Physics 9 — Monday, November 5, 2018

- For today, you read **Eric Mazur**'s chapter 22 (Electric Interactions) — PDF on Canvas.
- For Wed., read Giancoli ch16 (electric charge & electric field)
- I have a large number of supplemental chapters you could read for XC, if interested:
  - A newer Climate Change chapter from Muller's 2012 book *Energy for Future Presidents*.
  - Muller's chapters on chain reactions (exponential growth) and radioactivity.
  - Tutorials on using Mathematica to do math calculations, algebra, integrals, graphs, and much more.
  - Different chapters on architectural acoustics.
  - A more mathematical look at entropy & thermodymamics.
  - Giancoli's chapter on astronomy.

We have two ways to add energy to a gas: W and Q.

$$\Delta E_{\mathrm{thermal}} = W_{\mathrm{in}} + Q_{\mathrm{in}} - W_{\mathrm{out}} - Q_{\mathrm{out}}$$

W is due to (coherent) mechanical interactions, like a piston moving up and down or an electric motor pumping on the gas.

Q is due to (incoherent) thermal interactions, like the jiggling molecules of an adjacent heat bath (thermal reservoir) causing the molecules of the gas to reach the temperature of the heat bath.

- If $Q = 0$ the entropy of the gas does not change.
- If $Q > 0$ the entropy of the gas increases.
- If $Q < 0$ the entropy of the gas decreases.
- For heat transfer Q into the gas at constant temperature T,

$$\Delta S_{\mathrm{gas}} = \frac{Q}{k_B T}$$

$Q < 0$ is only possible if there is some compensating change in entropy somewhere else, such that the overall $\Delta S \geq 0$.

Rules for "steady devices" (engine, heat pump, etc.)

- Over one cycle $\Delta E_{\mathrm{thermal}} = W + Q = 0$ (the energy of the gas returns to the value at which it started):

$$W_{\mathrm{input}} + Q_{\mathrm{input}} = W_{\mathrm{output}} + Q_{\mathrm{output}}$$

- The total entropy cannot decrease. Since the gas returns to its initial state at the end of each cycle ($\Delta S_{\mathrm{cycle}} = 0$), this implies that the entropy of the environment cannot decrease:

$$\Delta S_{\mathrm{environment}} = \frac{Q_{\mathrm{output}}}{k_B T_{\mathrm{output}}} - \frac{Q_{\mathrm{input}}}{k_B T_{\mathrm{input}}} \geq 0$$

- Notice that moving heat from the device out to the environment increases S_{env}, while moving heat from the environment in to the device decreases S_{env}.

$$\frac{Q_{\mathrm{output}}}{T_{\mathrm{output}}} \geq \frac{Q_{\mathrm{input}}}{T_{\mathrm{input}}}$$

- If you're trying to do useful work with the heat (Q_{input}), some of the heat (Q_{output}) is always thrown away as a by-product.

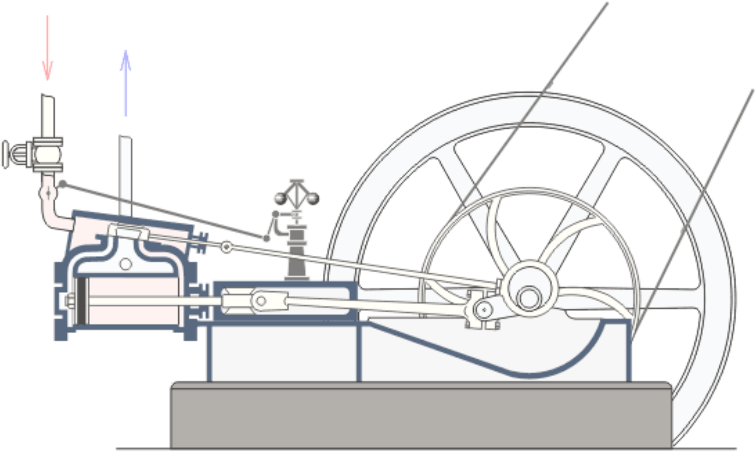
Engine efficiency

- What you "pay" to run the engine is $Q_{\rm input}$, which comes from e.g. burning fuel.
- What you want from the engine is mechanical work $W_{\rm output} - W_{\rm input} = -W$
  (Warning: Giancoli uses the opposite sign convention for W.)
- The **efficiency** of a heat engine is defined as

