

Physics 351, Spring 2015, Homework #9.

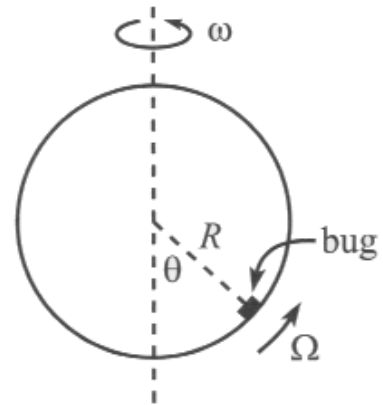
Due at start of class, Friday, March 27, 2015

Course info is at positron.hep.upenn.edu/p351

When you finish this homework, remember to visit the feedback page at
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to tell me how the homework went for you.

1. A high-speed train is traveling at a constant 150 m/s (about 335 mph) on a straight, horizontal track across the South Pole. Find the angle between a plumb line suspended from the ceiling inside the train and another inside a hut on the ground. In what direction is the plumb line on the train deflected?
2. If a negative charge $-q$ (an electron, for example) in an elliptical orbit around a fixed positive charge Q is subjected to a weak uniform magnetic field $\mathbf{B}$, the effect of $\mathbf{B}$ is to make the ellipse precess slowly — an effect known as **Larmor precession**. To prove this, write down the EOM of the negative charge in the field of Q and $\mathbf{B}$. Now rewrite it for a frame rotating with angular velocity $\mathbf{\Omega}$. [Remember that this changes both $d^2\mathbf{r}/dt^2$ and $d\mathbf{r}/dt$.] Show that by suitable choice of $\mathbf{\Omega}$ you can arrange that the terms involving $\dot{\mathbf{r}}$ cancel out, but that you are left with one term involving $\mathbf{B} \times (\mathbf{B} \times \mathbf{r})$. If $\mathbf{B}$ is weak enough this term can certainly be neglected. Show that in this case the orbit in the rotating frame is an ellipse (or hyperbola). Describe the appearance of the ellipse as seen in the original nonrotating frame.
3. A mass is dropped from a point directly above the equator. Consider the instant when the object has fallen a distance d . (a) If we consider only the centrifugal force, then you can quickly show that the correction to the effective g at this point (relative to the release point) is an increase by $\Omega^2 d$. (b) There is, however, a second-order Coriolis effect. What is the sum of these corrections? (c) The value of g_0 also varies with height, and this produces an increase in the effective g equal to $(2d/R)g_0$. Show that this is much larger than the above centrifugal and Coriolis effects.

4. A hoop of radius R is made to rotate at constant angular speed ω around a diameter, as shown in the figure. A small bug of mass m walks at constant angular speed Ω around the hoop. Let $\mathbf{F}$ be the total force that the hoop applies to the bug when the bug is at the angle θ shown, and let $F_{\perp}$ be the component of $\mathbf{F}$ that is perpendicular to the plane of the hoop. Find $F_{\perp}$ in two ways (ignore gravity in this problem): (a) Work in the lab frame: at the angle θ , find the rate of change of the bug's angular momentum around the rotation axis, and then consider the torque on the bug. (b) Work in the rotating frame of the hoop: at the angle θ , find the relevant fictitious force, and then take it from there.



5. Consider a frictionless puck on a horizontal turntable that is rotating counterclockwise with angular velocity Ω . (a) Write down Newton's second law for the coordinates x and y of the puck as seen by me standing on the turntable. (Be sure to include the centrifugal and Coriolis forces corresponding to the turntable's rotation, but ignore the earth's rotation.) (b) Solve the two equations by the trick of writing $\eta = x + iy$ and guessing a solution of the form $\eta = e^{-i\alpha t}$. [In this case — as in the case of the critically damped SHM in §5.4 — you get only one solution this way. The other has the same form (Eq. 5.43) we found for the second solution for the critically damped SHM.] Write down the general solution. (c) At time $t = 0$, I push the puck from position $\mathbf{r}_0 = (x_0, 0)$ with velocity $\mathbf{v}_0 = (v_{x0}, v_{y0})$ (all as measured by me on the turntable). Show that

$$\begin{aligned} x(t) &= (x_0 + v_{x0}t) \cos \Omega t + (v_{y0} + \Omega x_0)t \sin \Omega t \\ y(t) &= -(x_0 + v_{x0}t) \sin \Omega t + (v_{y0} + \Omega x_0)t \cos \Omega t \end{aligned}$$

(d) Use `ParametricPlot` to graph the orbits of the puck with $x_0 = \Omega = 1$ and the following initial velocities $\mathbf{v}_0$. (i) (0,1). (ii) (0,0). (iii) (0,-1). (iv) (-0.5,-0.5). (v) (-0.7,-0.7). (vi) (0,-0.1). If you have spare time, ponder (in terms of fictitious forces) the features of the small- t behavior for the last three cases.

