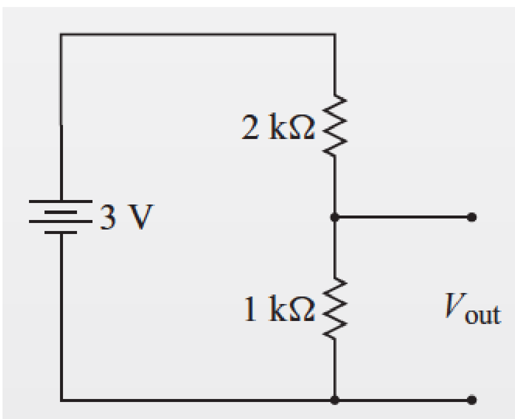
Physics 364, Fall 2014, reading due 2014-08-31.

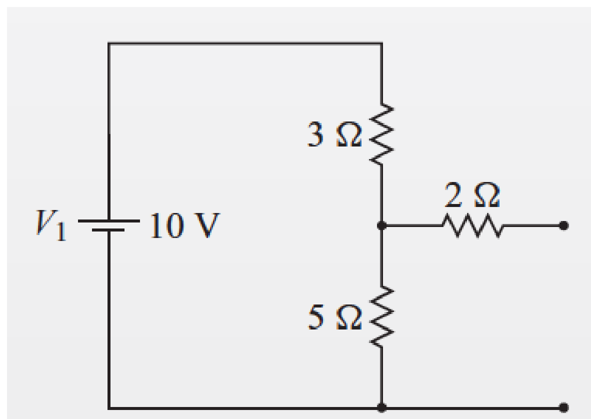
Email your answers to `ashmansk@hep.upenn.edu` by 11pm on Sunday

Course materials and schedule are at `positron.hep.upenn.edu/p364`

Assignment: (a) Read Chapter 1 of Eggleston's *Basic Electronics* textbook. (In case you don't yet have your own copy, I've put Chapter 1 up on Canvas.) You can skim section 1.2.3 (solving circuit problems) and section 1.3 (AC signals). Pay close attention to sections 1.2.1.3 (Thevenin and Norton) and 1.2.4 (input resistance). (b) You should also read through my notes (starting on next page). (c) Then email me your answers to the questions below.

1. The output (V_{out}) of the voltage divider shown in the left figure below is to be measured with voltmeters having input resistances of 1 MΩ, 10 kΩ, and 667 Ω. What voltage will each meter indicate? (Only one meter is connected at a time.) Most digital voltmeters nowadays have input resistances around 10 MΩ: maybe now you see why such a large value is helpful. Would you want a *current meter* to have a very large or a very small input resistance?





2. Find the Thevenin voltage (V_{th}) and Thevenin resistance (R_{th}) of the circuit shown in the right figure above. Explain to me how the R_{th} calculation is done using each of the two methods explained in the text.

3. What are you hoping to learn in Physics 364 this term? What is your reason for taking the course?

4. Is there anything from this reading assignment that you found confusing and would like me to try to clarify? If you didn't find anything confusing, what topic did you find most interesting?

5. How much time did it take you to complete this assignment?

The main purpose of electronics is to manipulate *signals,* which represent some desired information as a function of time.

For instance, imagine the signal path when you use a "walkie-talkie" radio to communicate with a friend. Electrochemical signals in your brain form thoughts, then some mental representation of words to be spoken. Neural signals travel along nerves to control your breathing, your mouth, and your speech organs, so that you utter the desired words.

