Physics 9 — Friday, November 2, 2018

- Turn in HW07. Pick up handout for HW08, due next Friday.
- For Monday, read **Eric Mazur**'s chapter 22 (Electric Interactions) — PDF on Canvas.
- I have a large number of supplemental chapters you could read for XC, if interested:
  - A newer Climate Change chapter from Muller's 2012 book *Energy for Future Presidents*.
  - Muller's chapters on chain reactions (exponential growth) and radioactivity.
  - Tutorials on using Mathematica to do math calculations, algebra, integrals, graphs, and much more.
  - Different chapters on architectural acoustics.
  - A more mathematical look at entropy & thermodymamics.
  - Giancoli's chapter on astronomy.
- What needs to be done for me (and maybe for some of you who are not architects) to install (legally) Rhinoceros and Grasshopper? **Pachyderm acoustics** is a free/open-source acoustical modeling software that runs on top of Rhino.

Entropy

The logarithm of [the number, Ω, of different basic states that contribute to a given macrostate] is called the *entropy* of that macrostate:

$$S = \ln(\Omega)$$

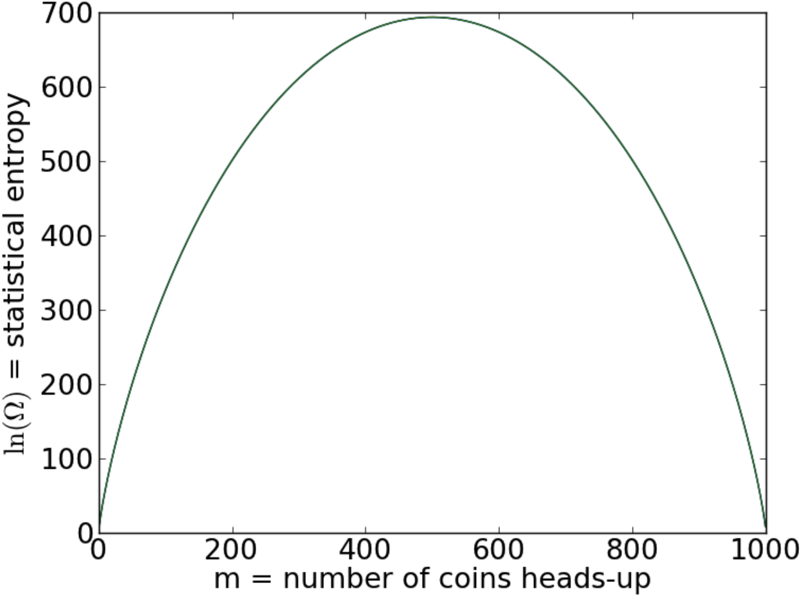
for the coin toss, the macrostate corresponds to knowing N and m, and Ω corresponds to $\frac{N!}{m!\ (N-m)!}$

Interesting nerd fact ("Stirling's formula"): as $N \to \infty$,

$$\ln(N!) \quad \to \quad N\ (\ln(N) - 1) \quad \approx \quad N\ \ln(N)$$

So the "statistical entropy," S, for m coins heads-up is

$$S = \ln(\Omega) = \ln\left(\frac{N!}{m!\ (N-m)!}\right) \approx N \ln N - m \ln m - (N-m) \ln(N-m)$$



$$S = \ln(\Omega) = \ln\left(\frac{N!}{m!\,(N-m)!}\right) \approx N\ln N - m\ln m - (N-m)\ln(N-m)$$

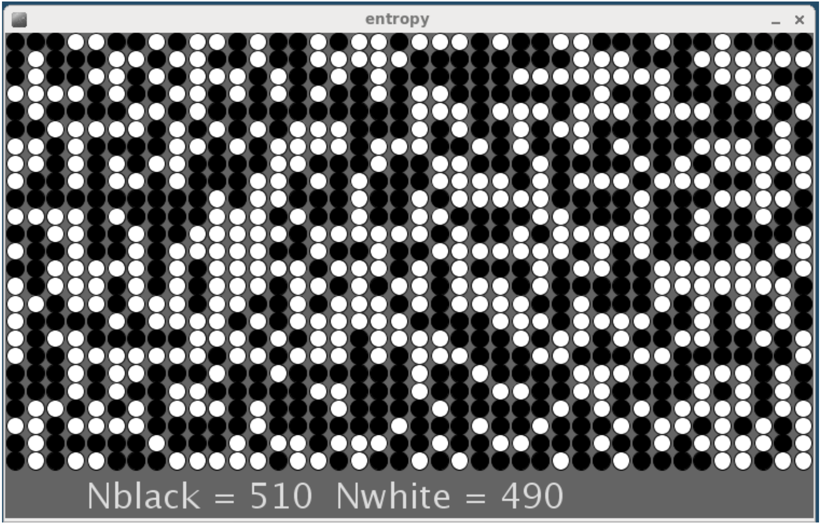
Suppose I start out with a jar of 1000 coins that are carefully arranged to be entirely heads-up, and I shake the jar for a long time. What will I find?

