycle consists of two isotherms ($\Delta T = 0$) and two isochores ($\Delta V = 0$). The cycle is $4 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow \ldots$. On which parts of the cycle is positive work done BY the gas ($W_{\mathrm{out}} > 0$)?



(A) $4 \rightarrow 1$ (B) $1 \rightarrow 2$ (C) $2 \rightarrow 3$ (D) $3 \rightarrow 4$

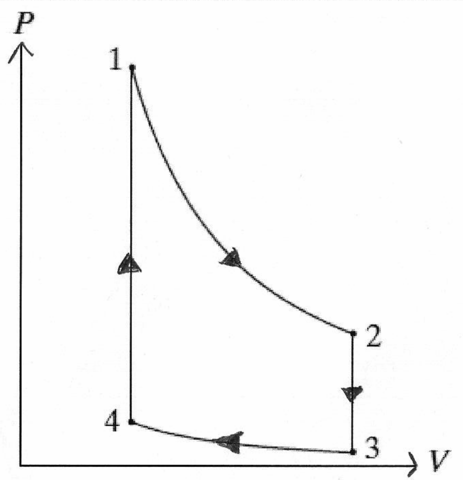
(E) $1 \rightarrow 2$ and $3 \rightarrow 4$ (F) $4 \rightarrow 1$ and $2 \rightarrow 3$

This cycle consists of two isotherms ($\Delta T = 0$) and two isochores ($\Delta V = 0$). The cycle is $4 \to 1 \to 2 \to 3 \to 4 \to \ldots$. Over one complete cycle, is W_{out} larger than, smaller than, or equal to W_{in}?

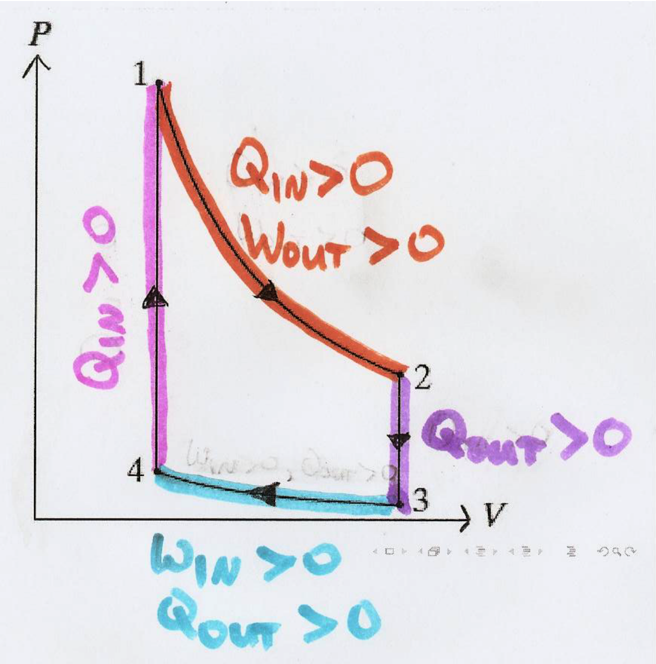


(A) $W_{\text{out}} > W_{\text{in}}$ (B) $W_{\text{out}} < W_{\text{in}}$ (C) $W_{\text{out}} = W_{\text{in}}$

This cycle consists of two isotherms ($\Delta T = 0$) and two isochores ($\Delta V = 0$). The cycle is $4 \to 1 \to 2 \to 3 \to 4 \to \ldots$. On which parts of the cycle is the internal energy of the gas constant?



(A) $4 \to 1$ (B) $1 \to 2$ (C) $2 \to 3$ (D) $3 \to 4$
(E) $1 \to 2$ and $3 \to 4$ (F) $4 \to 1$ and $2 \to 3$ (G) none


P
V
1
2
3
4
Q_IN > 0
W_OUT > 0
Q_IN > 0
Q_OUT > 0
W_IN > 0, Q_OUT > 0
W_IN > 0
Q_OUT > 0

Rules for “steady devices” (engine, heat pump, etc.)

