Physics 9 — Friday, November 16, 2018

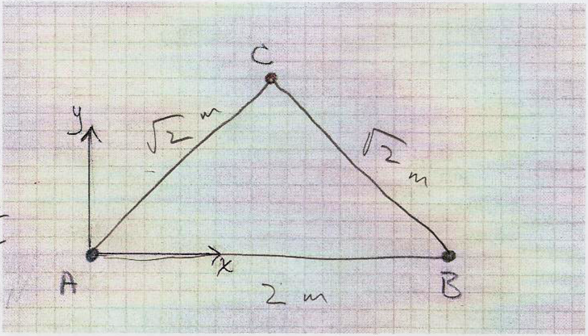
- Turn in HW9. Monday, I'll hand out HW10 (due 11/30, two weeks from today).
- For Monday, read Giancoli ch18 (electric currents): then we'll be ready to talk about both "volts" and "amps."
- Anyone still need a textbook?
- The main goals for the electricity segment (the last segment of the course) are for you to feel confident that you understand the meaning of electric potential (volts), electric current (amps), how these relate to energy and power, and also for you to understand the basic ideas of electric circuits (e.g. things wired in series vs in parallel). We'll get there soon.
- If you still have Wednesday's worksheet, then there is nothing to pick up at the back of the room. If you weren't here Wednesday, pick up the whole worksheet (stapled). If you were here but didn't bring back your worksheet, pick up the partial worksheet (single sheet).
- Next Wednesday I will do an optional class on Py-Processing.

We worked worksheet Question 1 out Wednesday, but we rounded up to $k \approx 10^{10}\,\mathrm{N\,m^2/C^2}$ to simplify the math, so $0.225\,\mathrm{N} \rightarrow 0.25\,\mathrm{N}$ and $0.45\,\mathrm{N} \rightarrow 0.5\,\mathrm{N}$.

Particles A, B, C carry identical positive charges $q_A = q_B = q_C = +10\ \mu\mathrm{C} = 1.0 \times 10^{-5}$ C. Particle A is located at $(x_A, y_A) = (0, 0)$ meters. Particle B is at $(x_B, y_B) = (+2.0, 0.0)$ meters. Particle C is at $(x_C, y_C) = (+1.0, +1.0)$ meters. The angles $\angle_{CAB}$ and $\angle_{CBA}$ are both 45°.

$$\frac{(9\times10^9)(10^{-5})^2}{(2)^2} = \frac{9}{4} = 0.225\,\mathrm{N}$$

$$\frac{(9\times10^9)(10^{-5})^2}{(\sqrt{2})^2} = \frac{9}{2} = 0.45\,\mathrm{N}$$

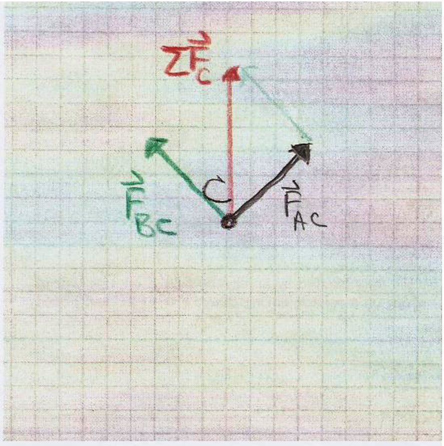


(1) What is the magnitude (in newtons) of the electric force between each pair of particles?

$F^E_{AB} =$ 0.225 N (≈ 0.25 N) $F^E_{AC} =$ 0.45 N (≈ 0.5 N) $F^E_{BC} =$ 0.45 N

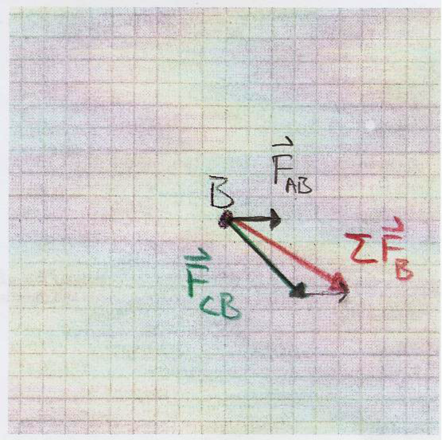
$F^E_{BA} =$ 0.225 N $F^E_{CA} =$ 0.45 N $F^E_{CB} =$ 0.45 N

(2) Draw arrows for the two electric forces that are acting ON particle C. (The electric force exerted by A on C is written $\vec{F}^E_{AC}$. The electric force exerted by B on C is written $\vec{F}^E_{BC}$.) Then draw an arrow for the vector sum of forces (a.k.a. the "net force") acting on particle C, which is written $\sum \vec{F}^E_C$. To make it easier to compare results, choose the length of your arrows so that the grid size on your force diagram is $0.1\ \mathrm{N}$. (Use the left grid below.)



We also worked out question 2 on Wednesday.

We also worked out question 3 on Wednesday. Then we very quickly did question 4. Since Q4 involves tricky use of unit vectors, let's repeat Q4 more carefully.



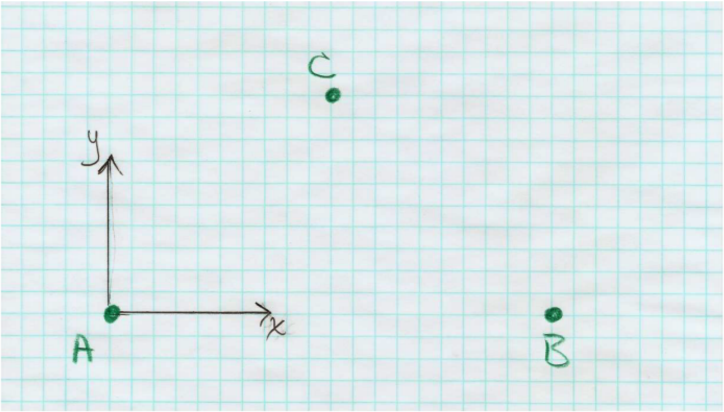
(3) Now draw (above right) arrows for the forces acting on particle B and their vector sum. Again use a grid size of $0.1\ \mathrm{N}$.

(4) Next, work out the Cartesian coordinates (F_x, F_y) for the forces acting on particle C and their vector sum.

$F^E_{AC,x} =$ $\qquad$ $F^E_{AC,y} =$

$F^E_{BC,x} =$ $\qquad$ $F^E_{BC,y} =$

$\sum F^E_{C,x} =$ $\qquad$ $\sum F^E_{C,y} =$



This may be helpful for Q4.

The electrostatic force due to b acting ON c is

$$\vec{F}^E_{bc} = k \, \frac{q_b \; q_c}{r^2_{bc}} \; \hat{r}_{bc}$$

where $k = 9.0 \times 10^9 \ \mathrm{N \cdot m^2/C^2} \ \approx \ 10^{10} \ \mathrm{N \cdot m^2/C^2}$.

The net force ON c due to a set of N charged objects is

$$\vec{F}^E_c = k \sum_{i=1}^{N} \frac{q_i \; q_c}{r^2_{ic}} \; \hat{r}_{ic}$$

where r_{ic} is the distance from object i to object c and $\hat{r}_{ic}$ is the **unit vector** pointing from object i toward object c.

(In practice, you usually draw $\hat{r}_{ic}$ equivalently as the unit vector pointing from object c away from object i, which is the same direction as the direction from i to c.)