Shaking will tend toward more probable (higher entropy) states. After shaking, I will find that the entropy has increased to the largest possible value (plus or minus very small fluctuations).

Once you reach "equilibrium," i.e. the $m_{\text{heads}} \approx 500$ state, you will never spontaneously go back to the $m_{\text{heads}} \approx 0$ state. It's just too improbable. Once you shake for long enough to get close to 500, you'll stay pretty close to 500 (roughly within $500 \pm \sqrt{500}$ or so).

Similarly, the *second law of thermodynamics* states that the entropy of a closed system will never decrease with time. (It can stay the same or can increase.) The 2nd law is just a statement that a closed system evolves toward the most probable macrostate.

Entropy



Entropy

Again, *second law of thermodynamics* ("the entropy law") states that the entropy of a closed system will never decrease with time. (It can stay the same or can increase.)

For a system that is not closed (i.e. it can exchange energy with its environment), the entropy law states that the combined entropy of the system + its environment will never decrease with time. (It can stay the same or can increase.) This is equivalent to treating "system + environment" as a (much larger) closed system.

Increases in entropy are associated with irreversible processes, like the dissipation of mechanical energy into heat when I drop a tennis ball and let it bounce until it comes to rest.

Reversible processes (i.e. processes for which a movie played either forward or backward looks like physics that is possible) correspond to zero change in entropy.

For a system that is in equilibrium with a thermal reservoir at temperature T, you can relate the system's change in entropy to the "energy transferred thermally" (a.k.a. heat) into the system:

$$\Delta S = \frac{Q}{k_B T}$$

or in other words,

$$Q = k_B T \Delta S$$

A net heat (Q) flow into a system increases the system's entropy. (Giancoli and Mazur have different conventions about the factor of k_B. Mazur uses "statistical entropy," which no units; Giancoli uses "thermodynamic entropy," which is in J/K (joules per kelvin).)

For an ideal gas, we learned

$$PV = Nk_B T$$

The thermal energy of the gas (if monatomic like He, Ar, etc.) is

$$E_{\rm thermal} = \frac{3}{2} Nk_B T = \frac{3}{2} PV$$

(Complication that we'll ignore: the $\frac{3}{2}$ becomes $\frac{d}{2}$ for the non-monatomic case.)

If I compress the gas without letting any heat escape, I will increase its thermal energy. You know from experience (e.g. bicycle pump) that work (W) is required to compress a gas: as I push down on a piston, I exert a force that opposes the gas pressure.

So we can increase the energy of a gas by doing mechanical work (W) on the gas, which decreases the gas's volume:

$$W_{(\mathrm{ON\ gas})} = -\int P\,\mathrm{d}V$$

$W > 0$ (work done ON the gas) increases the gas's energy and decreases the gas's volume.

The transfer of energy into a system via coherent mechanical movement (via an external force) is called **work**.

You also know from experience that you can increase the temperature (and therefore the energy) of the gas by **heating** it, i.e. by putting it in contact with an object of higher temperature.

The transfer of incoherent thermal energy into a system (usually because of a temperature difference) is called **heat** (Q).

FYI: Don't confuse "heat" (Q) with "thermal energy" (E_{thermal}, a.k.a. U). Because the word "heat" is used ambiguously, Mazur's book refers to Q as "energy transferred thermally."

So we have two ways to add energy to a gas: W and Q.

$$\Delta E_{\mathrm{thermal}} = W + Q$$

W is due to (coherent) mechanical interactions, like a piston moving up and down or an electric motor pumping on the gas.