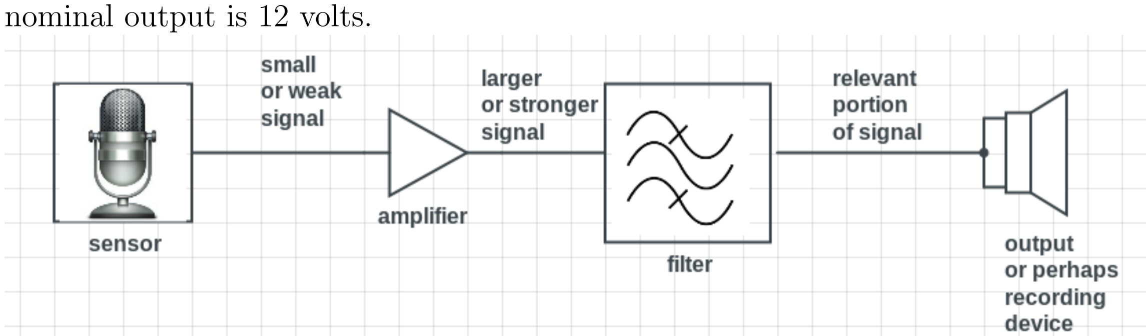
As a result, a longitudinal wave propagates through the air, with information represented as a time-dependent change in air pressure, $\Delta P(t)$, with respect to the ambient atmospheric pressure. When this traveling wave reaches your radio's microphone, the pressure disturbance causes a proportional change in the small displacement between the two parallel metal plates of a charged capacitor. (I'm assuming a capacitive microphone; there are other other types.) Since the capacitance varies while the stored charge stays roughly constant, a time-varying voltage $V(t)$ develops across the two plates of the microphone. If the displacement of the plates is much smaller than their normal separation, the voltage signal $V(t)$ is proportional to the acoustical signal $\Delta P(t)$. For human speech, the relevant frequencies for $\Delta P(t)$ are below 20 kHz.

This time-varying voltage signal, $V(t)$, is manipulated by circuits within your handset, which cause your radio to transmit an electromagnetic wave whose amplitude varies in proportion to the original acoustical signal. (This is for a simple AM radio. The way information is represented in the EM wave transmitted by a modern cell phone is much more complicated.)

Meanwhile, the antenna of your friend's radio responds to some portion of the incoming electric field. (You and your friend agreed in advance to tune your radios to the same channel, meaning the same small range of radio frequencies. The relevant frequencies for Citizens' Band radio are around 27 MHz. Your mobile phone uses radio frequencies $\mathcal{O}(10^9\ \text{Hz})$.) The radio's electronic circuitry amplifies the antenna signal, selects out the desired range of frequencies corresponding to the radio channel you are using, and then converts the time-varying amplitude of that radio-frequency signal into a voltage $V(t)$ that is once again proportional to the original $\Delta P(t)$ propagating from your mouth. This signal is amplified (i.e. increased in power) and converted into a time-varying current $I(t)$ that flows through a coil of wire in the speaker of your friend's radio. (I'm assuming a magnetic speaker; there are other kinds.) The current $I(t)$ in the coil of wire creates a magnetic field $B(t)$ that interacts with a small iron magnet in the speaker, causing the paper surface of the speaker to move back and forth in proportion to $I(t)$. This in turn produces a pressure variation $\Delta P(t)$ that propagates as a longitudinal wave through the air to your friend's ear. This acoustical signal in your friend's ear then excites nerve signals in his or her brain.

The point of this digression is to illustrate that a key function of electronic circuits is to manipulate electrical signals that somehow represent physical quantities that interest us. If you and I are talking by phone, then we are interested in air-pressure variations near each other's telephone handsets. The thermostat in my home observes one electrical signal that represents the ambient temperature and manipulates another electrical signal that turns the furnace on or off. A Positron Emission Tomography scanner analyzes electrical signals that represent the passage of X-ray photons through the scanner's detector modules. Scientific instruments use a wide range of sensors to convert physical quantities into electrical signals. In some cases, as in the capacitive microphone described above, the electrical signal takes the form of a voltage, $V(t)$. In other cases, as in the burst of electrons liberated when a visible photon strikes a photomultiplier tube, the electrical signal takes the form of a current, $I(t)$. Both current and voltage signals are very common as the immediate outputs of sensors, but these sensors are usually followed by amplifiers that produce voltage signals. So most of the electrical signals that you will measure and manipulate this semester will be voltages, $V(t)$.

In other physics courses, you learned to analyze circuits as closed loops around which charge-carriers flow. In electronics, we look at circuits not only as closed loops but also as sequences of operations performed on signals. In the sketch below, a sensor (e.g. a microphone) turns a real-world signal into a small or weak voltage or current. Next, an amplifier makes this signal larger and/or stronger. Then a filter picks out whatever feature of this signal we are most interested in. Finally, the signal may be recorded for later analysis or perhaps converted back into its real-world form (e.g. by a speaker). We are connecting the output of each box to the input of the next box, where the job of each box is somehow to manipulate the signal. By "large" or "small" signal, we mean its magnitude in volts or amps. We'll define later in these notes what we mean by a "weak" or "strong" signal, but briefly, a weak signal tends to become much smaller when it is connected a downstream box, while a strong signal can be connected to a downstream box without becoming appreciably smaller. We'll see in more precise terms in a moment that a 12-volt car battery is a much "stronger" voltage source than a string of eight AA batteries, even though in both cases the nominal output is 12 volts.