(4) Next, work out the Cartesian coordinates (F_x, F_y) for the forces acting on particle C and their vector sum.

$F^E_{AC,x} = (0.45\,N)\cos(45^\circ) = +0.318\,N$

$F^E_{AC,y} = (0.45\,N)\sin(45^\circ) = +0.318\,N$

$F^E_{BC,x} = -0.318\,N$

$F^E_{BC,y} = +0.318\,N$

$\sum F^E_{C,x} = 0\,N$

$\sum F^E_{C,y} = +0.636\,N$

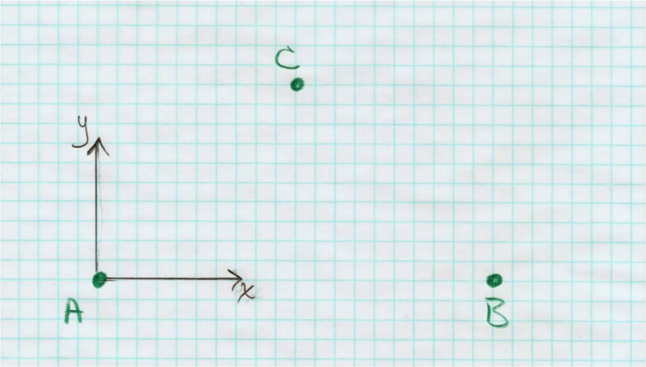
Rounding k from 9×10^9 up to 10^{10}, we would get $0.318\,\mathrm{N} \rightarrow 0.353\,\mathrm{N}$ and $0.636\,\mathrm{N} \rightarrow 0.707\,\mathrm{N}$.

Electric field due to N charged objects $Q_1, Q_2, \ldots, Q_N$

Just as the force experienced by a test charge q (positioned at point P) is the vector sum of the forces due to the other charges, the electric field $\vec{E}$ (evaluated at point P) due to N charged objects is the vector sum of the contributions from each charge:

$$\vec{E}(P) = \sum \frac{\vec{F}_{\text{on q}}}{q} = k \sum_{i=1}^{N} \frac{Q_i}{r_{iP}^2} \hat{r}_{iP}$$

That implies that the electric field due to N different source objects is just the *superposition* (i.e. the vector sum) of the electric fields due to the individual sources.



(5) Draw an arrow indicating the direction of the electric field $\vec{E}(P)$ at the point P given by $P_x = 1.0$ m, $P_y = 0.0$ m.

(6) Work out the Cartesian components (E_x, E_y) for the electric field $\vec{E}$ at the same point P.

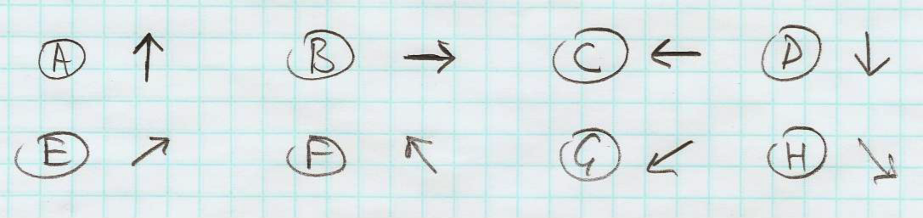
If we let $\vec{E}_A$, $\vec{E}_B$, and $\vec{E}_C$ be the electric field created by particle A, by particle B, and by particle C, respectively, then the combined electric field is the vector sum of these individual electric fields:

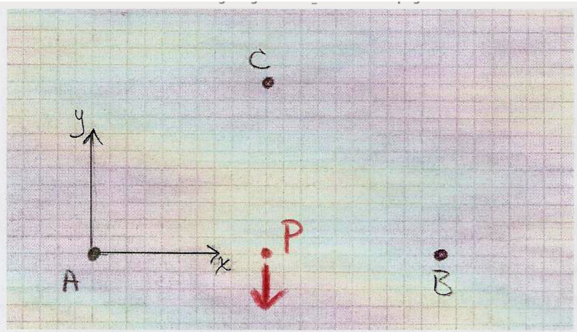
$$\vec{E}(P) = \vec{E}_A(P) + \vec{E}_B(P) + \vec{E}_C(P) = \sum_i k \frac{q_i}{r_{iP}^2} \hat{r}_{iP}$$

$E_x(P) =$

$E_y(P) =$

In which direction did you draw your $\vec{E}(P)$ arrow for question 5?



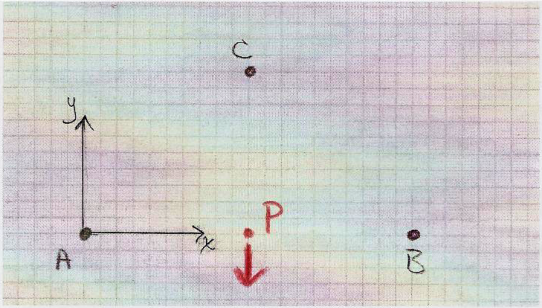


OK, now back to Question 6:

Work out the Cartesian components (E_x, E_y) for the electric field $\vec{E}$ at the same point P.

If we let $\vec{E}_A$, $\vec{E}_B$, and $\vec{E}_C$ be the electric field created by particle A, by particle B, and by particle C, respectively, then the combined electric field is the vector sum of these individual electric fields:

$$\vec{E}(P) = \vec{E}_A(P) + \vec{E}_B(P) + \vec{E}_C(P) = \sum_i k \frac{q_i}{r_{iP}^2} \hat{r}_{iP}$$



(5) Draw an arrow indicating the direction of the electric field $\vec{E}(P)$ at the point P given by $P_x = 1.0$ m, $P_y = 0.0$ m.

(6) Work out the Cartesian components (E_x, E_y) for the electric field $\vec{E}$ at the same point P.

If we let $\vec{E}_A$, $\vec{E}_B$, and $\vec{E}_C$ be the electric field created by particle A, by particle B, and by particle C, respectively, then the combined electric field is the vector sum of these individual electric fields:

$$\vec{E}(P) = \vec{E}_A(P) + \vec{E}_B(P) + \vec{E}_C(P) = \sum_i k \frac{q_i}{r_{iP}^2} \hat{r}_{iP}$$

$E_x(P) =$ 0 N/C

$E_y(P) =$ −90 kN/C

$$\frac{\left(9\times10^{-9}\ \frac{Nm^2}{C^2}\right)\left(10^{-5}\,C\right)}{(1.0\,m)^2} = 9\times10^{4}\ N/C$$

(7) Using your answer for part (6) and the equation $\vec{F}^E = q\vec{E}$ for the force exerted by an electric field $\vec{E}$ on a particle of charge q, what is the net electric force (magnitude and direction) acting on a particle having charge $q = -1$ μC and placed at point P ?

$F_x =$

$F_y =$

(7) Using your answer for part (6) and the equation $\vec{F}^E = q\vec{E}$ for the force exerted by an electric field $\vec{E}$ on a particle of charge q, what is the net electric force (magnitude and direction) acting on a particle having charge $q = -1\ \mu$C and placed at point P ?

$F_x =$ $0\ \text{N}$

$F_y =$ $q E_y = (-1.0\times10^{-6}\,\text{C})(-90\times10^{3}\,\frac{\text{N}}{\text{C}})$

$= +0.090\,\text{N} = +9\times10^{-2}\,\text{N}$

Rounding k from 9×10^9 up to 10^{10}, we would get $0.09\,\text{N} \to 0.10\,\text{N}$.