- Over one cycle $\Delta E_{\mathrm{thermal}} = W + Q = 0$ (the energy of the gas returns to the value at which it started):

$$W_{\mathrm{input}} + Q_{\mathrm{input}} = W_{\mathrm{output}} + Q_{\mathrm{output}}$$

- The total entropy cannot decrease. Since the gas returns to its initial state at the end of each cycle ($\Delta S_{\mathrm{cycle}} = 0$), this implies that the entropy of the environment cannot decrease:

$$\Delta S_{\mathrm{environment}} = \frac{Q_{\mathrm{output}}}{k_B T_{\mathrm{output}}} - \frac{Q_{\mathrm{input}}}{k_B T_{\mathrm{input}}} \geq 0$$

- Notice that moving heat from the device out to the environment increases S_{env}, while moving heat from the environment in to the device decreases S_{env}.

$$\frac{Q_{\mathrm{output}}}{T_{\mathrm{output}}} \geq \frac{Q_{\mathrm{input}}}{T_{\mathrm{input}}}$$

- If you're trying to do useful work with the heat (Q_{input}), some of the heat (Q_{output}) is always thrown away as a by-product.

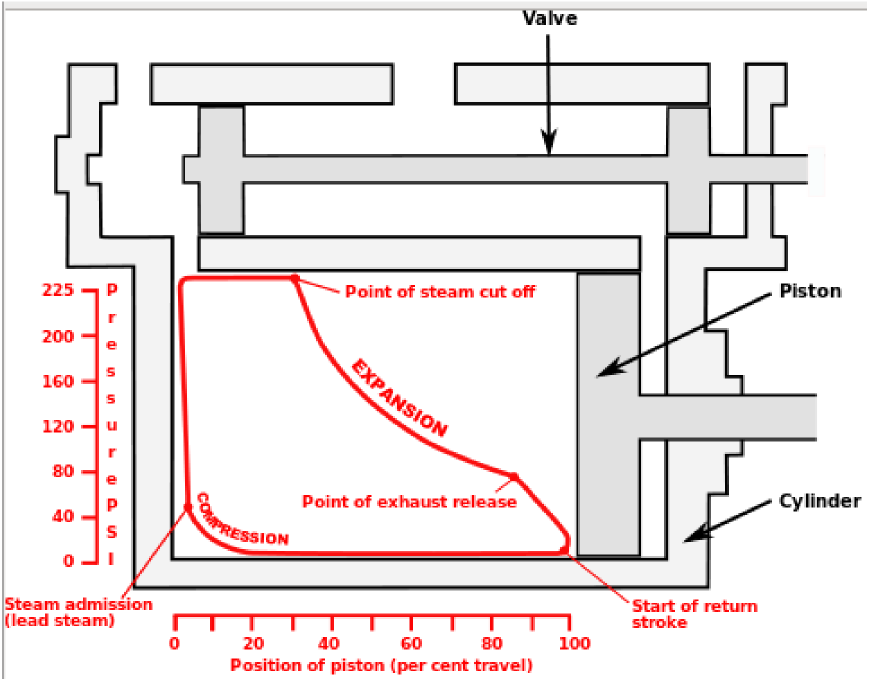
Engine efficiency

- What you "pay" to run the engine is $Q_{\rm input}$, which comes from e.g. burning fuel.
- What you want from the engine is mechanical work $W_{\rm output} - W_{\rm input} = -W$
  (Mazur and Giancoli have different sign conventions for W.)
- The **efficiency** of a heat engine is defined as

$$\boxed{\eta = \frac{W_{\rm output} - W_{\rm input}}{Q_{\rm input}}}$$

- An engine takes in $Q_{\rm input}$ at some high temperature $T_{\rm input}$ and exhausts waste heat $Q_{\rm output}$ at some low temperature $T_{\rm output}$. Engines need $T_{\rm input} > T_{\rm output}$ and work best when $T_{\rm input} \gg T_{\rm output}$.
- The entropy law says that engines can never do better than the theoretical $\Delta S_{\rm environment} = 0$ case ("reversible" engine):