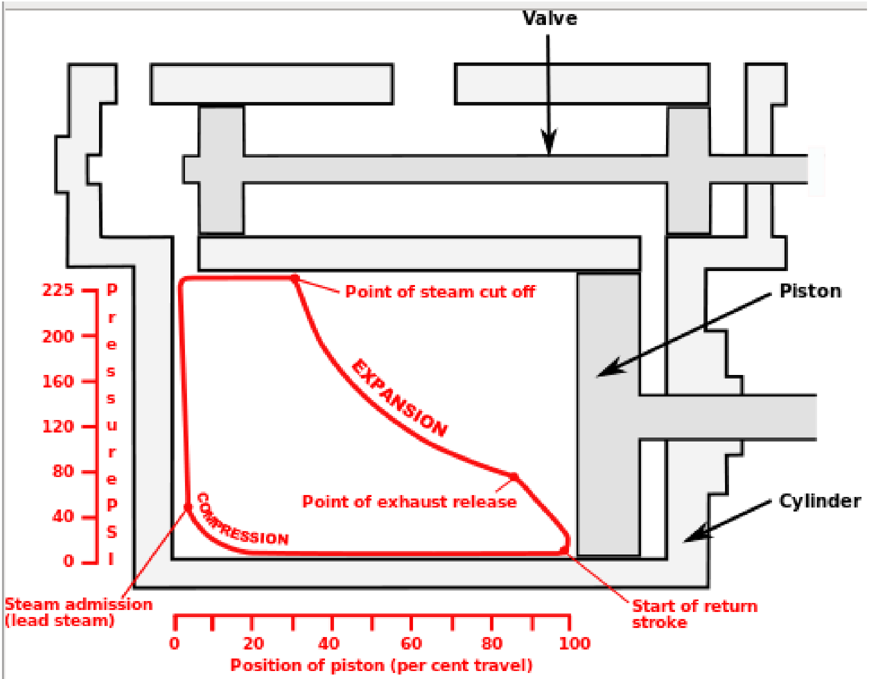
$$\boxed{\eta = \frac{W_{\rm output} - W_{\rm input}}{Q_{\rm input}}}$$

- An engine takes in $Q_{\rm input}$ at some high temperature $T_{\rm input}$ and exhausts waste heat $Q_{\rm output}$ at some low temperature $T_{\rm output}$. Engines need $T_{\rm input} > T_{\rm output}$ and work best when $T_{\rm input} \gg T_{\rm output}$.
- The entropy law says that engines can never do better than the theoretical $\Delta S_{\rm environment} = 0$ case ("reversible" engine):

$$\eta \le 1 - \frac{T_{\rm output}}{T_{\rm input}} \qquad \boxed{\text{efficiency} \le 1 - \frac{T_{\rm cold}}{T_{\rm hot}}}$$



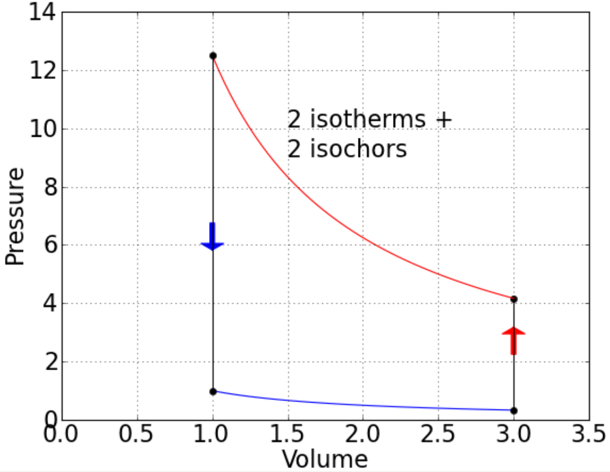
https://en.wikipedia.org/wiki/Steam_engine#/media/File:Steam_engine_in_action.gif


Valve
Piston
Cylinder
Point of steam cut off
EXPANSION
Point of exhaust release
COMPRESSION
Steam admission
(lead steam)
Start of return
stroke
225
200
160
120
80
40
0
Pressure pSI
0
20
40
60
80
100
Position of piston (per cent travel)

