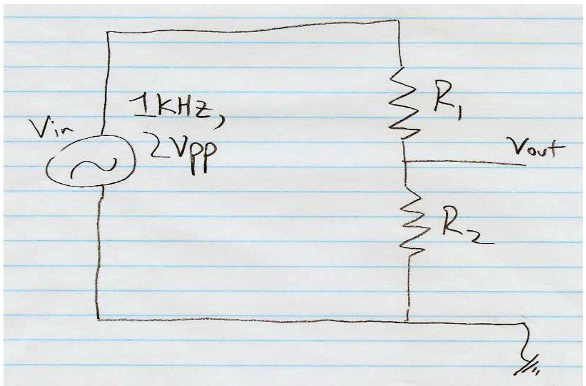
Physics 364 – fall 2010 – homework #2 – due Thursday, 2010-09-23

This week's homework will be simply to analyze on paper (and with LTspice, if you're able to run it) the circuits that we will build in Lab #2. You should get started on at least the first few of these circuits before Monday's lab, so that you will understand what you are building before you build it. We will spend part of Monday's lecture looking at the same (or similar) circuits, both on the blackboard and in LTspice.

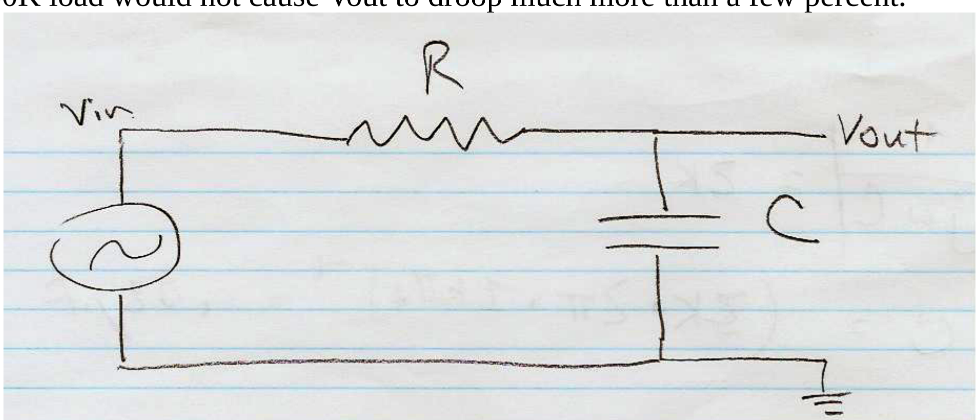
Problem 1. Draw the input and output of the following circuit using (a) R1=1K, R2=4K; (b) R1=2K, R2=3K; (c) R1=4K, R=1K; (d) R1=R2=2.5K. This is pretty simple and is just to remind you of why we call this circuit a voltage divider.



Problem 2. Look again at the circuit from problem 1(d). Does it make any difference that f=1kHz? Now replace R2 with a .068μF capacitor. Calculate |Vout/Vin| for f=0Hz (i.e. for DC), f=100Hz, f=1kHz, f=10kHz, f=1MHz. (Remember that $f=\omega/2\pi$.) At what limit (in frequency) is |Vout/Vin| maximized? What is this maximum value? At what frequency does |Vout/Vin| fall to $1/\sqrt{2}\approx 0.707$ of its maximum?

Since $20\cdot\log_{10}\sqrt{2}=3.010\cdots\approx 3$, a factor of √2 in amplitude (or a factor of 2 in power) is often called 3dB in electronics. The frequency at which a filter's output drops by 3dB (w.r.t. the output at the frequency that it "passes" most readily) is called $f_{3\mathrm{dB}}$. The filter in this problem – a *low pass* filter – passes low frequencies and attenuates high frequencies.