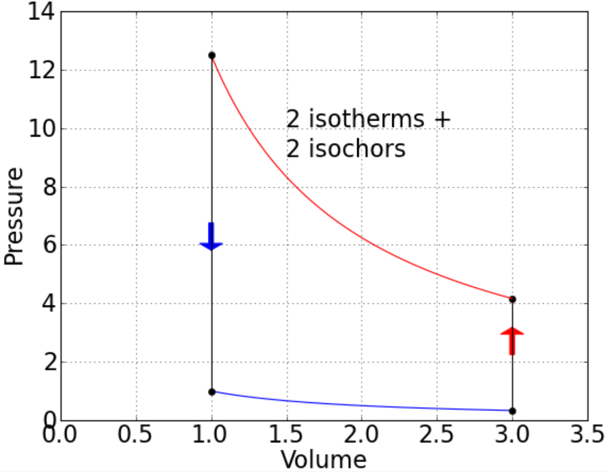
$$\eta \;\le\; 1 - \frac{T_{\rm output}}{T_{\rm input}} \qquad \boxed{\text{efficiency} \;\le\; 1 - \frac{T_{\rm cold}}{T_{\rm hot}}}$$


Valve
Piston
Cylinder
Point of steam cut off
EXPANSION
Point of exhaust release
COMPRESSION
Steam admission
(lead steam)
Start of return
stroke
225
200
160
120
80
40
0
Pressure pSI
0
20
40
60
80
100
Position of piston (per cent travel)

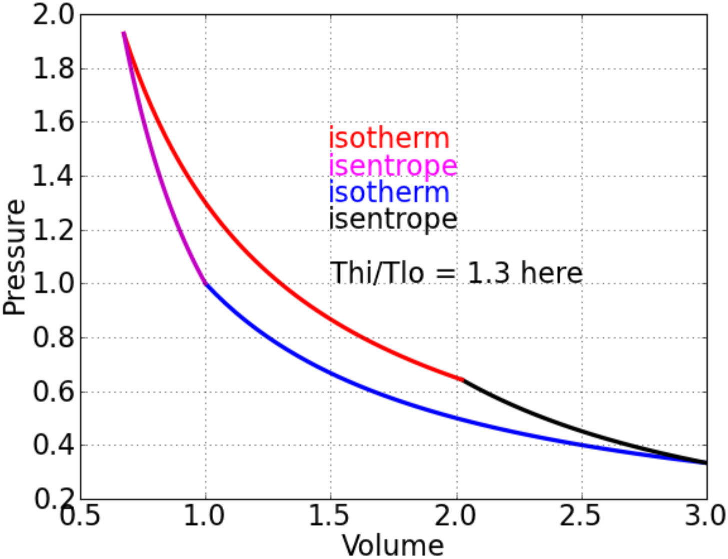
Refrigerator: PV diagram goes counterclockwise

Dumb example of fridge (but easy to illustrate with coffee cans)

Doing work on the gas ($W > 0$) or letting the gas do work ($W < 0$) at constant temperature T looks like $P_f V_f = P_i V_i$ (isotherm). (For a fridge, $W > 0$ happens at high T.)



This is a “reversible” ($\Delta S_{\mathrm{env}} = 0$) fridge cycle. Doing work on the gas ($W > 0$) or letting the gas do work ($W < 0$) at constant temperature T looks like $P_f V_f = P_i V_i$ (isotherm). Does anyone know the name of the cycle has two isotherms and two isentropes?



If I compress a gas (or let it expand) while the gas is sealed in a thermally insulated vessel (like a thermos bottle with a piston), so that the gas can't exchange heat with the environment, is that process called
(a) “isothermal” or
(b) “isentropic” (a.k.a. “adiabatic”)?

If I compress a gas (or let it expand) while the gas is in thermal contact with in a large container of water (a “heat bath” a.k.a. “thermal reservoir”), so that the gas maintains the same constant temperature as the reservoir, is that process called
(a) “isothermal” or
(b) “isentropic” (a.k.a. “adiabatic”)?

