

Physics 364, Fall 2014, Lab #4 **Name:** _____

(RC circuits — low-pass & high-pass filters, “integrator,” “differentiator”)

Wednesday, September 10 (section 401); Thursday, September 11 (section 402)

Course materials and schedule are at positron.hep.upenn.edu/p364

Part 1

Start Time: _____

voltage divider revisited

1.1 Using two resistors, draw and then build a voltage divider that will take a 1 kHz sine wave at $10 V_{pp}$ (volts “peak to peak,” which is $2\times$ the conventional definition of amplitude) as input and will produce a $5 V_{pp}$ sine wave as output. We actually have specific resistor value(s) in mind, but instead of telling you these values, we will instead say that your voltage divider should have $R_{\text{thevenin}} = 1 \text{ k}\Omega$. What value(s) did you choose?

1.2 Use the oscilloscope to observe $V_{\text{in}}(t)$ and $V_{\text{out}}(t)$. Make a rough sketch of the input and output waveforms, pointing out the relative amplitude and phase of V_{in} and V_{out} . (If your function generator and oscilloscope disagree by about a factor of two on the amplitude of V_{in} , check that the function generator's output is configured to drive a "high impedance" load rather than a $50\ \Omega$ load.)

1.3 Part 1 of this lab is a bit dull, so don't spend too much time on it. One thing you should do at this point is to continue getting to know the scope. Move the trigger threshold up and down and see how it affects the horizontal position of the traces on the screen. Why does this happen? Also, try using the vertical and horizontal cursors on the scope to check that the period and the amplitudes of the sine waves are what you expect.

Part 2

Start Time: _____

RC low-pass filter

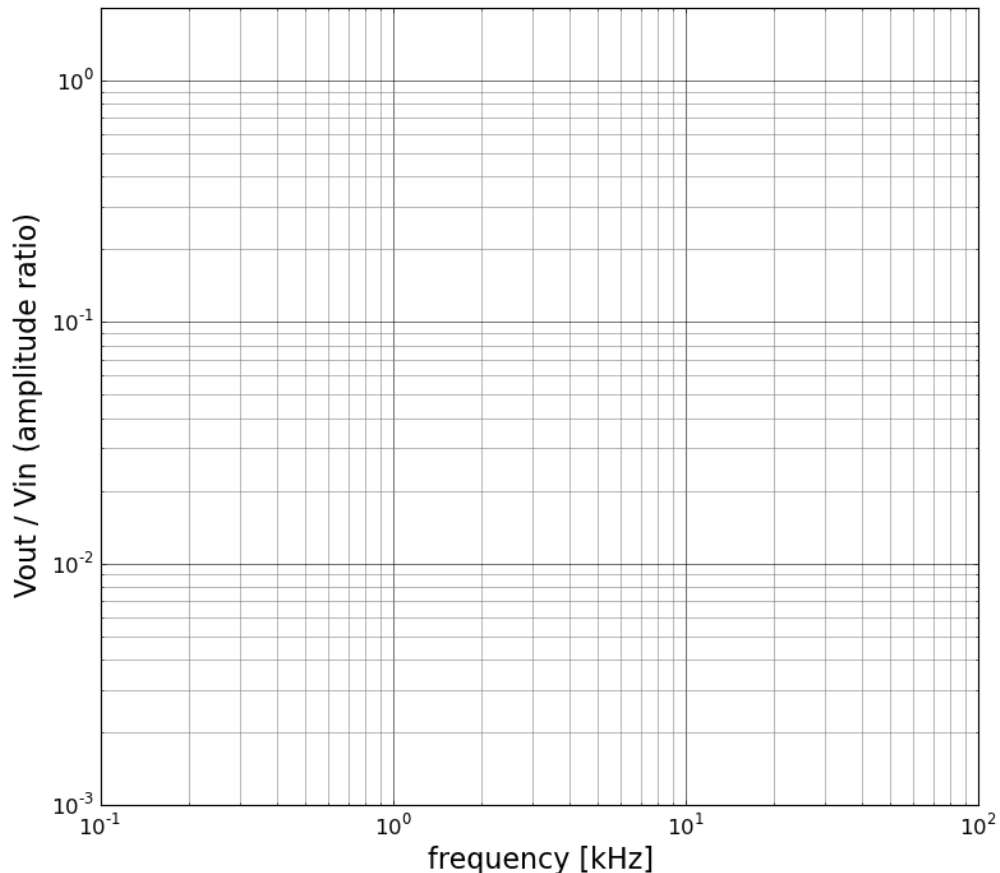
2.1 Now replace one of the two resistors with a capacitor to form a *low-pass* filter having $f_{3\text{dB}} \approx 1.6$ kHz. What capacitor value did you choose? (The parts drawers should have a cap that is within 10% of the desired value.) Which resistor did you replace? Draw your new schematic diagram.