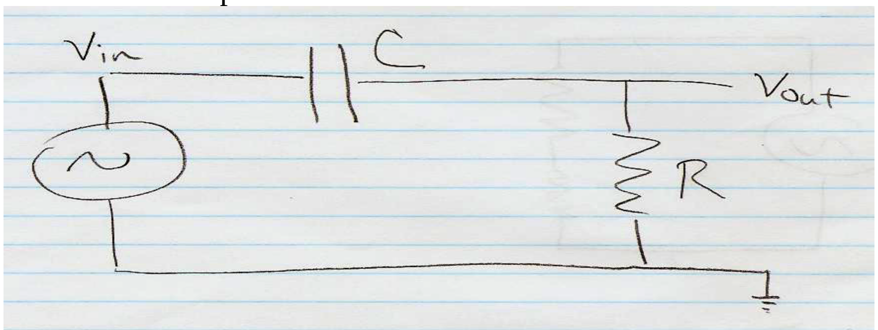
How does $f_{3\mathrm{dB}}$ depend on R and C? How would you choose R and C if you wanted to preserve audio frequencies (up to 10kHz or so) and to cut out unwanted signals at higher frequencies? Choose values such that a 100K load would not cause Vout to droop much more than a few percent.



Problem 3. Look again at the low-pass filter from problem 2, with R=2.5K and C=.068μF. What happens to |Vout/Vin| if you place a 250K load resistor in parallel with C? Now what happens if you place a 2.5K load resistor in parallel with C? What is |Vout/Vin| at DC now? What is f_{3dB} now?

Problem 4. Now interchange R and C from problem 2, so that you have the following circuit – a *high-pass* filter – with R=2.5K and C=.068μF. Calculate |Vout/Vin| for DC, f=100Hz, 1kHz, 10kHz, 1MHz. At what limiting case is |Vout/Vin| maximized? What is f_{3dB} ? Make a rough sketch of the frequency response (magnitude of Vout/Vin vs. frequency) on a log-log plot, pointing out pertinent features.

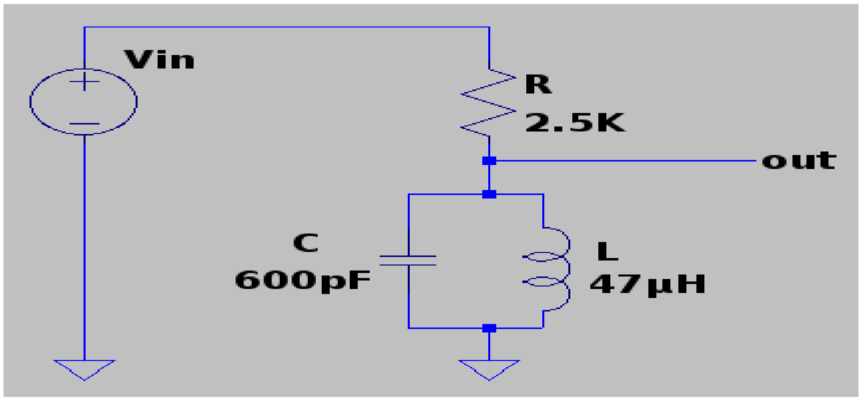
How would you choose R and C to reject 60 Hz noise while preserving as much as possible of the rest of the audio spectrum? (Note that a piano tuning fork rings at 440 Hz – the A above middle C, and that telephone equipment passes 300 to 3300 Hz.) Choose values such that a 100K load would not cause Vout to sag much more than a few percent from its unloaded value.



Problem 5. You probably see a pattern here – that we're drawing voltage dividers in which a resistance is replaced by an *impedance* that has non-trivial frequency dependence. Now let's make a *band-pass* filter, by replacing R2 in problem 1 with the parallel combination of a capacitor and an inductor. Take R=2.5K, C=600pF, L=47μH. What is f_0 , at the center of the pass band? What is the quality factor Q? What is the 3dB bandwidth, i.e. Δf between the two frequencies at which |Vout/Vin| is 3dB below the peak? On what frequency does WPEN (AM) transmit? How about KYW (AM)? And WPHT? (The last two are the only 50 kilowatt Philadelphia stations I found in Wikipedia.)