Motion in electric field.

Remember that the vector sum of forces acting ON an object causes the object to accelerate:

$$m\vec{a} = \sum \vec{F}$$

In an electric field $\vec{E}$, the force $\vec{F}$ on an object with charge q is

$$\vec{F} = q\vec{E}$$

If the force $\vec{F} = q\vec{E}$ is not balanced by any other force, a charged object will *accelerate* in an electric field:

$$\vec{a} = \frac{q}{m}\,\vec{E}$$

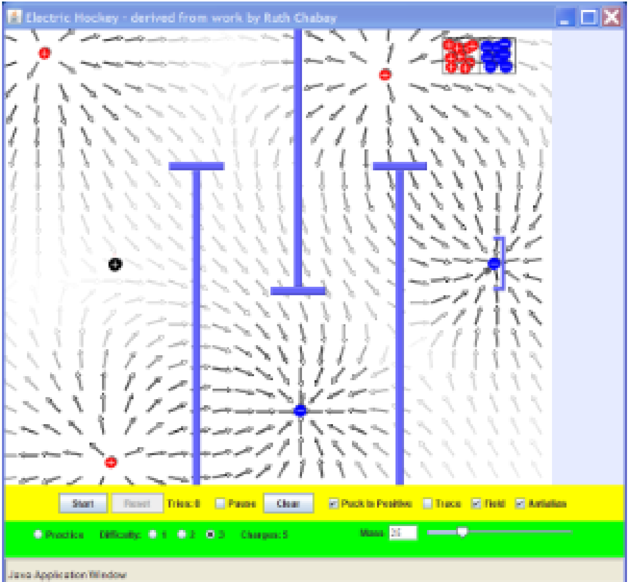
If some other force $\vec{F}_{\mathrm{other}}$ is also acting (e.g. gravity), then

$$\vec{a} = \frac{q}{m}\,\vec{E} + \frac{1}{m}\,\vec{F}_{\mathrm{other}}$$

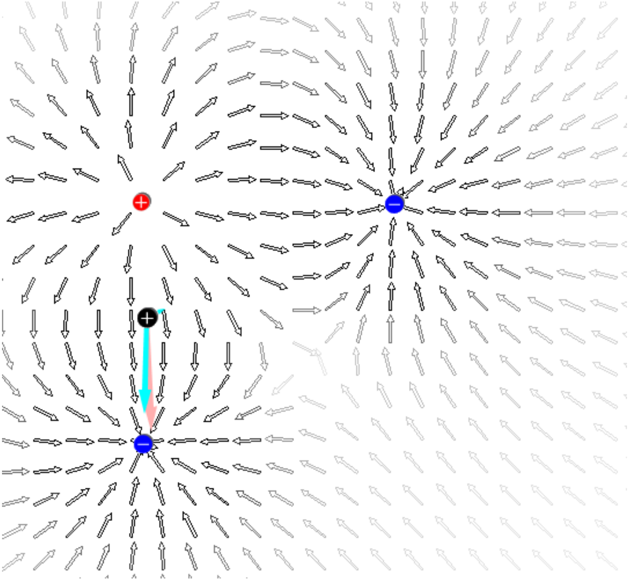
Electric field hockey: may help with $\vec{F} = q\vec{E}$

phet.colorado.edu/en/simulation/electric-hockey

http://www.youtube.com/watch?v=VuG4eG_KaUw



E.F.H can draw the electric field e.g. from upcoming HW10 problem 5. (The black ⊕ is the "test charge" and doesn't contribute to $\vec{E}$.)

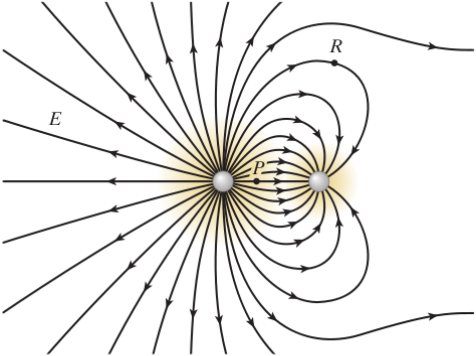


Someone else seems to have written an HTML5 version of Electric Field Hockey, which can be run without starting up a Java applet.

https://www.physicsclassroom.com/PhysicsClassroom/media/interactive/ElectricFieldHockey/index.html

"Flux" of electric field lines.

- $|\vec{E}|$ is proportional to the density of electric field lines. More closely spaced lines $\rightarrow$ bigger $|\vec{E}|$.
- Think of the field lines "flowing" (or radiating, like light) out from the $(+)$ charges and into the $(-)$ charges.
- The total "flux" of $\vec{E}$ through a hypothetical **closed** surface is proportional to the total charge enclosed by that surface.

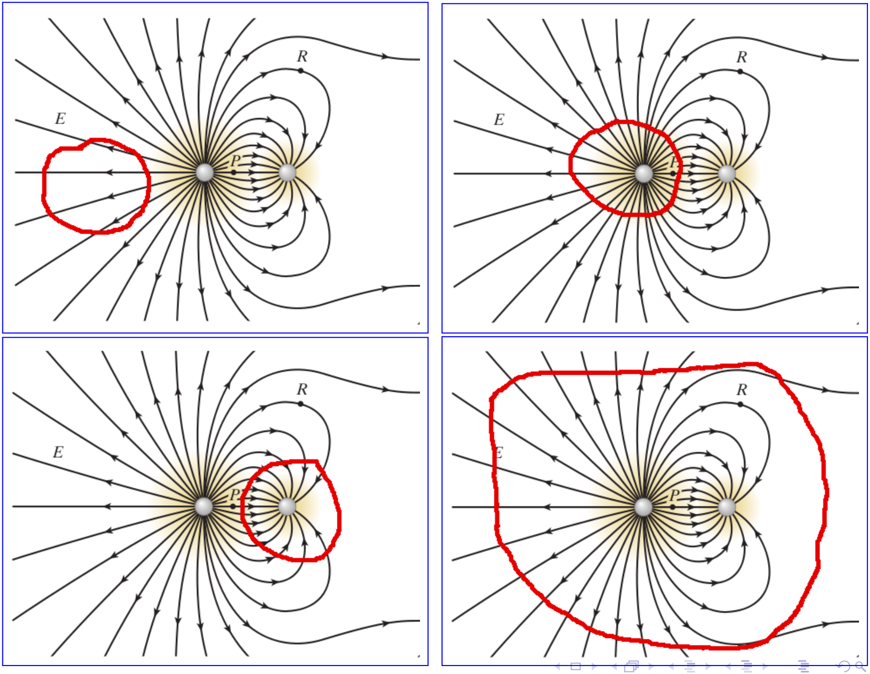


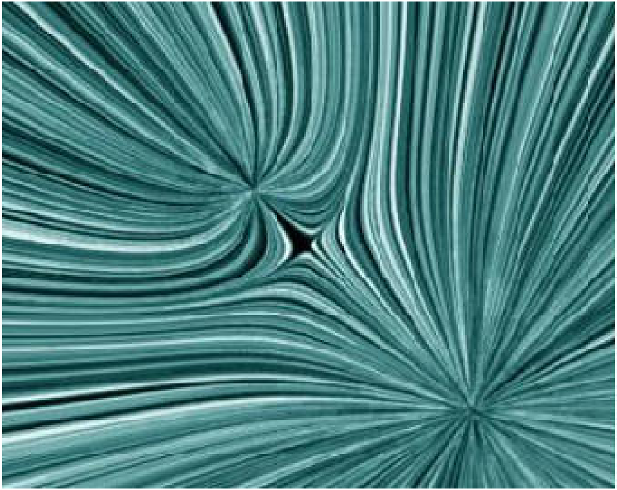
Where is $|\vec{E}|$ largest here?