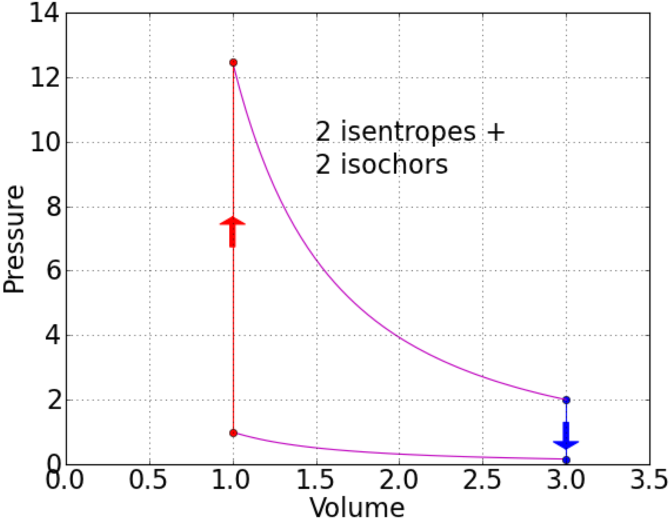
Q is due to (incoherent) thermal interactions, like the jiggling molecules of an adjacent heat bath (thermal reservoir) causing the molecules of the gas to reach the temperature of the heat bath.

- If $Q = 0$ the entropy of the gas does not change.
- If $Q > 0$ the entropy of the gas increases.
- If $Q < 0$ the entropy of the gas decreases.
- For heat transfer Q into the gas at constant temperature T,

$$\Delta S_{\mathrm{gas}} = \frac{Q}{k_B T}$$

$Q < 0$ is only possible if there is some compensating change in entropy somewhere else, such that the overall $\Delta S \geq 0$.

Heating ($Q > 0$) or cooling ($Q < 0$) the gas while $W = 0$ looks like $\Delta V = 0$ (isochore, "constant volume"). Doing work on the gas ($W > 0$) or letting the gas do work ($W < 0$) while $Q = 0$ looks like $P_f V_f^{(5/3)} = P_i V_i^{(5/3)}$ (isentrope, "constant entropy"). Giancoli says "adiabatic" where Mazur says "isentropic."



In the **unlikely event** that anyone wants to know where the $\frac{5}{3}$ in $PV^{(5/3)}$ comes from, it's because (for a monatomic gas, $d = 3$)

$$0 = \Delta S = N \ln\left(\frac{V_f}{V_i}\right) + \frac{3}{2} N \ln\left(\frac{T_f}{T_i}\right)$$

then using the ideal gas law, $PV \propto T$,

$$0 = \Delta S = N \ln\left(\frac{V_f}{V_i}\right) + \frac{3}{2} N \ln\left(\frac{P_f V_f}{P_i V_i}\right)$$

$$0 = \Delta S = \frac{5}{2} N \ln\left(\frac{V_f}{V_i}\right) + \frac{3}{2} N \ln\left(\frac{P_f}{P_i}\right)$$

$$1 = \left(\frac{V_f}{V_i}\right)^5 \times \left(\frac{P_f}{P_i}\right)^3$$

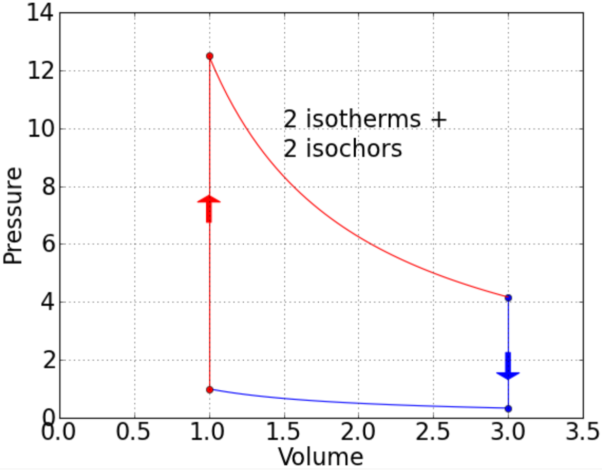
so finally (for an isentropic process on a monatomic gas)

$$P_i V_i^{(5/3)} = P_f V_f^{(5/3)}$$

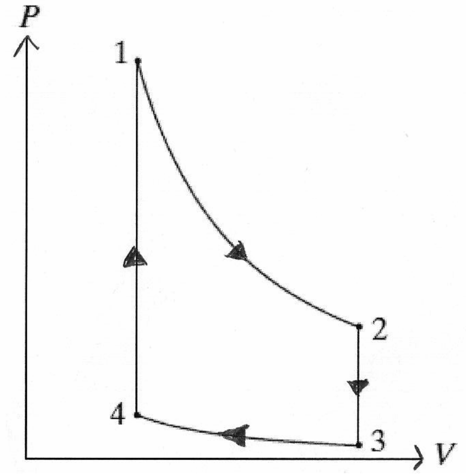
The starting point was Mazur eq 19.61. But you don't need to know this!

Heating ($Q > 0$) or cooling ($Q < 0$) the gas while $W = 0$ looks like $\Delta V = 0$ (isochore).