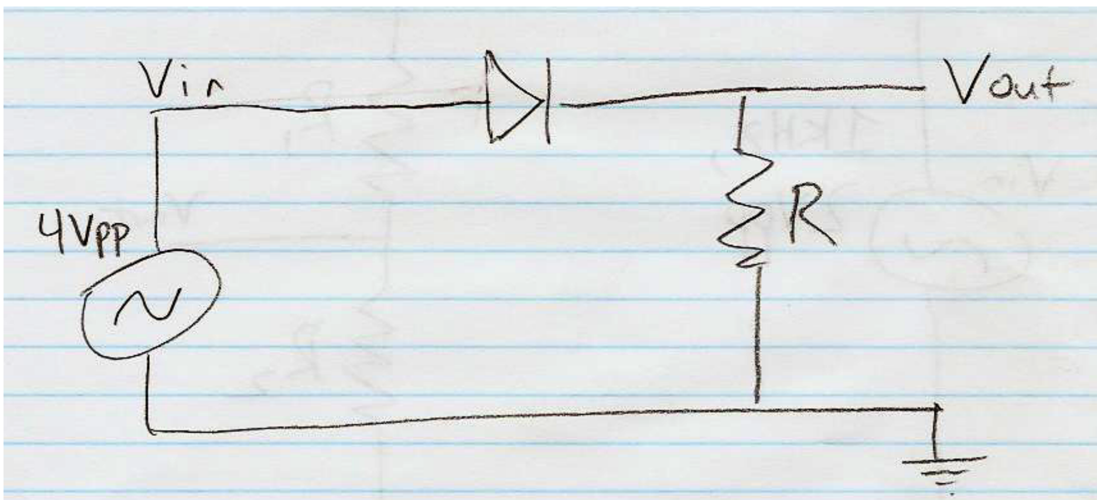
Suppose I have a drawer full of 33pF and 47pF capacitors. What is an easy way to change the capacitance from 600pF to 633pF or 647pF?

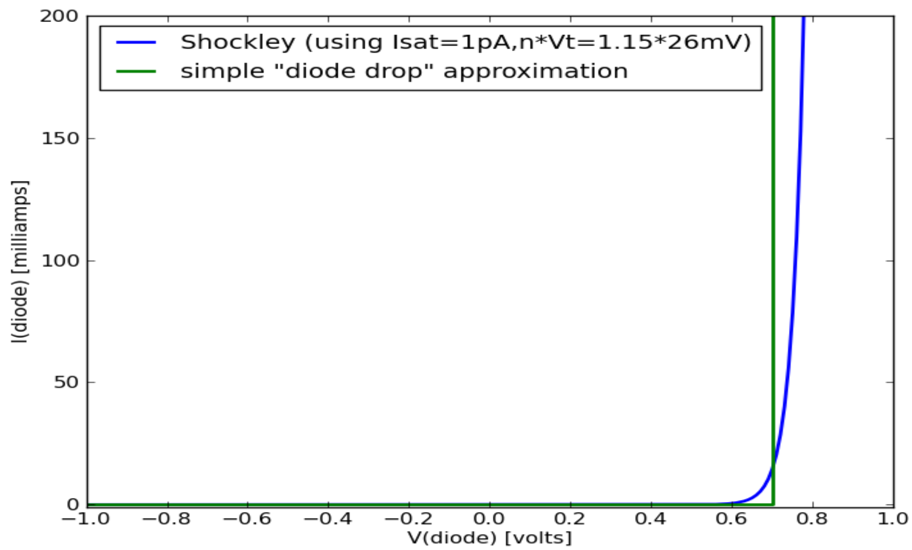
How can I double the bandwidth without changing the center frequency? How can I move f_0 to 1.21 MHz without changing the bandwidth?



<u>Problem 6.</u> Let's again modify the voltage divider from problem 1(d), but this time replacing R1 with a diode, such as 1N914. Using a 4Vpp sine wave (i.e. 2 volts in amplitude) for Vin, **sketch Vin and Vout**. You will probably find it convenient in this case to use the simplified "diode drop" approximation to the diode's I-V curve, i.e. an open circuit for Vdiode<0.7V and a constant drop of 0.7V once the diode turns on. (The green curve in the graph below.) This is often a useful first approximation in analyzing diode circuits and in some cases is all you need. We will see circuits later in the course in which you will need to keep in mind the exponentially growing blue curve.