What are the signs of the two particles' charges?

If you draw a circle that encloses no net charge, what is the net flux through the circle?

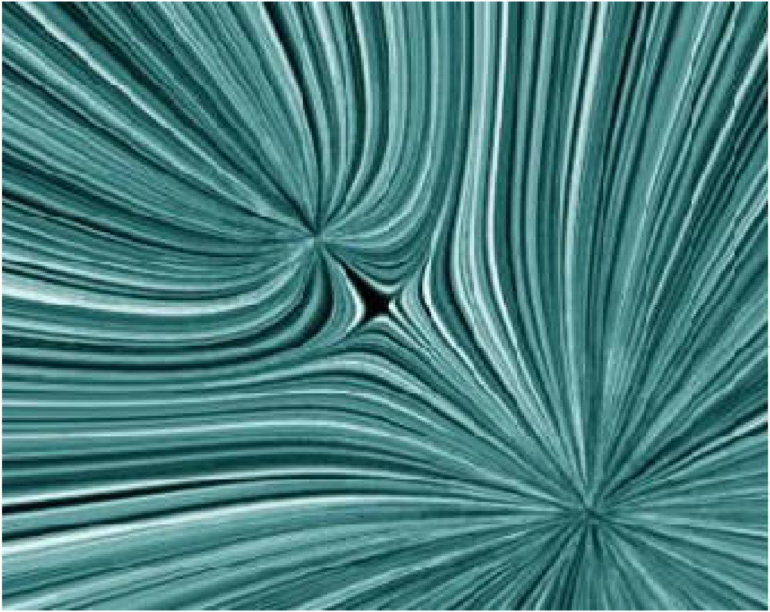
Is net charge enclosed by each circle $+$, $-$, or 0 ?



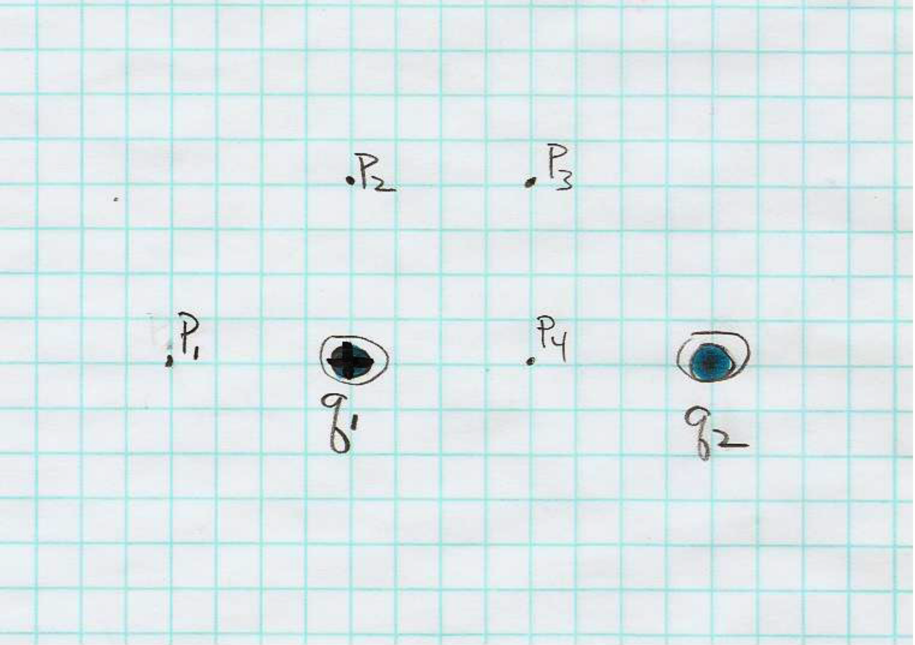


The image shows the electric field around two charged particles, using the “grass seed” representation of field lines. From this picture you can conclude that

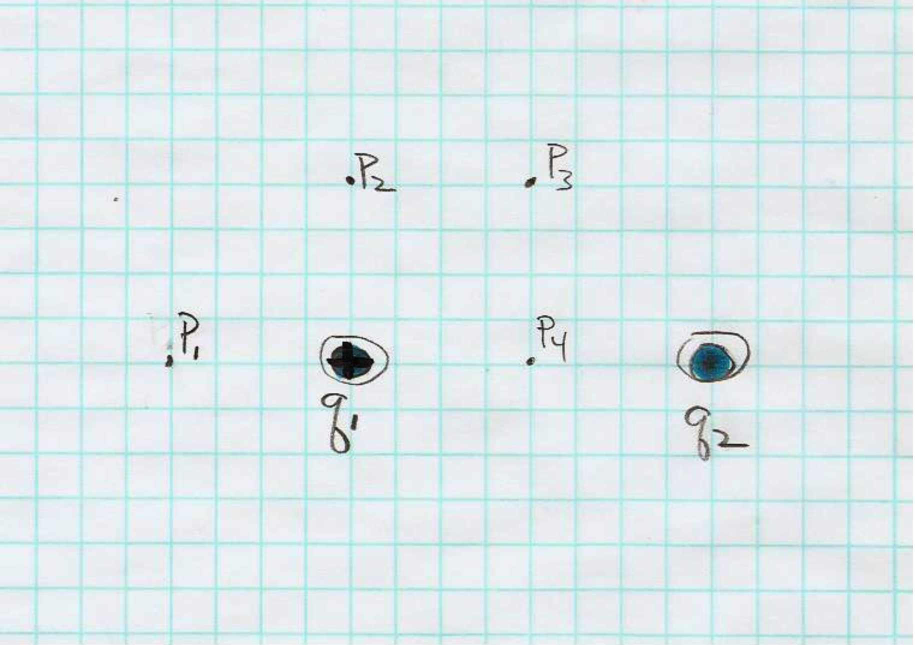
(A) The two charges attract one another.
(B) The two charges repel one another.
(C) Not enough information to tell whether they repel or attract.



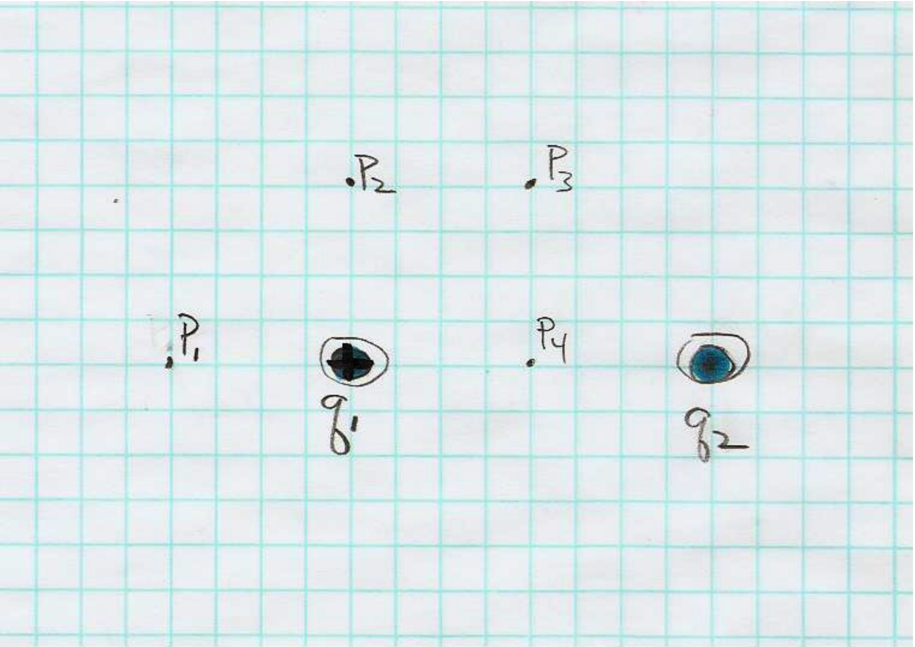
(Follow-up: Where are the two particles located? Can you say what the signs of the charges are? Can you tell whether the charge magnitudes are the same or different?)