6. A thin rod (of width zero, but not necessarily uniform) is pivoted freely at one end about the horizontal z axis, being free to swing in the xy plane (x horizontal, y vertically down). Its mass is m , its CM is a distance a from the pivot, and its moment of inertia (about the z axis) is I . (a) Write down the EOM $\dot{L}_z = \Gamma_z$ and, assuming the motion is confined to small angles (measured

from the downward vertical), find the period of this compound pendulum. (“Compound pendulum” is traditionally used to mean any pendulum whose mass is distributed — as contrasted with a “simple pendulum,” whose mass is concentrated at a single point on a massless arm.) (b) What is the length of the “equivalent” simple pendulum, that is, the simple pendulum with the same period?

7. Consider the rod of Problem 6. The rod is struck sharply with a horizontal force F which delivers an impulse $F \Delta t = \xi$ a distance b below the pivot. (a) Find the rod’s angular momentum about the pivot, and hence the rod’s momentum, just after the impulse. (b) Find the impulse η delivered to the pivot. (c) For what value of b (call it b_0) is $\eta = 0$? (The distance b_0 defines the so-called “sweet spot.” If the rod were a tennis racquet and the pivot your hand, then if the ball hits the sweet spot, your hand would experience no impulse.)

8. A rigid body comprises 8 equal masses m at the corners of a cube of side a , held together by massless struts. (a) Use the definitions (Eq. 10.37 and 10.38) $I_{xx} = \sum m_\alpha (y_\alpha^2 + z_\alpha^2)$ and $I_{xy} = -\sum m_\alpha x_\alpha y_\alpha$ (and cyclic permutations) to find the moment of inertia tensor $\mathbf{I}$ for rotation about a corner O of the cube. (Use axes along the three edges through O .) (b) Find the inertia tensor of the same body but for rotation about the center of the cube. (Again use axes parallel to the edges.) Explain why in this case certain elements of $\mathbf{I}$ could be expected to be zero.

9. Uhoh — the errata (“Important errors,” page 484) to David Morin’s textbook state that HW8/q4 should be **replaced** (!) with the following problem, which I want you to solve so that you’re not led astray. At a polar angle θ , a projectile is fired eastward with speed v_0 at an angle α above the ground. Show that the southward (in the northern hemisphere) and eastward deflections due to the Coriolis force are (to first order in Ω)

$$d_{\text{south}} = \frac{4\Omega v_0^3}{g^2} \cos \theta \cos \alpha \sin^2 \alpha$$

$$d_{\text{east}} = \frac{4\Omega v_0^3}{g^2} \sin \theta \left(\cos^2 \alpha \sin \alpha - \frac{1}{3} \sin^3 \alpha \right).$$

Hint: The first term in d_{east} arises because the flight time is modified due to the vertical component of the Coriolis force.

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XC00*. Optional/extra-credit. If there are extra-credit problems from HW4,5,6,7,8 that you didn't have time to do sooner, you can feel free to turn them in with HW10 for full credit. Just clearly indicate for Tanner which problem you're solving.

XC1*. Optional/extra-credit. (a) Show that if an object is thrown with initial velocity $\mathbf{v}_0$ from a point O on the earth's surface at colatitude θ , then (as you worked out last week, in less generality) to first order in Ω its orbit is

$$\begin{aligned}x &= v_{x0}t + \Omega(v_{y0} \cos \theta - v_{z0} \sin \theta)t^2 + \frac{1}{3}\Omega g t^3 \sin \theta \\y &= v_{y0}t - \Omega(v_{x0} \cos \theta)t^2 \\z &= v_{z0}t - \frac{1}{2}gt^2 + \Omega(v_{x0} \sin \theta)t^2\end{aligned}$$

Use this result to do the following: A naval gun shoots a shell at colatitude θ in a direction that is an angle α above the horizontal and due east, with muzzle speed v_0 . (b) Ignoring the earth's rotation (and air resistance), find how long (t) the shell would be in the air and how far away (R) it would land. If $v_0 = 500$ m/s and $\alpha = 20^\circ$, what are t and R ? (c) A naval gunner spots an enemy ship due east at the range R of part (b) and, forgetting about the Coriolis effect, aims his/her gun exactly as in part (b). Find by how far north or south, and in which direction, the shell will miss the target, in terms of Ω , v_0 , α , θ , and g . (It will also miss in the east-west direction, but we save this complication for XC2.) (d) If the incident occurs at latitude 50° north ($\theta = 40^\circ$), what is this distance? (e) What if the latitude is 50° south ($\theta = 140^\circ$)? This problem is a serious issue in long-range gunnery: In a battle near the Falkland Islands (a.k.a. Islas Malvinas) in World War I, the British navy consistently missed German ships by many tens of yards because they apparently forgot that the Coriolis effect in the southern hemisphere is opposite to that in the north.

XC2*. Optional/extra-credit. For problem XC1(d) and (e), find the distance by which the shell misses its target in both the north-south and east-west directions. [In this case, you should account for the fact that the time of flight is affected by the Coriolis effect — a complication that we ignored last week in HW8.] How much is your calculated deflection affected by whether or not you account for the time-of-flight correction? (**Oops!** While the TOF correction to the N-S deflection is a correction to a correction, i.e. $\mathcal{O}(\Omega^2)$, the TOF correction to the eastward range is a correction ($\mathcal{O}(\Omega)$) to a value that starts off with no Ω dependence, so it is significant.)