Doing work on the gas ($W > 0$) or letting the gas do work ($W < 0$) at constant temperature T looks like $P_f V_f = P_i V_i$ (isotherm). (For an engine, $W > 0$ happens at low T.)

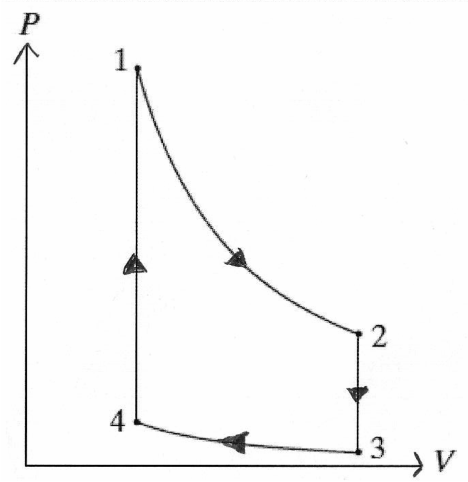


This cycle consists of two isotherms ($\Delta T = 0$) and two isochores ($\Delta V = 0$). The cycle is $4 \to 1 \to 2 \to 3 \to 4 \to \ldots$ On which parts of the cycle is positive work done ON the gas ($W_{\text{in}} > 0$)?



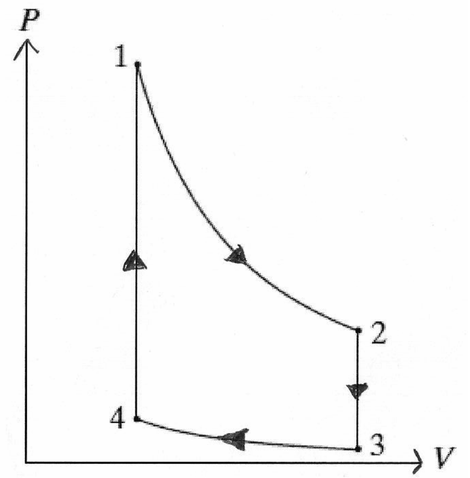
(A) $4 \to 1$ (B) $1 \to 2$ (C) $2 \to 3$ (D) $3 \to 4$
(E) $1 \to 2$ and $3 \to 4$ (F) $4 \to 1$ and $2 \to 3$

This cycle consists of two isotherms ($\Delta T = 0$) and two isochores ($\Delta V = 0$). The cycle is $4 \to 1 \to 2 \to 3 \to 4 \to \ldots$. On which parts of the cycle is positive work done BY the gas ($W_{\text{out}} > 0$)?



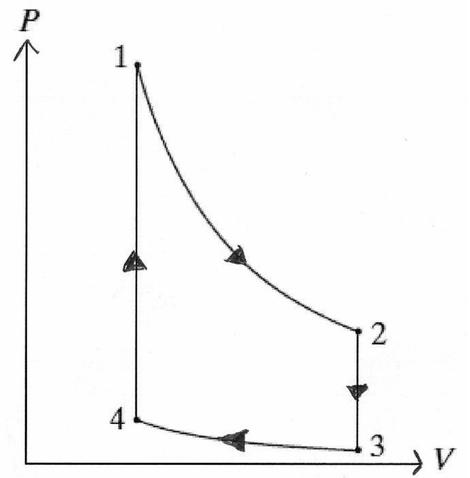
(A) $4 \to 1$ (B) $1 \to 2$ (C) $2 \to 3$ (D) $3 \to 4$
(E) $1 \to 2$ and $3 \to 4$ (F) $4 \to 1$ and $2 \to 3$

This cycle consists of two isotherms ($\Delta T = 0$) and two isochores ($\Delta V = 0$). The cycle is $4 \to 1 \to 2 \to 3 \to 4 \to \ldots$. Over one complete cycle, is W_{out} larger than, smaller than, or equal to W_{in}?