Using **superposition,** draw $\vec{E}$ at points $P_1 \ldots P_4$ if $q_2 = +q_1$.



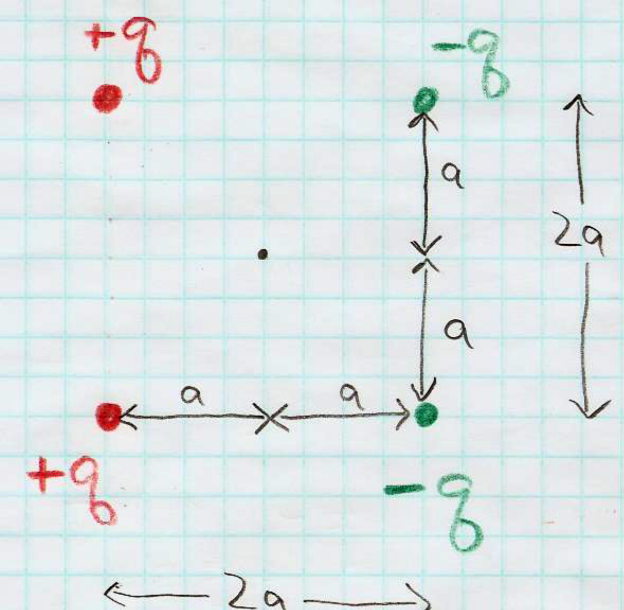
Using **superposition,** draw $\vec{E}$ at points $P_1 \ldots P_4$ if $q_2 = +2q_1$.



Using **superposition,** draw $\vec{E}$ at points $P_1 \ldots P_4$ if $q_2 = -q_1$.

https://phet.colorado.edu/sims/html/charges-and-fields/latest/charges-and-fields_en.html

Is anyone interested in working through this example? Four charged particles are arranged in a square (side length $2a$), as shown. Find & draw the electric field at the center of the square.

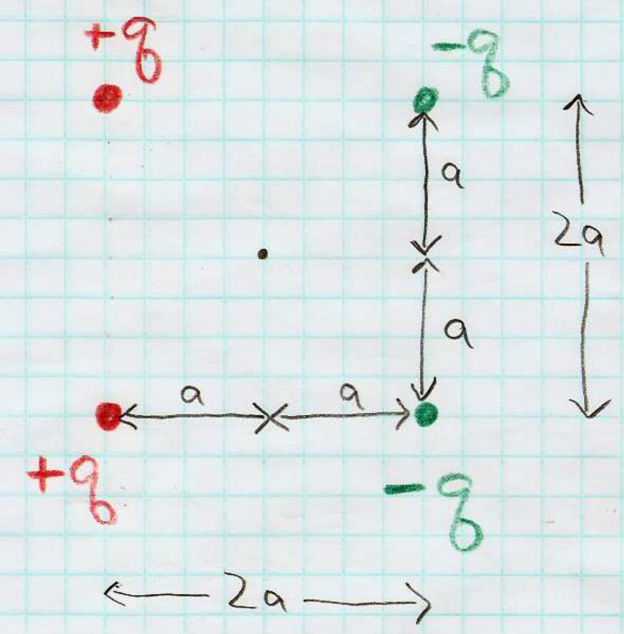


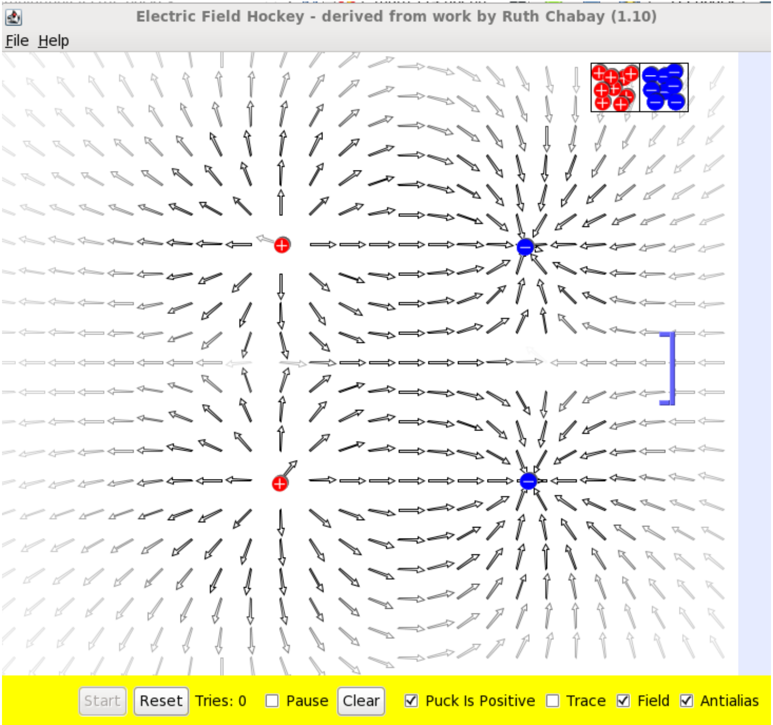
(We'll see on next page that Electric Field Hockey can draw the E field diagram for charge configurations like this.)

$$E_x = \sqrt{2}\,\frac{kq}{a^2}$$

$$E_y = 0$$

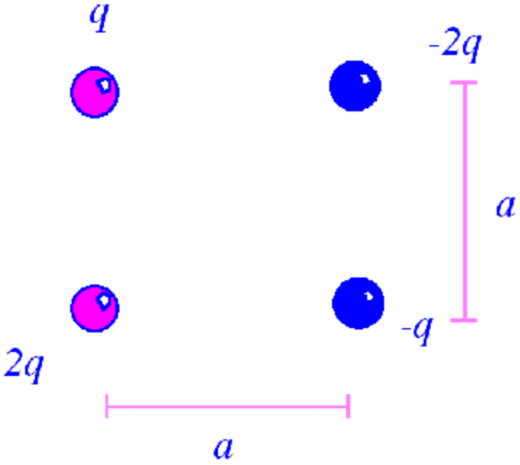
$$\vec{E} = \sqrt{2}\,\frac{kq}{a^2}\,\hat{i}$$