In any case, to analyze a circuit, you need to keep track of two quantities: **current** and **voltage**.

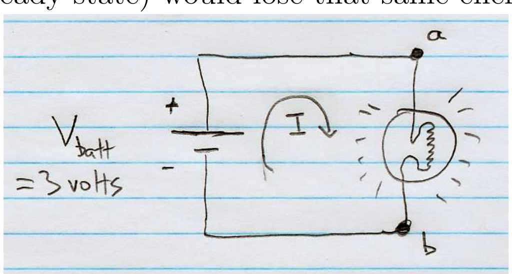
Electric **current** (symbol I or i, unit A = ampere or "amp") is the flow of electric charge: 1 A = 1 C/s, where -1.602×10^{-19} C is the charge on one electron. The flow of conventional (positive) current is opposite the flow of electrons. An upward flow of 6.24×10^{18} electrons per second through a wire is a downward current of one amp.

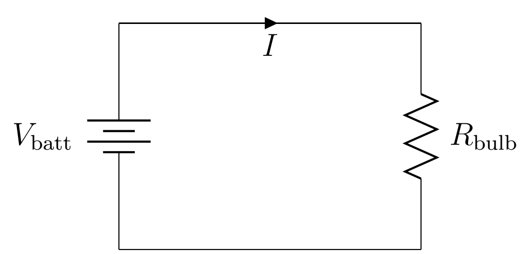
Voltage (symbol V or v, unit V = volt), a.k.a. electric potential, is electrical energy per unit charge: 1 V = 1 J/C. To move one coulomb of charge across a potential difference of one volt, you must provide one joule of energy.

An analogy with water flow can aid intuition: current is analogous to the rate of flow through a pipe (e.g. in liters per second); and voltage is analogous to the pressure (or elevation[1]) difference between the upstream and downstream ends of the pipe.

Just as friction (or viscosity) dissipates energy when water flows through a pipe, electrical **resistance** (symbol R, unit Ω = ohm) dissipates energy when electric current flows through a circuit. For a given pressure difference, more water flows through a meter-long fire hose than through a mile-long garden hose. So **Ohm's law** seems plausible: to maintain a current flow I through a two-terminal circuit element of resistance R requires a voltage difference $V = IR$ between its two terminals.[2] It also makes sense that for material of resistivity ρ, a wire of cross-section A and length ℓ has resistance $R = \rho\ell/A$.

I show below (left) a simple example of a circuit: a $V_{\text{batt}} = 3.0$ V battery is wired to a flashlight bulb. By convention, current is the flow of positive charge. So we imagine positive charge-carriers flowing clockwise around the circuit, from the battery's + terminal, through the bulb, and back to the battery's − terminal. A positive charge-carrier would gain energy as it moved upward through the battery and then (in a steady state) would lose that same energy as it moved downward through the bulb.





If you imagine the wires to be thick copper at room temperature and the bulb's filament to be extremely thin tungsten at $T \approx 3000$ K (for most materials, ρ increases with T), the resistance of the copper wires is negligible in comparison to that of the filament. We usually draw "schematic" circuit diagrams in which idealized wires have zero resistance and pertinent electrical properties like resistance (and next week

[1] More precisely, the "pressure head" corresponding to height difference Δh is $\Delta P = \rho g \Delta h$.

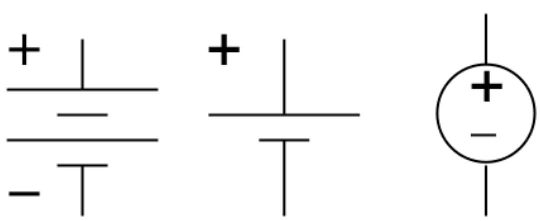
[2] For a simple physical model of electrical resistance, see section 4.3 of Purcell's E&M textbook or try `en.wikipedia.org/wiki/Drude_model`.