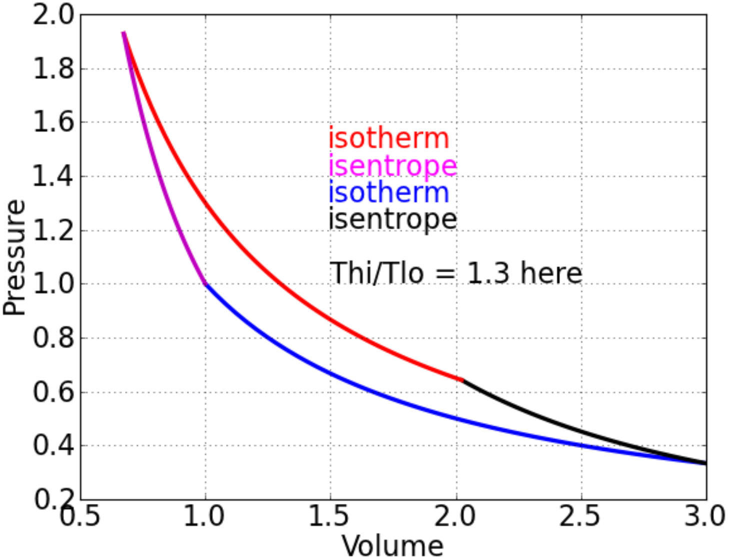
Refrigerator: PV diagram goes counterclockwise

Dumb example of fridge (but easy to illustrate with coffee cans)

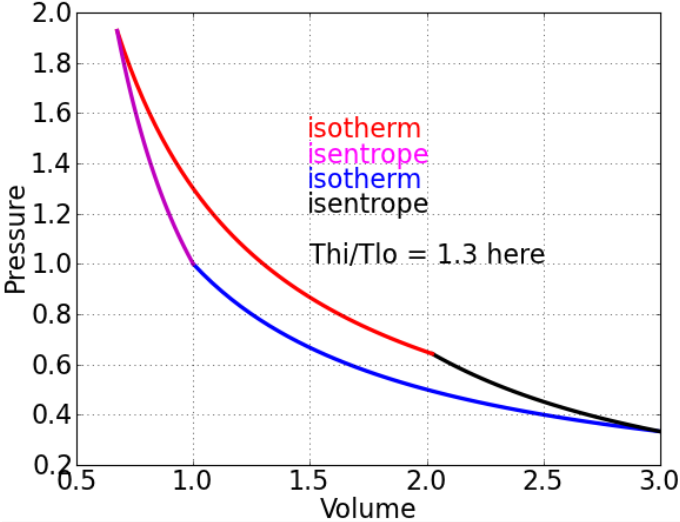
Doing work on the gas ($W > 0$) or letting the gas do work ($W < 0$) at constant temperature T looks like $P_f V_f = P_i V_i$ (isotherm). (For a fridge, $W > 0$ happens at high T.)



This is a "reversible" ($\Delta S_{\mathrm{env}} = 0$) fridge cycle. Doing work on the gas ($W > 0$) or letting the gas do work ($W < 0$) at constant temperature T looks like $P_f V_f = P_i V_i$ (isotherm). Does anyone know the name of the cycle has two isotherms and two isentropes?



By the way, if you run this same (Carnot) cycle clockwise, you get the Carnot heat engine, which is of theoretical interest (because it has optimal efficiency) but not of practical interest (because the net work done per cycle is very small — as you can see from the fact that the enclosed area between the curves is so small).



If I compress a gas (or let it expand) while the gas is sealed in a thermally insulated vessel (like a thermos bottle with a piston), so that the gas can't exchange heat with the environment, is that process called
(a) "isothermal" or
(b) "isentropic" (a.k.a. "adiabatic")?

If I compress a gas (or let it expand) while the gas is in thermal contact with in a large container of water (a "heat bath" a.k.a. "thermal reservoir"), so that the gas maintains the same constant temperature as the reservoir, is that process called
(a) "isothermal" or
(b) "isentropic" (a.k.a. "adiabatic")?