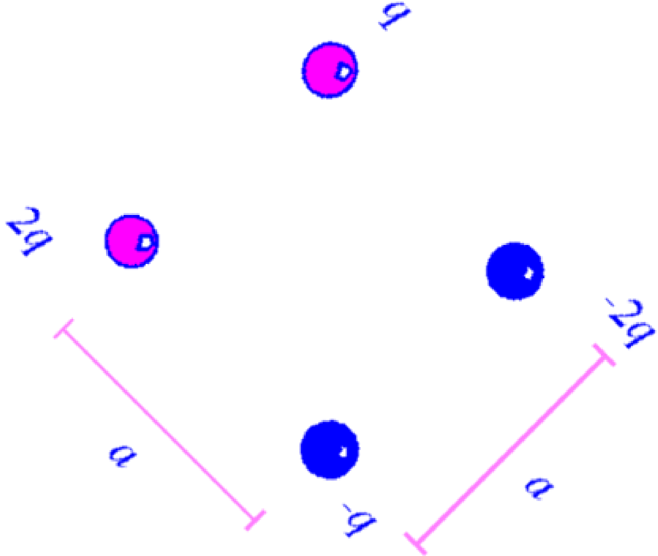


Electric Field Hockey - derived from work by Ruth Chabay (1.10)
File Help
Start
Reset
Tries: 0
Pause
Clear
Puck Is Positive
Trace
Field
Antialias

Is anyone interested in working through this example? Four charged particles are arranged in a square, as shown. Find (and draw) the electric field at the center of the square. (Let's instead solve this problem in a friendlier coordinate system.)



Four charged particles are arranged in a square, as shown. Find (and draw) the electric field at the center of the square. For convenience, define $d = a/\sqrt{2}$, which is the distance of each particle to the center of the square.



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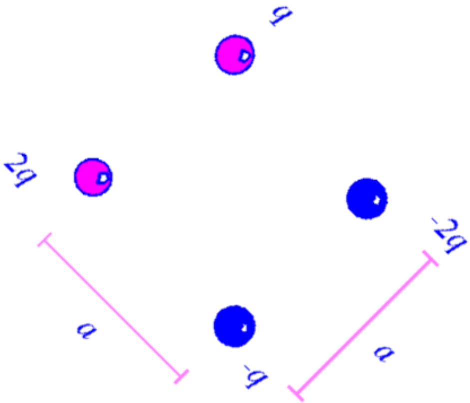
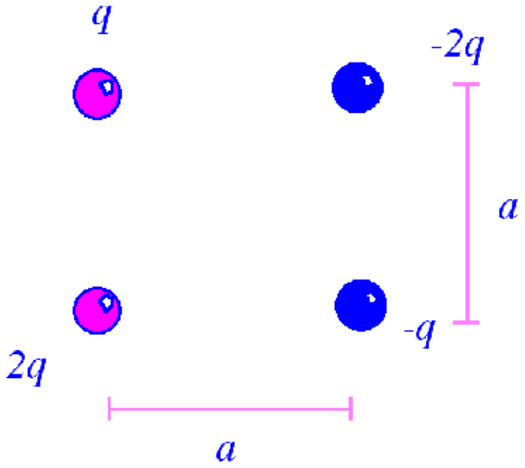


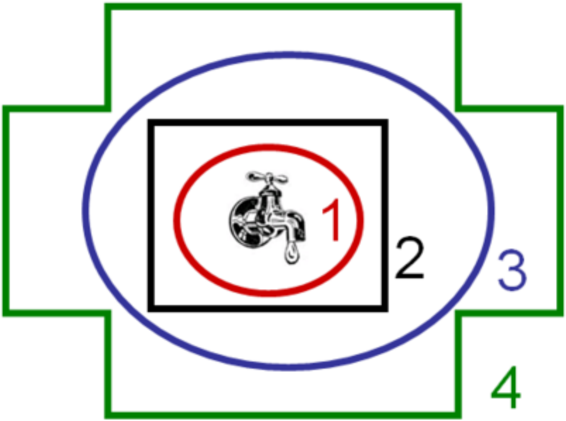
$$E_x = +4\,\frac{kq}{d^2} \qquad E_y = -2\,\frac{kq}{d^2} \qquad |E| = \sqrt{20}\,\frac{kq}{d^2} = 4\sqrt{5}\,\frac{kq}{a^2}$$

Four charged particles are arranged in a square, as shown. Find (and draw) the electric field at the center of the square. (Here was the messier solution to the original problem.)



$$E_x = 6\sqrt{2}\,\frac{kq}{a^2} \qquad E_y = 2\sqrt{2}\,\frac{kq}{a^2} \qquad |\vec{E}| = 4\sqrt{5}\,\frac{kq}{a^2}$$

A faucet tap is turned on at the center. Rank order which closed line has the most (to the least) water flowing across the line per unit time. (The faucet has been on for a long time.)