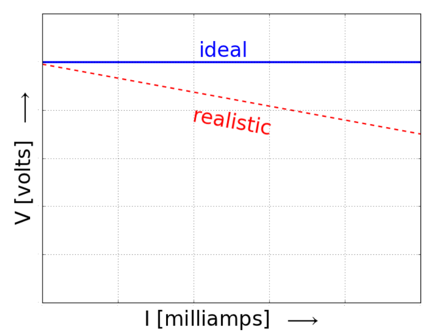
capacitance and inductance) are lumped into discrete components. If a real wire has non-negligible resistance, we'll draw it as a separate resistor, with a label like R_{wire}.

Conservation of electric charge requires that (in the steady state) the same current I that flows up through the battery must flow down through the resistor. If the light bulb has resistance $R_{\text{bulb}} = 10\ \Omega$, then $I = V_{ab}/R_{\text{bulb}}$, where V_{ab} is the voltage measured between the points marked a and b. Conservation of energy requires that the voltage drop $V_{ab} = IR_{\text{bulb}}$ across the bulb must equal the voltage V_{batt} supplied by the battery. Looping around in the direction of I, we write $V_{\text{batt}} - IR_{\text{bulb}} = 0$, so $I = (3.0\text{ V})/(10\ \Omega) = 0.3$ A.

We usually talk about the current *through* a wire or *through* a two-terminal component like a battery or a lamp or a resistor; and we usually talk about the voltage *between* two points of a circuit or *across* the two terminals of a component. Thinking this way may remind you in the lab that you need to interrupt a circuit to measure a current, and that you need to have in mind the two points between which you are measuring a voltage (even when one of those two points has been conveniently defined to be "ground"). Just as it only makes sense to talk about differences in potential energy (an overall offset doesn't change the physics), it also makes sense only to talk about differences in electric potential (voltage). But it is often convenient to define a point in your circuit called "ground" or "Earth" to be at zero volts.[3]

A battery is a real-world example of a **voltage source**. On a schematic diagram, the three symbols shown below (left) are used (interchangeably) to represent ideal voltage sources.



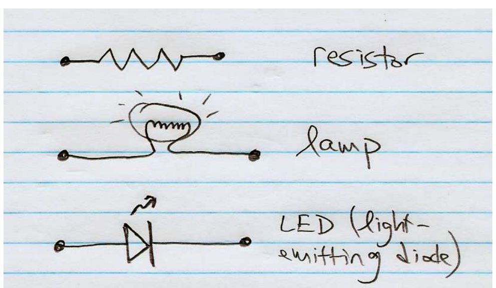


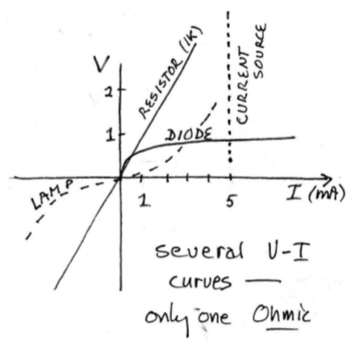
As shown in the V-I curve above (right), an ideal 3.0 V voltage source would unconditionally maintain a potential difference of 3.0 V between its two terminals, regardless of the magnitude of the current required by whatever circuit the voltage source is driving. Whether you connect a 100 Ω resistance or an 0.01 Ω resistance, you get the nominal 3.0 V across the two terminals of an ideal voltage source. If you can imagine trying to use flashlight batteries to start your car, you can see that real-world voltage sources are not ideal: if you try to draw too much current from a real battery

[3]The name "ground" arises because the metal chassis of an electrical appliance is typically wired (via the electrical outlet) to a metal pipe driven into the ground beneath the building, for safety.

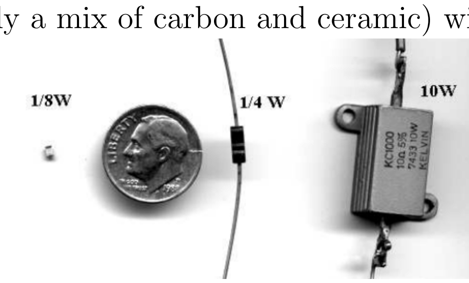
(i.e. by connecting a very small resistance across its terminals), the voltage supplied by the battery will droop below its nominal value. (Think of what happens when you start your car while the headlights are already on: the headlights dim while the starter motor is cranking, because the voltage supplied by the battery momentarily falls below its nominal 12 V.) The V-I curve of a real-world voltage source will sag or droop (people really say this!) downward as you try to draw too large a current: the slope $|\mathrm{d}V/\mathrm{d}I|$ (which has dimensions of resistance) is nonzero for a realistic voltage source. We'll see in a moment that the simplest way to give the V-I curve this slope is to pretend that inside the battery, there is a small resistor in series with an ideal voltage source.