Refrigerator "coefficient of performance"

- What you "pay" to run the fridge is $W = W_{\rm input} - W_{\rm output}$, which comes from the "compressor's" electric motor.
- What you want from the fridge is the cooling: $Q_{\rm input}$
- The **coefficient of performance** of a fridge is defined as

$$\boxed{\mathrm{COP}_{\rm cooling} = \frac{Q_{\rm input}}{W}}$$

- A fridge performs best when the kitchen ($T_{\rm output}$) is not too much warmer than the desired fridge temperature ($T_{\rm input}$)! Unlike an engine, a fridge prefers a small ΔT.
- The entropy law dictates that fridges can never do better than the theoretical $\Delta S_{\rm env} = 0$ case (ideal "reversible" fridge):

$$\mathrm{COP}_{\rm cooling} \le \frac{T_{\rm input}}{T_{\rm output} - T_{\rm input}} \qquad \boxed{\mathrm{COP}_{\rm cooling} \le \frac{T_{\rm cold}}{T_{\rm hot} - T_{\rm cold}}}$$

Heat pump "coefficient of performance"

- What you "pay" to run the heat pump is W, which comes from an electric motor — same as with a fridge.
- What you want from heat pump is the **heating**: Q_{output}
- The **coefficient of performance** of a heat pump is

$$\text{COP}_{\text{heating}} = \frac{Q_{\text{output}}}{W}$$

- A heat pump performs best when the house (T_{output}) is not too much warmer than the heat-exchange-coil (T_{input})! A heat pump prefers a small ΔT. (So bury coil underground.)
- The entropy law dictates that heat pumps can never do better than the theoretical $\Delta S_{\text{env}} = 0$ case (ideal "reversible" pump):

$$\text{COP}_{\text{heating}} \le \frac{T_{\text{output}}}{T_{\text{output}} - T_{\text{input}}} \qquad \boxed{\text{COP}_{\text{heating}} \le \frac{T_{\text{hot}}}{T_{\text{hot}} - T_{\text{cold}}}}$$

- **Confusingly,** T_{output} (to which heat is output) is the indoor temperature, and T_{input} (from which heat is input) is the outdoor temperature!

A heat pump has a COP of 3.0 and is rated to do work at 1500 W. How much heat can it add to a room per second?

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answer: $4500\,\mathrm{W}$

Suppose that a heat pump operates with $T_{\mathrm{hot}} = 35^{\circ}\mathrm{C}$ and $T_{\mathrm{cold}} = 0^{\circ}\mathrm{C}$. What is the theoretical maximum COP that one could ever hope to achieve for these T_H and T_C values?

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$$COP_{\mathrm{heating}} = \frac{Q_{\mathrm{hot}}}{W} = \frac{Q_{\mathrm{hot}}}{Q_{\mathrm{hot}} - Q_{\mathrm{cold}}} \le \frac{T_{\mathrm{hot}}}{T_{\mathrm{hot}} - T_{\mathrm{cold}}}$$

$$COP_{\mathrm{heating}} \le \frac{308\mathrm{K}}{308\mathrm{K} - 273\mathrm{K}} \le 8.8$$

but in practice under these conditions the best heat-pump systems achieve $COP_{\mathrm{heating}} \approx 4.5$ in laboratory tests and more like 3.5 in real-life application.

It is still a useful guideline that COP_{heating} gets worse as the indoor/outdoor temperature difference increases.

That is why a "ground source" heat pump usually achieves a much better COP than an "air source" heat pump.

A freezer has a COP of 3.8 and uses 200 W of electrical power. How long would it take to freeze an ice-cube tray that contains 600 g of water at $0^\circ\mathrm{C}$? The latent heat of fusion for water is $L = 333\,\mathrm{kJ/kg}$.

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$$t = \frac{m\,L}{COP \times 200\,W} = 260\,\mathrm{s} \approx 4.3\,\mathrm{minutes}$$

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This seems unrealistically fast for an ordinary fridge, but dedicated ice-cube makers use metal rods to maximize the thermal contact between the fridge and the water.