To check that your choice (of which resistor to replace) makes sense, look at the voltage-divider equation

$$\frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

and remember that a capacitor looks like a short-circuit ($Z \rightarrow 0$) at high frequency and looks like an open-circuit ($Z \rightarrow \infty$) at low frequency. There's no calculation to do here — just mentally check that the low- and high-frequency limits make sense.

2.2 Now measure $|\frac{V_{out}}{V_{in}}|$ for $f = 0$ Hz (i.e. for DC, or at least very close to DC), and in factor-of-two steps in frequency from $f = 200$ Hz up through $f \approx 100$ kHz. At what frequency does $|\frac{V_{out}}{V_{in}}| = 1/\sqrt{2}$? Do you see that beyond f_{3dB} , the response falls (asymptotically) by 6dB/octave? In other words, for each doubling in frequency, the amplitude should decrease by a factor of two. Make a log-log graph, below, of $|\frac{V_{out}}{V_{in}}|$ vs. f and see if you find the expected shape: pretty flat for $f \ll f_{3dB}$ and falling with a slope of -1 (a factor of 10 in amplitude for each factor of 10 in frequency) for $f \gg f_{3dB}$; if you're short on time, just sketch roughly what you would expect the graph to look like.



2.3 Measure the phase difference between $V_{\text{out}}(t)$ and $V_{\text{in}}(t)$ for three frequencies: for $f \ll f_{3\text{dB}}$, for $f = f_{3\text{dB}}$, and for $f \gg f_{3\text{dB}}$. At what frequency is the phase shift 45° ? Where you find a large phase shift, which waveform is ahead of the other one — in other words, which direction does the phase shift go? (Another way to ask this question is, “Does the output *lead* or *lag* the input?”) How did you measure the relative phase? (There are several ways to do it. Ask for help if you don’t know how to do it. The classic method is to look at the time difference between the two waveforms’ positive-slope zero crossings, then multiply that Δt by $\frac{360^\circ}{\text{period}}$.)

2.4 [If you haven't reached this section by 3:15pm, just skip it and move on to Part 3.] Finally, what is the worst-case *Thevenin impedance* (more commonly called *output impedance*) of the low-pass filter? Remember that a better (stronger, stiffer) voltage source has a smaller R_{thev} (which we can generalize to \mathbf{Z}_{thev} when capacitors and inductors are included). So the worst case (for $|\mathbf{Z}_{\text{thev}}|$ of a voltage source) would be the largest possible value. Hint: this worst case happens in the $f \rightarrow 0$ limit, because $|\mathbf{Z}_{\text{cap}}| \propto 1/f$. The way to work this out is to write down an expression for Z_{thev} of the RC filter (in analogy to what you would write for a voltage divider) and then to take the $f \rightarrow 0$ limit, which is a very simple expression.

With your above answer in mind, do you think that V_{out} of your circuit will change appreciably if it is required to drive a 100 k Ω load connected to its output? (The whole point of a filter is to send a signal (with modified frequency content) to some downstream circuit fragment. What we're asking here is how picky our filter will be about what sort of load it is willing to drive, before its output begins to droop — and other bad things happen, too.)

Go ahead and connect a 100 k Ω load resistor; then draw the new schematic diagram; then check whether V_{out} has changed appreciably for $f = \frac{1}{2\pi RC}$ and for $f \ll \frac{1}{2\pi RC}$. You should find that the filter has no trouble driving a 100 k Ω load.

Now (finally!) if you replace the 100 k Ω load resistor with a 2 k Ω resistor instead, you should find that two unwanted things happen: first, V_{out} is considerably smaller, even at very low frequencies; second (this is more insidious), the “corner frequency” $f_{3\text{dB}}$ is shifted when too small a load resistance is connected. (For a filter whose largest $|\frac{V_{\text{out}}}{V_{\text{in}}}|$ is different from 1.0, you need to define $f_{3\text{dB}}$ as the frequency at which $|\frac{V_{\text{out}}}{V_{\text{in}}}|$ is a factor $\frac{1}{\sqrt{2}}$ smaller than its maximum value. This will also be the frequency at which the phase shift is 45 $^\circ$.) What is the new $f_{3\text{dB}}$ for your overloaded RC filter? (We say overloaded because we violated the rule-of-thumb that R_{in} of the downstream load should be at least $10 \times R_{\text{thев}}$ of the upstream circuit fragment, if we want to be able to analyze circuit fragments independently.) You should first measure the new $f_{3\text{dB}}$ and then see if you can figure out how to calculate it; the calculation is based on a similar trick to the one that you often use in calculating $R_{\text{thев}}$.