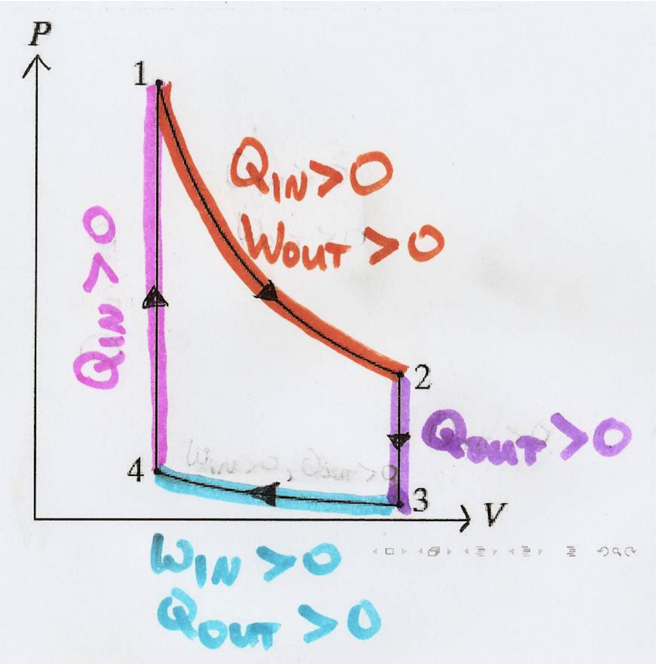


(A) $W_{\text{out}} > W_{\text{in}}$ (B) $W_{\text{out}} < W_{\text{in}}$ (C) $W_{\text{out}} = W_{\text{in}}$

This cycle consists of two isotherms ($\Delta T = 0$) and two isochores ($\Delta V = 0$). The cycle is $4 \to 1 \to 2 \to 3 \to 4 \to \ldots$. On which parts of the cycle is the internal energy of the gas constant?



(A) $4 \to 1$ (B) $1 \to 2$ (C) $2 \to 3$ (D) $3 \to 4$
(E) $1 \to 2$ and $3 \to 4$ (F) $4 \to 1$ and $2 \to 3$ (G) none


P
V
1
2
3
4
QIN>0
WOUT>0
QIN>0
QOUT>0
WIN>0
QOUT>0

Rules for "steady devices" (engine, heat pump, etc.)

- Over one cycle $\Delta E_{\text{thermal}} = W + Q = 0$ (the energy of the gas returns to the value at which it started):

$$W_{\text{input}} + Q_{\text{input}} = W_{\text{output}} + Q_{\text{output}}$$

- The total entropy cannot decrease. Since the gas returns to its initial state at the end of each cycle ($\Delta S_{\text{cycle}} = 0$), this implies that the entropy of the environment cannot decrease:

$$\Delta S_{\text{environment}} = \frac{Q_{\text{output}}}{k_B T_{\text{output}}} - \frac{Q_{\text{input}}}{k_B T_{\text{input}}} \geq 0$$

- Notice that moving heat from the device out to the environment increases S_{env}, while moving heat from the environment in to the device decreases S_{env}.

$$\frac{Q_{\text{output}}}{T_{\text{output}}} \geq \frac{Q_{\text{input}}}{T_{\text{input}}}$$

- If you're trying to do useful work with the heat (Q_{input}), some of the heat (Q_{output}) is always thrown away as a by-product.

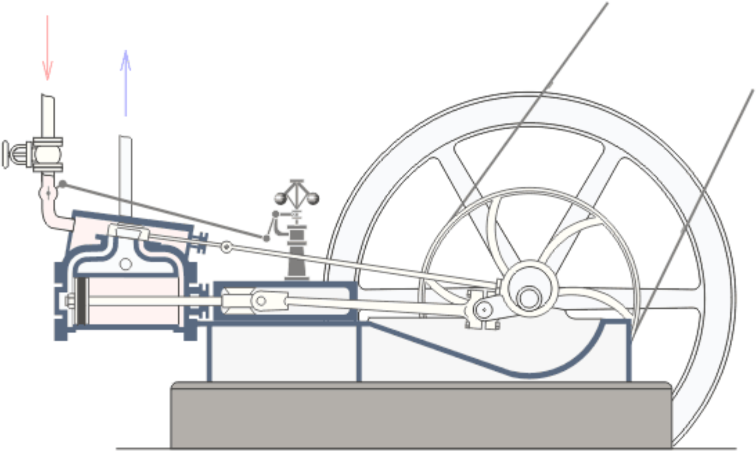
Engine efficiency

- What you "pay" to run the engine is Q_{input}, which comes from e.g. burning fuel.
- What you want from the engine is mechanical work $W_{\text{output}} - W_{\text{input}} = -W$
  (Mazur and Giancoli have different sign conventions for W.)
- The **efficiency** of a heat engine is defined as