Note that

$$COP_{\mathrm{cooling}} = \frac{Q_{\mathrm{cold}}}{W} = \frac{Q_{\mathrm{cold}}}{Q_{\mathrm{hot}} - Q_{\mathrm{cold}}} \leq \frac{T_{\mathrm{cold}}}{T_{\mathrm{hot}} - T_{\mathrm{cold}}}$$

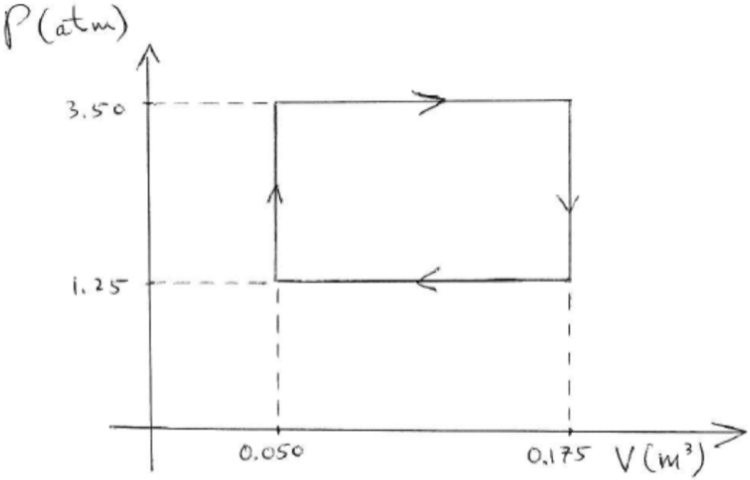
So in particular the "ideal COP_{cooling}" has T_{cold} in the numerator, while the "ideal COP_{heating} has T_{hot} in the numerator.

One more confusing detail. In “US customary units,” one uses the (Seasonal) Energy Efficiency Ratio (SEER, or EER), where heat is measured in BTU, while work is measured in watt-hours.

Since a British Thermal Unit is 1055 J, a COP of 1.0 corresponds to an SEER of 3.4.

So if a very good COP value these days is around 4.0, then a very good SEER is around 13.6.

4*. The PV diagram for a certain (somewhat contrived, to keep the math simple) heat engine cycle is shown below. What is the efficiency ($\eta = (W_{\text{out}} - W_{\text{in}})/Q_{\text{in}}$) of this engine, if it exhausts $Q_{\text{out}} = 43.5$ kJ of thermal energy per cycle? (Remember 1 atm = 101325 N/m^2.)



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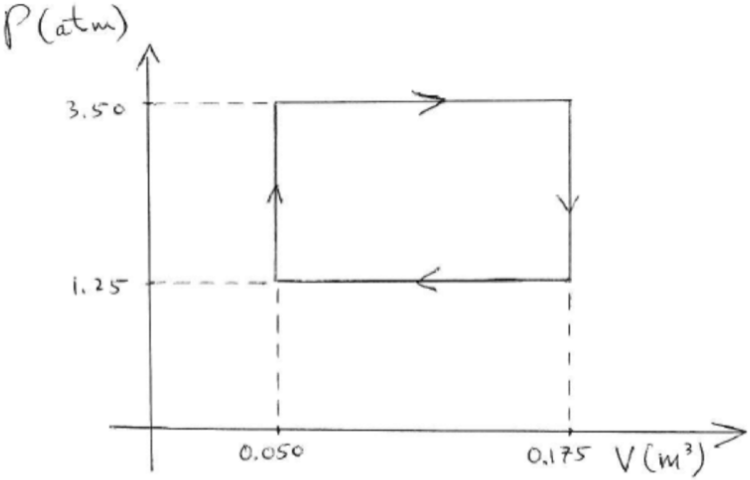

Answer: $W_{\text{out}} = 44.3$ kJ, $W_{\text{in}} = 15.8$ kJ,
$Qin = W_{\text{out}} + Q_{\text{out}} - W_{\text{in}} = 72.0$ kJ. $\eta = 0.396$.

In one cycle, a steady device transfers 1200×10^3 J of energy from a thermal reservoir at 600 K to a thermal reservoir at 300 K. (This is a pretty useless device: since $Q_{\rm out} = Q_{\rm in}$, we must have $W = 0$.) Find the change in entropy (after one complete cycle),

(a) For the device, and

(b) for the environment.

(Mazur writes $\Delta S = Q/(k_B T)$. Giancoli writes $\Delta S = Q/T$. Let's use Giancoli's convention here, since it makes the math easier.)