Keep your low-pass filter intact for the next part.

Part 3

Start Time: _____

RC “integrator” (such as it is)

3.1 Now remove the load resistor from the RC low-pass filter from Part 2, so that it is back to the state it started in in Part 2.1. Supply a 10 V_{pp} **square wave** at 100 Hz as $V_{\text{in}}(t)$. You should see the familiar $1 - e^{-t/RC}$ shape as the capacitor charges up (though it will go from -5 V up to $+5\text{ V}$ instead of starting at 0 V), and the familiar $e^{-t/RC}$ shape as it discharges. Measure how long V_{out} takes to climb 63% of the way up from its minimum, and how long it takes to fall back down to just 37% of the way up from the minimum. Do you find the expected time constant? (To observe the rise time, you may need to use the “horizontal” knob to adjust the scope’s time scale.)

3.2 At the bottom of page 2 of this week's reading notes, we saw that $\frac{d}{dt}V_{\text{out}} = \frac{1}{RC}(V_{\text{in}} - V_{\text{out}})$, so in the limiting case $|V_{\text{out}}| \ll |V_{\text{in}}|$, we get

$$V_{\text{out}} \approx \frac{1}{RC} \int V_{\text{in}} dt.$$

Since you already know that this circuit is a low-pass filter with $f_{3\text{dB}} \approx 1.6$ kHz, you should be able to arrange to have $|V_{\text{out}}| \ll |V_{\text{in}}|$ just by adjusting the square-wave frequency to be considerably larger than 1.6 kHz. Try gradually increasing the frequency until V_{out} looks like a more and more plausible integral of V_{in} . Once the integration looks OK, try changing the shape from square to triangle, and see whether the circuit seems to be integrating. You already tried “integrating” a sine wave in Part 2: you got back a sine wave with its phase shifted 90° , which is just (minus) a cosine wave.

Part 4**Start Time:** _____**RC high-pass filter / “differentiator”**

4.1 Now interchange the resistor and capacitor from Part 3 so that you have a *high-pass* filter. Before measuring it, think about what you expect a log-log graph of $|V_{\text{out}}/V_{\text{in}}|$ vs. f to look like. We think you already know where $f_{3\text{dB}}$ will be. Now go ahead and measure $|V_{\text{out}}/V_{\text{in}}|$ for sinusoidal $V_{\text{in}}(t)$ at $f = 12$ kHz, 6 kHz, 3 kHz, 1.5 kHz, 800 Hz, 400 Hz, 200 Hz, and 100 Hz. Make V_{in} quite large (e.g. 10 V_{pp}) so that you can still see V_{out} even where the filter’s response is very small. (Don’t bother to graph the data this time.)

For the lowest frequencies, does V_{out} fall a factor of two for each octave in frequency? How about the phase shift? $V_{\text{in}}(t)$ and $V_{\text{out}}(t)$ should be in phase with one another for the frequencies at which $|V_{\text{out}}| \approx |V_{\text{in}}|$ (i.e. the highest frequencies), and they should be 90° apart at $f \ll f_{3\text{dB}}$. When they are $\approx 90^\circ$ apart, which one is ahead of the other? (Remember that the textbook describes the high-pass filter also as a “positive phase-shifter.”) At what frequency do you expect $V_{\text{in}}(t)$ and $V_{\text{out}}(t)$ to be 45° apart?

4.2 Now try a square-wave $V_{\text{in}}(t)$ at a frequency $f \ll f_{3\text{dB}}$, e.g. around 150 Hz, so that $\frac{d}{dt}V_{\text{out}} \ll \frac{d}{dt}V_{\text{in}}$. In this limit (notes page 3),

$$V_{\text{out}} \approx RC \frac{dV_{\text{in}}}{dt}.$$

Does the circuit appear to be differentiating V_{in} ? What happens when you give it a triangle-wave input instead? How about a sine?