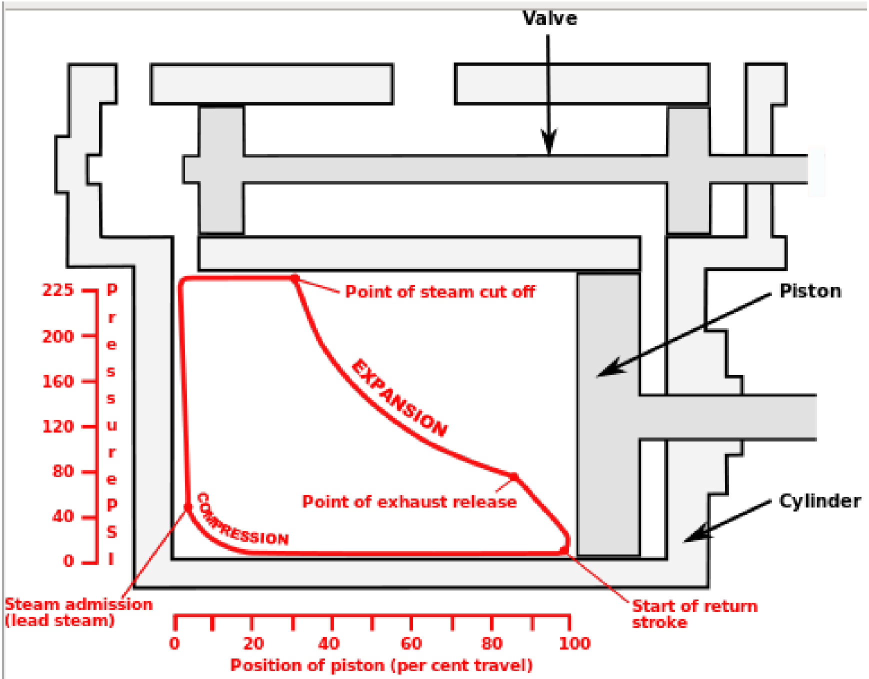
$$\boxed{\eta = \frac{W_{\text{output}} - W_{\text{input}}}{Q_{\text{input}}}}$$

- An engine takes in Q_{input} at some high temperature T_{input} and exhausts waste heat Q_{output} at some low temperature T_{output}. Engines need $T_{\text{input}} > T_{\text{output}}$ and work best when $T_{\text{input}} \gg T_{\text{output}}$.
- The entropy law says that engines can never do better than the theoretical $\Delta S_{\text{environment}} = 0$ case ("reversible" engine):

$$\eta \ \leq \ 1 - \frac{T_{\text{output}}}{T_{\text{input}}} \qquad \boxed{\text{efficiency} \ \leq \ 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}}$$



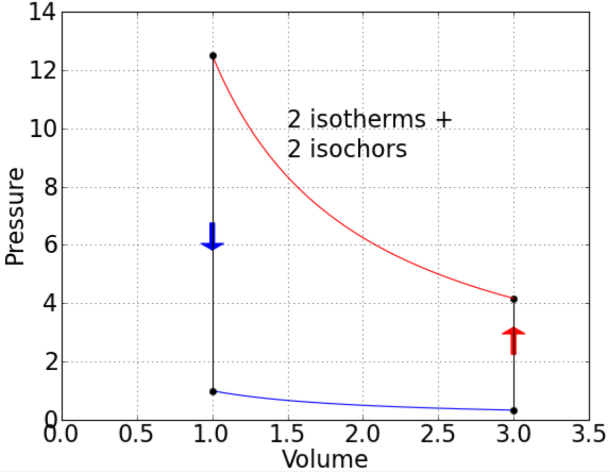
https://en.wikipedia.org/wiki/Steam_engine#/media/File:Steam_engine_in_action.gif


Valve
Piston
Cylinder
Point of steam cut off
EXPANSION
Point of exhaust release
COMPRESSION
Steam admission
(lead steam)
Start of return
stroke
225
200
160
120
80
40
0
PressurePSI
0
20
40
60
80
100
Position of piston (per cent travel)

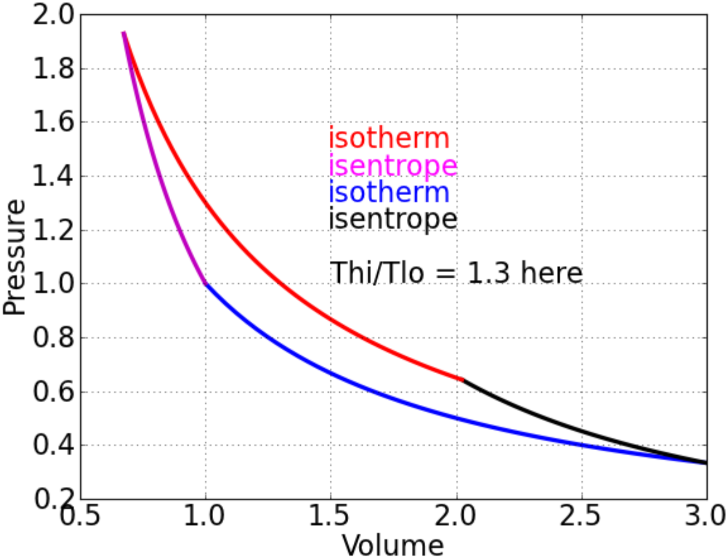
Refrigerator: PV diagram goes counterclockwise

Dumb example of fridge (but easy to illustrate with coffee cans)

Doing work on the gas ($W > 0$) or letting the gas do work ($W < 0$) at constant temperature T looks like $P_f V_f = P_i V_i$ (isotherm). (For a fridge, $W > 0$ happens at high T.)