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Answer: $\Delta S_{\mathrm{env}} = \frac{Q_{\mathrm{output}}}{T_{\mathrm{output}}} - \frac{Q_{\mathrm{input}}}{T_{\mathrm{input}}} = \frac{1200\ \mathrm{kJ}}{300\ \mathrm{K}} - \frac{1200\ \mathrm{kJ}}{600\ \mathrm{K}} = 2000\ \mathrm{J/K}$

About HW problem 2 for HW08 (Reynolds number):

If **the same flow (volume per unit time)** of a fluid passes through both wide and narrow sections of the pipe/duct/river/etc., the narrow section is more likely to be turbulent, hence more likely to be noisy. If you only partially close off an HVAC duct, so that (approximately) the same air flow must pass (at higher speed) through a reduced area, the air flow makes more noise.

Reynolds number:

$$Re = \frac{2r\overline{v}\rho}{\eta} \propto r\overline{v}$$

Flow rate (volume/time):

$$Q = A\overline{v} = \pi r^2 \overline{v} \propto r^2 \overline{v}$$

Suppose $r_1 = R$, $\overline{v}_1 = V$, and $r_2 = 2R$. To get same flow, $Q_2 = Q_1$, you need $\overline{v}_2 = (V/4)$. Then

$$\frac{Q_2}{Q_1} = \frac{r_2^2\,\overline{v}_2}{r_1^2\,\overline{v}_1} = \frac{(2R)^2(V/4)}{(R)^2(V)} = 1$$

But

$$\frac{Re_2}{Re_1} = \frac{r_2\,\overline{v}_2}{r_1\,\overline{v}_1} = \frac{(2R)(V/4)}{(R)(V)} = \frac{1}{2}$$

Same flow through wider duct has smaller Reynolds number, so is less turbulent. Consistent with pictures of river. (This argument depends on same flow rate: if making the pipe bigger just allows a bigger flow, then you're not reducing turbulence.)

comparing electric and gravitational forces

- We learned at the beginning of the semester (and you probably also had learned in an earlier physics course) that the force of gravity:
  - is a field force that acts at a distance
  - is always "attractive" (never "repulsive")
  - grows weaker with distance R, like $1/R^2$
  - is proportional to the product of the two masses
  - causes a dropped object to accelerate toward Earth's center
  - is the "weight" that architectural structures must support
  - causes Earth to orbit the Sun, the Moon to orbit Earth, etc.

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The gravitational force due to b acting ON a has magnitude

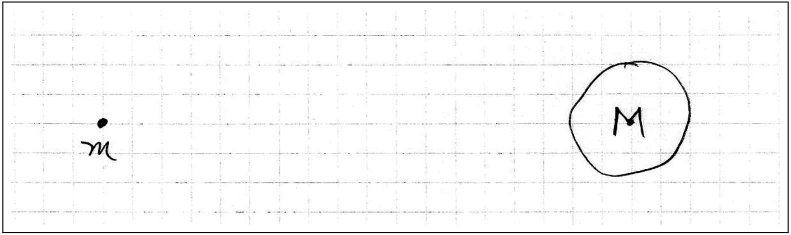
$$\vec{F}^{G}_{b,a} = G \; \frac{m_a \; m_b}{r^2_{ab}} \; \hat{r}_{ab}$$

The force acting ON a points from a toward b.

Object 1, of mass m, sits to the left of object 2, of mass M. The gravitational force between objects 1 and 2 has magnitude

$$F = \frac{GmM}{r_{12}^2}$$

where r_{12} is the distance between object 1 and object 2. What is the *direction* of the gravitational force **acting on object 1**? (Pretend that the objects are in outer space, so you can ignore Earth's gravity.)