Refrigerator "coefficient of performance"

- What you "pay" to run the fridge is W, which comes from the "compressor's" electric motor.
- What you want from the fridge is the cooling: $Q_{\rm input}$
- The **coefficient of performance** of a fridge is defined as

$$\boxed{{\rm COP}_{\rm cooling} = \frac{Q_{\rm input}}{W}}$$

- A fridge performs best when the kitchen ($T_{\rm output}$) is not too much warmer than the desired fridge temperature ($T_{\rm input}$)! Unlike an engine, a fridge prefers a small ΔT.
- The entropy law dictates that fridges can never do better than the theoretical $\Delta S_{\rm env} = 0$ case (ideal "reversible" fridge):

$${\rm COP}_{\rm cooling} \le \frac{T_{\rm input}}{T_{\rm output} - T_{\rm input}} \qquad \boxed{{\rm COP}_{\rm cooling} \le \frac{T_{\rm cold}}{T_{\rm hot} - T_{\rm cold}}}$$

Heat pump "coefficient of performance"

- What you "pay" to run the heat pump is W, which comes from an electric motor — same as with a fridge.
- What you want from heat pump is the **heating**: Q_{output}
- The **coefficient of performance** of a heat pump is

$$\mathrm{COP}_{\mathrm{heating}} = \frac{Q_{\mathrm{output}}}{W}$$

- A heat pump performs best when the house (T_{output}) is not too much warmer than the heat-exchange-coil (T_{input})! A heat pump prefers a small ΔT. (So bury coil underground.)
- The entropy law dictates that heat pumps can never do better than the theoretical $\Delta S_{\mathrm{env}} = 0$ case (ideal "reversible" pump):

$$\mathrm{COP}_{\mathrm{heating}} \le \frac{T_{\mathrm{output}}}{T_{\mathrm{output}} - T_{\mathrm{input}}} \qquad \boxed{\mathrm{COP}_{\mathrm{heating}} \le \frac{T_{\mathrm{hot}}}{T_{\mathrm{hot}} - T_{\mathrm{cold}}}}$$

- **Confusingly,** T_{output} (to which heat is output) is the indoor temperature, and T_{input} (from which heat is input) is the outdoor temperature!

About HW problem 2 for today (Reynolds number):

If **the same flow (volume per unit time)** of a fluid passes through both wide and narrow sections of the pipe/duct/river/etc., the narrow section is more likely to be turbulent, hence more likely to be noisy. If you only partially close off an HVAC duct, so that (approximately) the same air flow must pass (at higher speed) through a reduced area, the air flow makes more noise.

Reynolds number:

$$Re = \frac{2r\overline{v}\rho}{\eta} \propto r\overline{v}$$

Flow rate (volume/time):

$$Q = A\overline{v} = \pi r^2 \, \overline{v} \propto r^2 \, \overline{v}$$

Suppose $r_1 = R$, $\overline{v}_1 = V$, and $r_2 = 2R$. To get same flow, $Q_2 = Q_1$, you need $\overline{v}_2 = (V/4)$. Then

$$\frac{Q_2}{Q_1} = \frac{r_2^2 \, \overline{v}_2}{r_1^2 \, \overline{v}_1} = \frac{(2R)^2(V/4)}{(R)^2(V)} = 1$$

But

$$\frac{Re_2}{Re_1} = \frac{r_2 \, \overline{v}_2}{r_1 \, \overline{v}_1} = \frac{(2R)(V/4)}{(R)(V)} = \frac{1}{2}$$

Same flow through wider duct has smaller Reynolds number, so is less turbulent. Consistent with pictures of river. (This argument depends on same flow rate: if making the pipe bigger just allows a bigger flow, then you're not reducing turbulence.)

Physics 9 — Wednesday, October 31, 2018

- HW07 due Friday in class.
- HW help sessions: Wed 4–6pm DRL **4C2** (Bill), Thu 6:30–8:30pm DRL 2C8 (Grace)
- There was no required reading for today, but if you're interested I have a newer Climate Change chapter from Muller's 2012 book *Energy for Future Presidents* that you could read for XC.