While we're on the topic of V-I curves, I show below the schematic symbols (left) and the V-I curves (right) for a few two-terminal circuit elements that we will use this week. Notice that only the resistor obeys Ohm's law. As you apply a larger voltage to the lamp, the dissipated power $P = VI$ increases, making the filament (somewhat insulated by the glass bulb) even hotter, thus increasing the resistance. We'll discuss diodes in much more detail a couple of weeks from now, but for now just notice (a) the curve is non-linear, (b) current can flow in one direction but not the other, and (c) when a current of at least several milliamps is flowing in the forward direction (left-to-right as I drew the schematic symbol), the voltage drop across the diode is something like 0.7 volts.

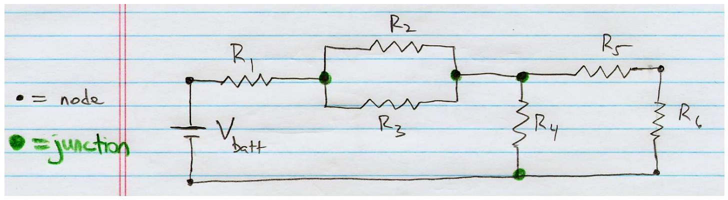




By the way, resistors come in different shapes and sizes, depending on their power ratings. (See figure below.) A resistor needs to be large enough to transfer efficiently to its surroundings whatever power ($P = IV = V^2/R = I^2R$) it dissipates. If heat is carried away so slowly that the temperature rises too far, the resistivity of the internal material (usually a mix of carbon and ceramic) will change—or worse!



To analyze circuits that are more complicated than a single battery wired to a lamp (e.g. the circuit shown below), some formal rules can be helpful. Consider a circuit to be a set of two-terminal components connected by ideal ($R = 0$) wires. A point where two or more wires or terminals meet is called a **node**. A point where three or more wires or terminals meet is called a **junction**. A circuit segment that contains no junctions is called a **branch**.



Then conservation of charge implies **Kirchoff's current law** (KCL): $\sum I = 0$ for a given node or junction. The algebraic sum of the currents flowing into a node or junction must be zero. It follows that the steady-state current must be equal at all locations along a branch. So when solving for the currents in a circuit, only one current per branch is needed. (Hence the single I for the battery+lamp circuit.)

Since voltage is electrostatic energy per unit charge, conservation of energy implies[4] **Kirchoff's voltage law** (KVL): that $\sum V = 0$ around any closed path. Looping around a closed path in the circuit, the sum of the voltage gains minus the voltage drops equals zero. Hence $V_{\rm batt} - IR_{\rm bulb} = 0$ for the battery+lamp.

The most common brute-force method to solve circuits is first to label each branch with a current (drawing an arrow to keep track of each current's sign); then to use KCL at each junction to eliminate redundant currents; then to use KVL around closed paths to write one equation per remaining current. In practice, the bruce-force methods (described in Eggleston section 1.2.3) are seldom needed except for complicated problems involving multiple voltage and current sources.

Two **resistors in series** have combined resistance $R_{\rm series} = R_1 + R_2$ and two **resistors in parallel** have $R_\parallel = 1/(\frac{1}{R_1} + \frac{1}{R_2}) = \frac{R_1 R_2}{R_1+R_2}$, as you can prove using KCL and KVL. We will often write $R_1 \parallel R_2$ (read "R_1 parallel R_2") as shorthand for $\frac{R_1 R_2}{R_1+R_2}$.