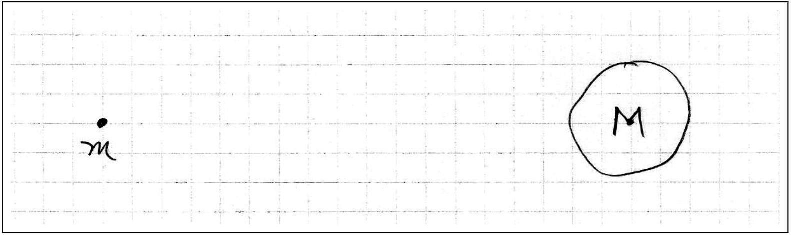


(A) up (B) down (C) right (D) left

Object 1, of mass m, sits to the left of object 2, of mass M. The gravitational force between objects 1 and 2 has magnitude

$$F = \frac{GmM}{r_{12}^2}$$

where r_{12} is the distance between object 1 and object 2. What is the *direction* of the gravitational force **acting on object 2**? (Pretend that the objects are in outer space, so you can ignore Earth's gravity.)



(A) up (B) down (C) right (D) left

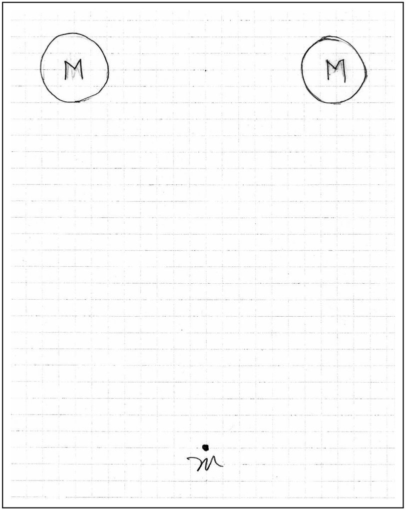
comparing electric and gravitational forces

The net gravitational force due to two objects b and c acting ON a is the vector sum of the individual forces ON a:

$$\vec{F}^{G}_{ON\ a} = G\,\frac{m_a\ m_b}{r_{ab}^2}\,\hat{r}_{ab} + G\,\frac{m_a\ m_c}{r_{ac}^2}\,\hat{r}_{ac}$$

where $\hat{r}_{ab}$ is the unit vector pointing from a toward b, and $\hat{r}_{ac}$ is the unit vector pointing from a toward c.

Objects 1 and 2, both of mass M, are at the top of the picture, separated from each other horizontally. Object 3, of mass m, is at the bottom of the picture, directly below the midpoint of objects 1 and 2. What is the direction of the gravitational force acting **on object 3** (the bottom object)? (Ignore Earth's gravity.)



(A) up

(B) down

(C) right

(D) left

(E) diagonal

comparing electric and gravitational forces

The electrical force:

- is a field force that acts at a distance
- can be either "attractive" or "repulsive"
- grows weaker with distance R, like $1/R^2$ (like gravity)
- is a long-range force, like gravity
- is proportional to the product of the two electric charges
- is what binds electrons to their atomic nuclei
- is stronger than gravity by a factor of roughly 10^{42} : that's 1,000,000,000,000,000,000,000,000,000,000,000,000,000,000
- provides the energy content of food, fuel, chemical reactions
- is the underlying cause of everyday "contact" forces

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The electrostatic force due to b acting ON a is

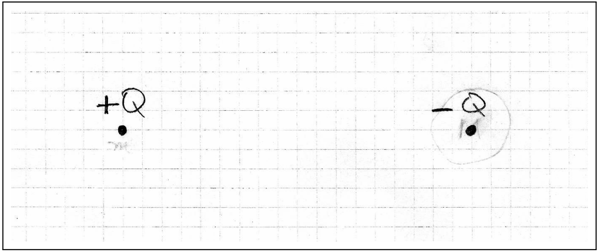
$$\vec{F}^{E}_{b,a} = -k\ \frac{q_a\ q_b}{r_{ab}^2}\ \hat{r}_{ab}$$

The force ON a points away from b for like charges and points toward b for opposite charges. (Opposites attract.)

Consider the two particles carrying identical electric charges (of the same sign), shown below. The force $\vec{F}_{12}^{E}$ (the force exerted BY 1 acting ON 2) points

(A) up (B) down (C) right (D) left (E) other direction

An object carrying positive charge, $+Q$, is placed to the left of an object carrying negative charge, $-Q$. The direction of the electrostatic force acting ON the left-hand charge is



(A) up (B) down (C) right (D) left (E) other direction

When the electric charge on each of two objects is doubled, the electric force between the two objects is

(A) The same.

(B) Doubled.

(C) Halved.

(D) Quadrupled.

(E) None of the above.