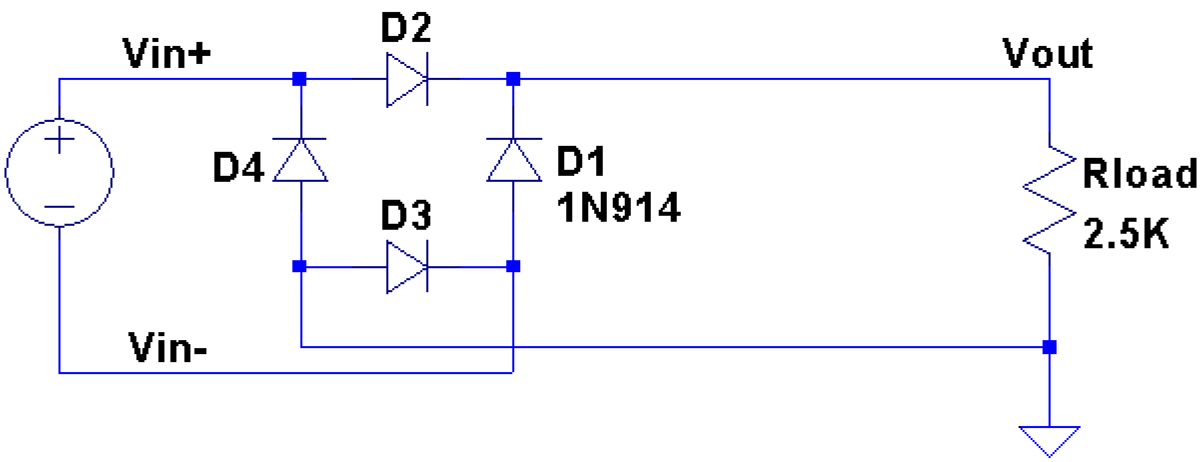
Now suppose that Vin is no longer a single sine wave, but instead is a 1 MHz sine wave modulated by a 1kHz sine wave, e.g. Vin(t) = 2 volts · (1.0 – 0.25 sin(2πt · 1kHz)) * cos(2πt · 1MHz). **Now sketch Vin and Vout for a few milliseconds.** This is a (simple) example of amplitude modulation. In real life, the 1kHz sine wave would be replaced with something more complicated, e.g. an audio signal.





<u>Problem 7.</u> The electricity that comes out of most wall outlets in North America takes the form of a sinusoidal voltage source, 110 volts rms, 60 Hz. What is the amplitude of a 110 volt rms sine wave? Imagine that you are trying to turn this AC line voltage into 5 volts DC to power your mp3 player. We'll first use a transformer to get the voltage into a more convenient (and safer) range, say 10 volts in amplitude. Now we need to turn this into a voltage whose average value exceeds +5V. (You'll learn later in the course how to turn an unregulated 6 or 7 volt power supply into a regulated 5 volt supply.)

We could start with the circuit in problem 6, but it has a significant drawback: it uses only half of each 60Hz cycle. Consider instead the circuit drawn below. Using the simple "diode drop" approximation, **sketch Vin and Vout for a few cycles**. Do you see why this is called a *full-wave* rectifier, in contrast with the *half-wave* rectifier of problem 6? (Technically, a *full-wave bridge rectifier*, because the diodes are arranged in a pattern called a bridge.)



Now add a capacitor (the *filter* capacitor) in parallel with the load. Remember the capacitor equation Q=CV, so dV/dt=I/C. So if you were to charge up the filter capacitor to some maximum value and then disconnect it from the diode bridge, it would discharge according to $-\mathrm{d}V_{\mathrm{out}}/\mathrm{d}t = I_{\mathrm{load}}/C_{\mathrm{filter}} = V_{\mathrm{out}}/R_{\mathrm{load}}C_{\mathrm{filter}}$, so Vout would decay exponentially with time constant $R_{\mathrm{load}}C_{\mathrm{filter}}$. The diode bridge is squirting charge into the capacitor at a rate $I \propto |\cos(2\pi\, t \cdot 60\mathrm{Hz})|$, which peaks every 8.33ms.

How large a capacitor do you need to keep Vout from drooping more than 5–10% between peaks? To simplify the calculation, pretend that the rectified sine wave is a series of very narrow spikes. This will overestimate the required capacitance a bit – i.e. will make your design *conservative*.