This is a “reversible” ($\Delta S_{\mathrm{env}} = 0$) fridge cycle. Doing work on the gas ($W > 0$) or letting the gas do work ($W < 0$) at constant temperature T looks like $P_f V_f = P_i V_i$ (isotherm). Does anyone know the name of the cycle has two isotherms and two isentropes?



If I compress a gas (or let it expand) while the gas is sealed in a thermally insulated vessel (like a thermos bottle with a piston), so that the gas can't exchange heat with the environment, is that process called
(a) "isothermal" or
(b) "isentropic" (a.k.a. "adiabatic")?

If I compress a gas (or let it expand) while the gas is in thermal contact with in a large container of water (a "heat bath" a.k.a. "thermal reservoir"), so that the gas maintains the same constant temperature as the reservoir, is that process called
(a) "isothermal" or
(b) "isentropic" (a.k.a. "adiabatic")?

Refrigerator "coefficient of performance"

- What you "pay" to run the fridge is W, which comes from the "compressor's" electric motor.
- What you want from the fridge is the cooling: Q_{input}
- The **coefficient of performance** of a fridge is defined as

$$\boxed{\text{COP}_{\text{cooling}} = \frac{Q_{\text{input}}}{W}}$$

- A fridge performs best when the kitchen (T_{output}) is not too much warmer than the desired fridge temperature (T_{input})! Unlike an engine, a fridge prefers a small ΔT.
- The entropy law dictates that fridges can never do better than the theoretical $\Delta S_{\text{env}} = 0$ case (ideal "reversible" fridge):

$$\text{COP}_{\text{cooling}} \leq \frac{T_{\text{input}}}{T_{\text{output}} - T_{\text{input}}} \qquad \boxed{\text{COP}_{\text{cooling}} \leq \frac{T_{\text{cold}}}{T_{\text{hot}} - T_{\text{cold}}}}$$

Heat pump "coefficient of performance"

- What you "pay" to run the heat pump is W, which comes from an electric motor — same as with a fridge.
- What you want from heat pump is the **heating**: Q_{output}
- The **coefficient of performance** of a heat pump is

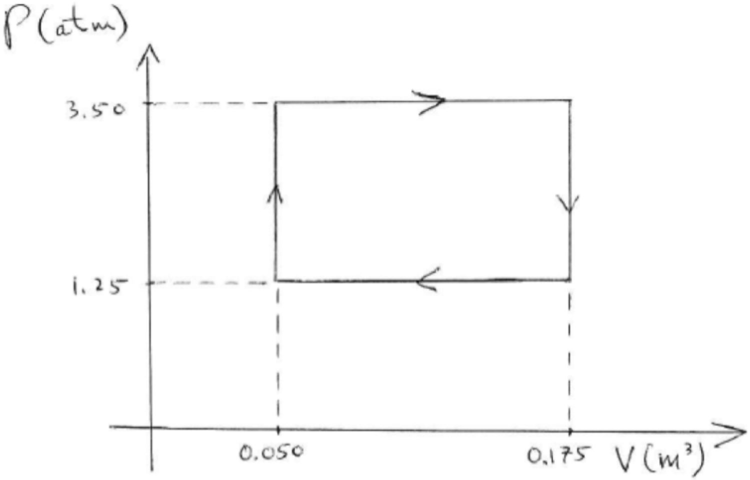
$$\text{COP}_{\text{heating}} = \frac{Q_{\text{output}}}{W}$$

- A heat pump performs best when the house (T_{output}) is not too much warmer than the heat-exchange-coil (T_{input})! A heat pump prefers a small ΔT. (So bury coil underground.)
- The entropy law dictates that heat pumps can never do better than the theoretical $\Delta S_{\text{env}} = 0$ case (ideal "reversible" pump):

$$\text{COP}_{\text{heating}} \le \frac{T_{\text{output}}}{T_{\text{output}} - T_{\text{input}}} \qquad \boxed{\text{COP}_{\text{heating}} \le \frac{T_{\text{hot}}}{T_{\text{hot}} - T_{\text{cold}}}}$$

- **Confusingly,** T_{output} (to which heat is output) is the indoor temperature, and T_{input} (from which heat is input) is the outdoor temperature!