Two small objects are placed a large distance apart. Each object carries a positive charge. If the distance between the two objects is tripled, then the strength of the electrostatic repulsion between them will decrease by a factor of

(A) 3

(B) 6

(C) 8

(D) 9

(E) 12

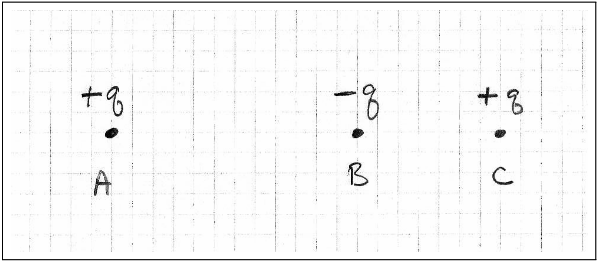
comparing electric and gravitational forces

The net electrostatic force of due to two objects b and c acting ON a is the vector sum of the individual forces ON a:

$$\vec{F}^{E}_{ON\ a} \; = \; -\,k\,\frac{q_a\;q_b}{r^2_{ab}}\,\hat{r}_{ab}\;-\;k\,\frac{q_a\;q_c}{r^2_{ac}}\,\hat{r}_{ac}$$

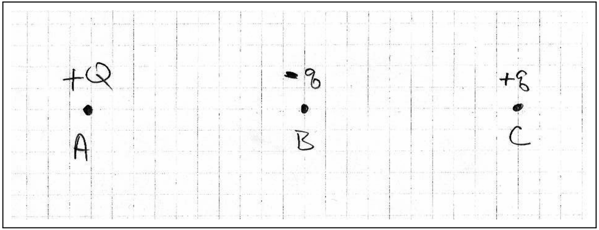
where $\hat{r}_{ab}$ is the unit vector pointing from a toward b, and $\hat{r}_{ac}$ is the unit vector pointing from a toward c.

Three charged objects, A,B,C, are placed in a horizontal row. Objects A and C are positively charged. Object B is negatively charged. All three charges have the same magnitude. What is the direction of the electric force exerted on object A ?



(A) up (B) down (C) right (D) left (E) zero (cancels)

Three charged objects, A,B,C, are placed in a horizontal row, equally spaced. Object B is negatively charged. Objects A and C are positively charged. The charge on object A is much larger than the charges on objects B and C. What is the direction of the electric force exerted on object B ?



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Electrostatic forces

i.e. forces due to electric charges that are not in motion

The electrostatic force due to b acting ON a is

$$\vec{F}^{\text{electric}}_{b\ ON\ a} = k\ \frac{q_a\ q_b}{r_{ab}^2}\ \hat{r}_{b\to a}$$

(or in Mazur's more abbreviated notation)

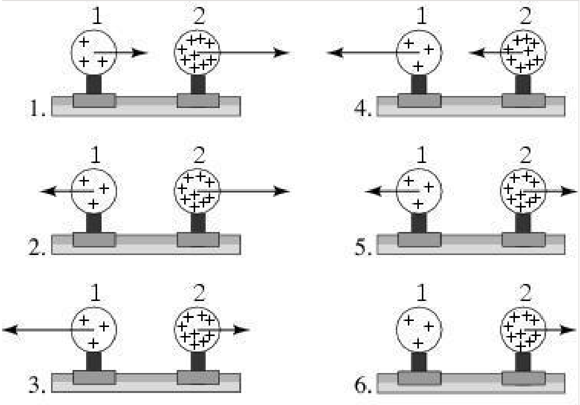
$$\vec{F}^{E}_{ba} = k\ \frac{q_a\ q_b}{r_{ab}^2}\ \hat{r}_{ba}$$

where $k = 9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}$.

The force ON a points away from b for like charges and points toward b for opposite charges. (Opposites attract, likes repel.)

(Ask yourself how you would rewrite this equation to get the force due to a acting on b.)

Two uniformly charged spheres are firmly fastened to (and electrically insulated from) frictionless pucks on an air table. The charge on sphere 2 is three times the charge on sphere 1. Which force diagram correctly shows the magnitude and direction of the electrostatic forces?



(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6 (G) none

How would you change the last picture if both spheres were negatively charged?

How would you change the last picture if one sphere were positively charged and one were negatively charged?

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