(A) $1 > 2 > 3 > 4$

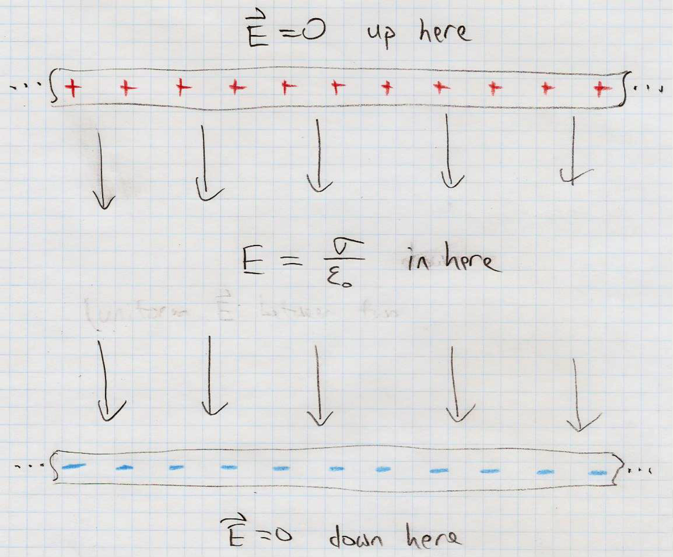
(B) $1 = 2 > 3 > 4$

(C) $1 = 3 > 2 > 4$

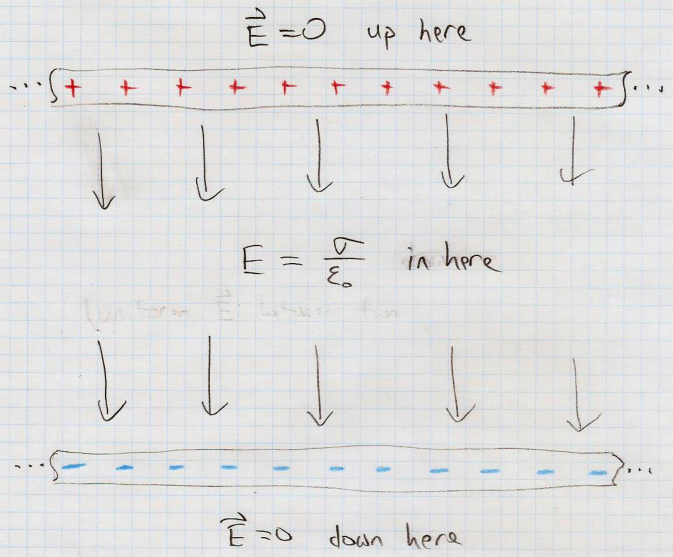
(D) $4 > 3 > 2 > 1$

(E) $1 = 2 = 3 = 4$

(F) none of the above

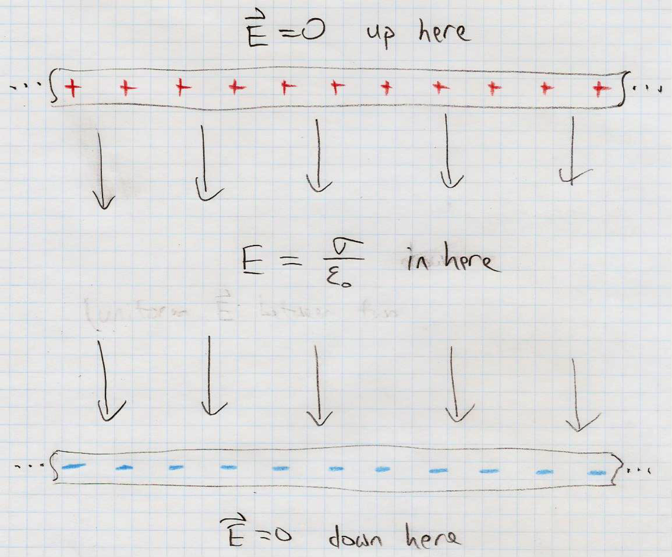


The most commonly used way to create a uniform electric field is to use the area between two large, parallel, oppositely-charged planes of uniform charge-per-unit-area, $\sigma = Q/A$.



Notice that if you do this, a positive particle will “fall” in the direction that $\vec{E}$ points, just as a rock will fall in the direction gravity points — toward Earth’s surface. To lift up a positive particle, you would have to add energy (do + work).

Suppose I move a charged particle vertically upward in the region where $\vec{E}$ is uniform and points downward. The work-per-unit-charge that I have to do to move the particle up is:

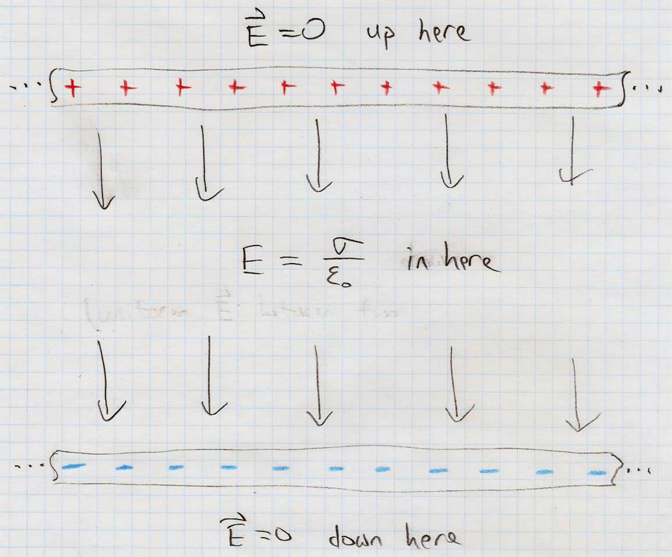


(A) positive

(B) negative

(C) zero

Suppose I move a charged particle vertically **downward** in the region where $\vec{E}$ is uniform and points downward. The work-per-unit- charge that I have to do to move the particle is:

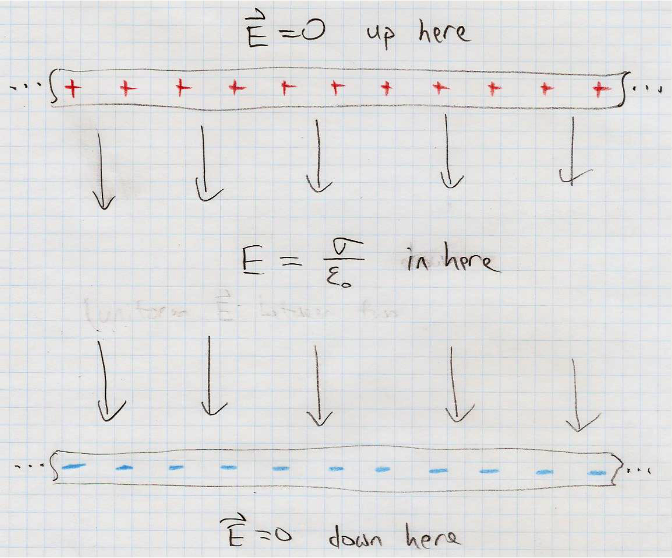


(A) positive

(B) negative

(C) zero

Suppose I move a charged particle **horizontally** in the region where $\vec{E}$ is uniform and points downward. The work-per-unit-charge that I have to do to move the particle is:

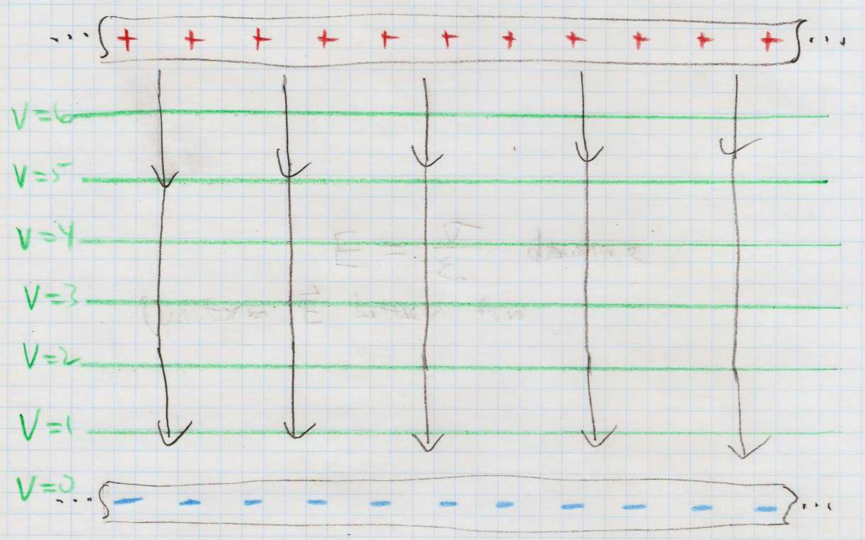


(A) positive

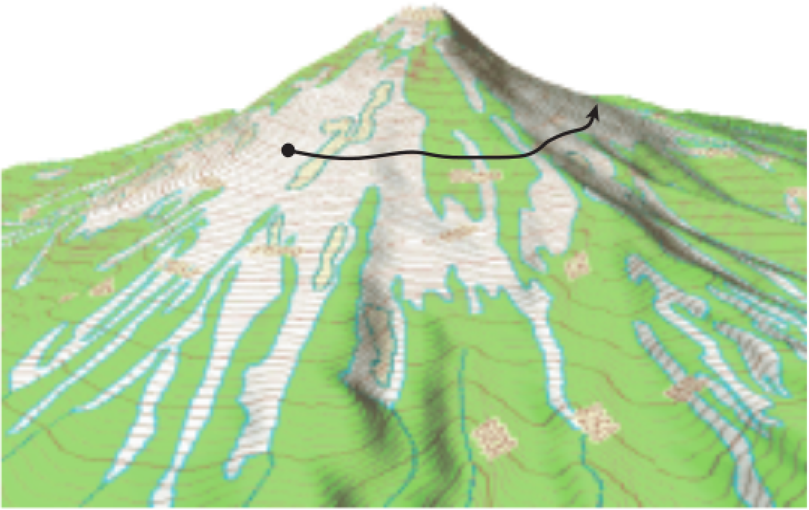
(B) negative

(C) zero

Electrostatic potential is analogous to altitude. Gravity points in the direction in which altitude decreases most quickly. $\vec{E}$ points in the direction in which "voltage" decreases most quickly. Equipotential lines are perpendicular to $\vec{E}$.