Returning to the messy six-resistor circuit drawn above, you can write down immediately that the combined resistance seen by the battery is

$$R_{\rm comb} = R_1 + (R_2 \parallel R_3) + (R_4 \parallel (R_5 + R_6))$$

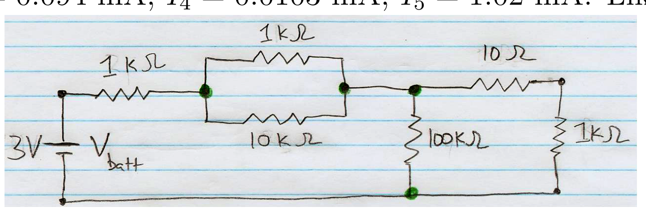
so the current through R_1 is $I_1 = V_{\rm batt}/R_{\rm comb}$. The currents through R_2 and R_3 must sum to I_1, with each of the two currents inversely proportional to the corresponding resistance. (Two resistors in parallel form a "current divider.") So the top current

[4] More correctly, KVL is a consequence of Faraday's law of induction: $\oint \vec{E} \cdot {\rm d}\vec{\ell} = -\frac{\rm d}{{\rm d}t}\Phi_B$. If your circuit loop encloses a time-varying magnetic flux, you need to include the corresponding voltage. To avoid unwanted magnetic "pickup," sensitive circuits are built compactly and often send signals around on tightly-coupled *differential pairs* of wires.

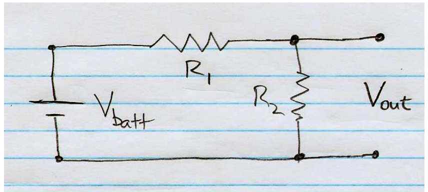
is $I_2 = \frac{1/R_2}{(1/R_2)+(1/R_3)} I_1 = \frac{R_3}{R_2+R_3} I_1$, and the bottom is $I_3 = \frac{1/R_3}{(1/R_2)+(1/R_3)} I_1 = \frac{R_2}{R_2+R_3} I_1$. Then you can use the same current-divider trick for I_4 (through R_4) and I_5 (through both R_5 and R_6, since they are on the same branch): you get $I_4 = \frac{(R_5+R_6)}{R_4+(R_5+R_6)} I_1$ and $I_5 = \frac{R_4}{R_4+(R_5+R_6)} I_1$. But I will never make you do anything this tedious.

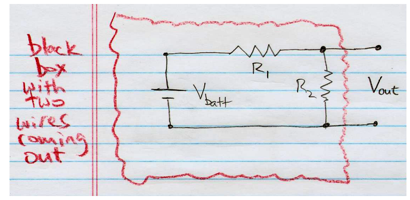
In this course, I am more interested in developing your intuition about circuits than in training you to make lengthy calculations on paper. You can save a lot of time in the lab by **learning a few shortcuts** to make mental calculations, with a goal of getting answers that are good to something like $\pm 10\%$. Toward that end, remember that for identical resistors in parallel, $R \parallel R = R/2$, that $R \parallel R \parallel R = R/3$, etc. And if you keep in mind that $2R \parallel 2R = R$, then you can work out in your head that $R \parallel 2R = 2R \parallel 2R \parallel 2R = 2R/3$. So putting 5 Ω in parallel with 10 Ω gives you 3.3 Ω: you knew it had to be smaller than 5 Ω (the smaller resistor by itself) and bigger than 2.5 Ω (two fives in parallel). For very different resistor values in series, $R_{\text{small}} + R_{\text{big}} \approx R_{\text{big}}$ (plus a small correction), and for very different resistor values in parallel, $R_{\text{small}} \parallel R_{\text{big}} \approx R_{\text{small}}$ (minus a small correction). For two very different resistors in series, R_{big} dominates; for two very different resistors in parallel, R_{small} dominates. As you work in the lab, train your eye to look for these simplifications.

Below I've redrawn the messy six-resistor circuit with some numerical values: $R_1 = R_2 = R_6 = 1$ kΩ (that's 1000 Ω), $R_3 = 10$ kΩ, $R_4 = 100$ kΩ, $R_5 = 10$ Ω, and $V_{\text{batt}} = 3$ V. Suppose we want to know the current drawn from the battery. Looking at R_2 and R_3, notice that 1 k$\Omega \parallel 10$ k$\Omega \approx 1$ kΩ (minus about a 10% correction). Then look at R_5 and R_6 and notice that $10\ \Omega + 1$ k$\Omega \approx 1$ kΩ (plus about a 1% correction). Then notice that $R_4 = 100$ kΩ can be neglected in parallel with 1 kΩ. So $R_{\text{comb}} \approx 3$ kΩ, and $I_1 = V_{\text{batt}}/R_{\text{comb}} \approx 1$ mA. (For comparison, using the exact expression we worked out above, I get $R_{\text{comb}} = 2.91$ kΩ and $I_1 = 1.03$ mA. Is the added precision worth the effort?) Continuing along, I_3 and I_2 must be about 0.1 mA and 0.9 mA, respectively; and I_4 (since it sees $100\times$ the resistance) must carry about $\frac{1}{100}$ of the current: $I_4 \approx 0.01$ mA and $I_5 \approx 1$ mA. (Using the exact expressions, I get $I_2 = 0.94$ mA, $I_3 = 0.094$ mA, $I_4 = 0.0103$ mA, $I_5 = 1.02$ mA. Like the shortcut?)