4*. The PV diagram for a certain (somewhat contrived, to keep the math simple) heat engine cycle is shown below. What is the efficiency ($\eta = (W_{\text{out}} - W_{\text{in}})/Q_{\text{in}}$) of this engine, if it exhausts $Q_{\text{out}} = 43.5$ kJ of thermal energy per cycle? (Remember 1 atm = 101325 N/m^2.)



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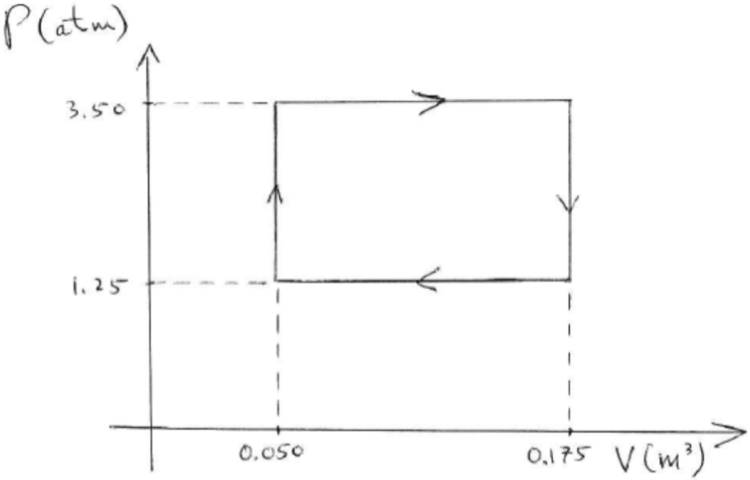

Answer: $W_{out} = 44.3$ kJ, $W_{in} = 15.8$ kJ,
$Qin = W_{out} + Q_{out} - W_{in} = 72.0$ kJ. $\eta = 0.396$.

In one cycle, a steady device transfers 1200×10^3 J of energy from a thermal reservoir at 600 K to a thermal reservoir at 300 K. (This is a pretty useless device: since $Q_{\mathrm{out}} = Q_{\mathrm{in}}$, we must have $W = 0$.) Find the change in entropy (after one complete cycle),

(a) For the device, and

(b) for the environment.

(Mazur writes $\Delta S = Q/(k_B T)$. Giancoli writes $\Delta S = Q/T$. Let's use Giancoli's convention here, since it makes the math easier.)

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Answer: $\Delta S_{\mathrm{env}} = \frac{Q_{\mathrm{output}}}{T_{\mathrm{output}}} - \frac{Q_{\mathrm{input}}}{T_{\mathrm{input}}} = \frac{1200\ \mathrm{kJ}}{300\ \mathrm{K}} - \frac{1200\ \mathrm{kJ}}{600\ \mathrm{K}} = 2000\ \mathrm{J/K}$

About HW problem 2 for HW08 (Reynolds number):

If **the same flow (volume per unit time)** of a fluid passes through both wide and narrow sections of the pipe/duct/river/etc., the narrow section is more likely to be turbulent, hence more likely to be noisy. If you only partially close off an HVAC duct, so that (approximately) the same air flow must pass (at higher speed) through a reduced area, the air flow makes more noise.

Reynolds number:

$$Re = \frac{2r\overline{v}\rho}{\eta} \propto r\overline{v}$$

Flow rate (volume/time):

$$Q = A\overline{v} = \pi r^2 \overline{v} \propto r^2 \overline{v}$$

Suppose $r_1 = R$, $\overline{v}_1 = V$, and $r_2 = 2R$. To get same flow, $Q_2 = Q_1$, you need $\overline{v}_2 = (V/4)$. Then

$$\frac{Q_2}{Q_1} = \frac{r_2^2\,\overline{v}_2}{r_1^2\,\overline{v}_1} = \frac{(2R)^2(V/4)}{(R)^2(V)} = 1$$

But

$$\frac{Re_2}{Re_1} = \frac{r_2\,\overline{v}_2}{r_1\,\overline{v}_1} = \frac{(2R)(V/4)}{(R)(V)} = \frac{1}{2}$$

Same flow through wider duct has smaller Reynolds number, so is less turbulent. Consistent with pictures of river. (This argument depends on same flow rate: if making the pipe bigger just allows a bigger flow, then you're not reducing turbulence.)

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