Contour lines on a topo map are always perpendicular to gravity. Contour lines are lines of constant elevation. Moving along a contour line, you do no work against gravity. Along a contour line, G.P.E. (per unit mass) is constant.



Equipotential lines (constant V) are perpendicular to $\vec{E}$. Moving along an equipotential, you do no work against $\vec{E}$. Along an equipotential, E.P.E. (per unit charge) is constant.

Equipotential lines (constant V) are perpendicular to $\vec{E}$. Moving along an equipotential, you do no work against $\vec{E}$. Along an equipotential, E.P.E. (per unit charge) is constant.

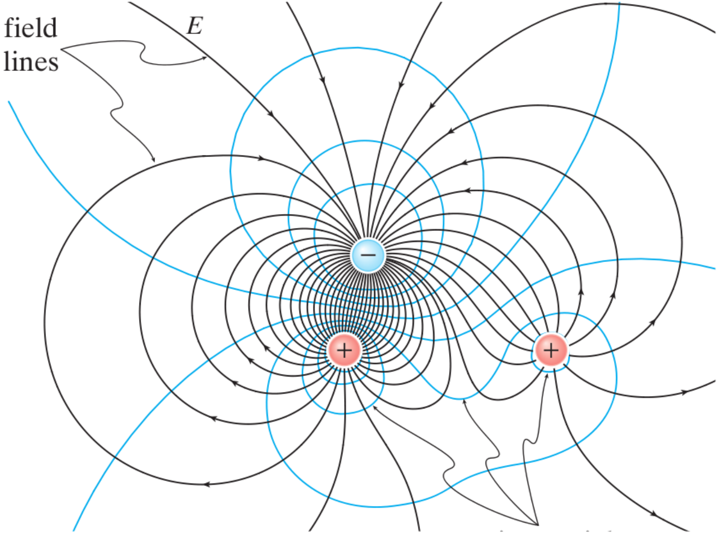
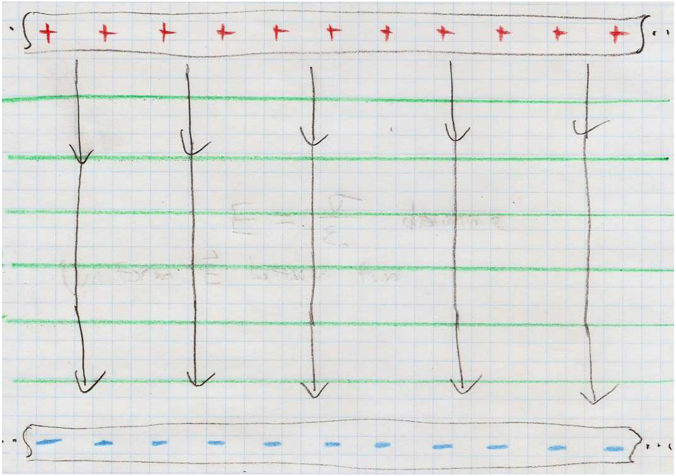


Figure 25.9 Field lines and equipotentials for three stationary charged particles.

I am standing in a uniform electric field, of magnitude 1 N/C, which points downward. I climb up 1 meter. What is the potential difference, $V_{1\to 2} = V_2 - V_1$, between my old location and my new location? (Note: 1 N/C is the same as 1 volt per meter.)



(A) $V_{1\to 2} = +1$ volt

(B) $V_{1\to 2} = -1$ volt

(C) $V_{1\to 2} = 0$ volts

The "potential difference" between point a and point b is **minus** the work-per-unit-charge done by the electric field in moving a test particle from a to b.

$$V_{ab} = -\frac{1}{q}\int_a^b \vec{F}^E \cdot \mathrm{d}\vec{\ell} = -\int_a^b \vec{E} \cdot \mathrm{d}\vec{\ell}$$

More intuitively, V_{ab} is (**plus**) the work-per-unit-charge that an external agent (like me) would have to do to move a particle from a to b. I would be working against the electric field to do this.

But a much easier-to-remember definition of voltage is "electric potential energy per unit charge."

Just as $\vec{E}$ is electric force per unit charge, V is electric potential energy per unit charge.

$$V = \frac{U^E}{q}$$

Moving a **positive** particle to higher V means moving it to a position of higher electric potential energy.

Near Earth's surface, gravitational potential energy is

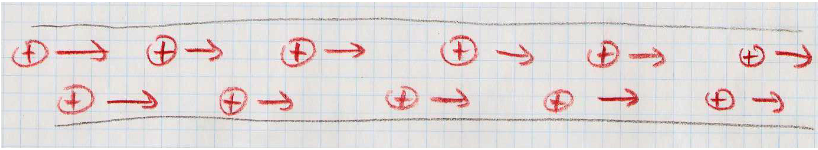
$$U^G = m\ g\ h$$

G.P.E. per unit mass would be just $(U/m) = gh$, which is proportional to altitude. Moving an object (no matter what mass) along a contour of equal gh does not require doing any work against gravity, and does not change the object's G.P.E.

In a uniform downward-pointing electric field, electric potential energy is

$$U^E = q\ E\ y$$

E.P.E. per unit charge would be just $V = (U/q) = E\ y$. So if $\vec{E}$ is uniform and points down, then potential (or "voltage") V is analogous to altitude. Moving perpendicular to $\vec{E}$ does not require doing any work against $\vec{E}$, and does not change E.P.E. So "equipotential" lines (constant V) are always perpendicular to $\vec{E}$.

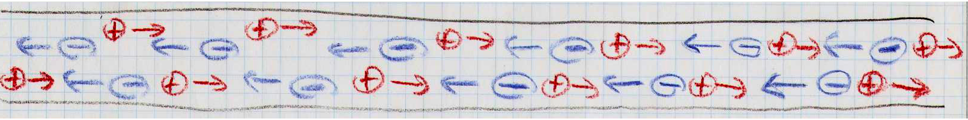


Inside a wire, positively charged particles are moving to the right. What is the direction of the electric current (symbol I, unit = ampere, or "amp") ?

(A) up
(B) down
(C) left
(D) right
(E) into the page
(F) out of the page
(G) zero

Inside a wire, negatively charged particles are moving to the right. What is the direction of the electric current?

(A) up
(B) down
(C) left
(D) right
(E) into the page
(F) out of the page
(G) zero



Inside a wire, positively charged particles are moving to the right. An equal number of negatively charged particles is moving to the left, at the same speed. The electric current is

(A) flowing to the right

(B) flowing to the left

(C) zero

Physics 9 — Friday, November 16, 2018

- Turn in HW9. Monday, I'll hand out HW10 (due 11/30, two weeks from today).
- For Monday, read Giancoli ch18 (electric currents): then we'll be ready to talk about both "volts" and "amps."
- Anyone still need a textbook?
- The main goals for the electricity segment (the last segment of the course) are for you to feel confident that you understand the meaning of electric potential (volts), electric current (amps), how these relate to energy and power, and also for you to understand the basic ideas of electric circuits (e.g. things wired in series vs in parallel). We'll get there soon.
- If you still have Wednesday's worksheet, then there is nothing to pick up at the back of the room. If you weren't here Wednesday, pick up the whole worksheet (stapled). If you were here but didn't bring back your worksheet, pick up the partial worksheet (single sheet).
- Next Wednesday I will do an optional class on Py-Processing.