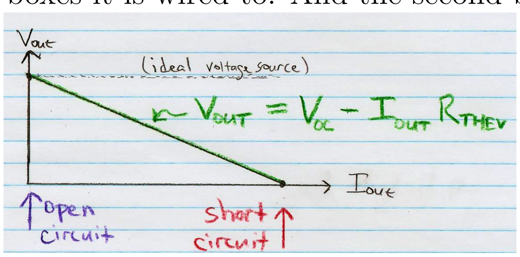


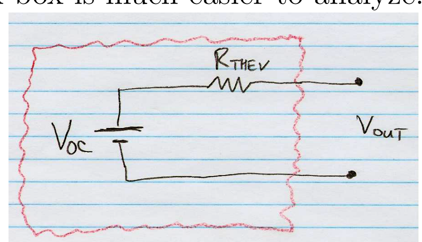
Now consider the **voltage divider** drawn below (left). Pause to convince yourself that the voltage V_{out} measured between the two terminals of R_2 is $V_{\text{out}} = \frac{R_2}{R_1+R_2} V_{\text{batt}}$ (assuming that no current is drawn by whatever device you are using to measure V_{out}). The way to intuit this is to think (a) the same current flows through R_1 and R_2; (b) the two voltage drops must add up to V_{batt}; (c) the two voltage drops are, respectively, IR_1 and IR_2. So the voltage drop across R_2 is a fraction $R_2/(R_1+R_2)$ of V_{batt}.





Now consider the same voltage divider hidden away inside a black box (above right), with two wires coming out. Suppose that I market this black box to you as a battery. How good a battery is it? Let's draw its V-I curve. If $I_{\rm out} = 0$ (no current flows out of the black box), then $V_{\rm out}(I_{\rm out} = 0) = \frac{R_2}{R_1+R_2} V_{\rm batt} \equiv V_{OC}$.[5] We call this the "open circuit" voltage (V_{OC}). Now, if we connect a finite load resistance $R_{\rm load}$[6] between the two wires, some finite current will flow. That flow of current through $R_{\rm load}$ must increase the current flowing through R_1, which in turn increases the voltage drop across R_1, and hence must decrease $V_{\rm out}$. In the extreme case, try $R_{\rm load} = 0$: a *short circuit*. (The open-circuit case was $R_{\rm load} = \infty$.) Then $V_{\rm out} = 0$, and $I_{\rm out}(V_{\rm out} = 0) = V_{\rm batt}/R_1 \equiv I_{SC}$. (Minus) the slope of the V-I curve (shown below, left) is $R_{\rm thevenin} \equiv V_{OC}/I_{SC} = \frac{R_1 R_2}{R_1+R_2}$. This V-I curve is the same as the curve that you would get from the modified black box (below, right) that contains a single (ideal) voltage source V_{OC} in series with a single resistor $R_{\rm thev}$. As long as you fix V_{OC} and $R_{\rm thev}$ to the values computed above, the outside world can't tell which of the two black boxes it is wired to. And the second black box is much easier to analyze.





Thevenin's theorem states that any two-terminal combination of (ideal and independent) voltage sources, current sources, and resistors has the same V-I curve as a single voltage source $V_{\rm OC}$ (a.k.a. $V_{\rm thev}$) in series with a single resistor $R_{\rm thev}$. There are two ways to compute $R_{\rm thev}$. The first method, used in the previous paragraph, is to divide V_{OC} by I_{SC}. The second method (usually easier if you are looking at the schematic diagram) is this: (a) replace all voltage sources with short-circuits; (b) replace all current sources with open-circuits; (c) the resistance between the two terminals (considering only what is inside the box, not the external load) will be $R_{\rm thev}$. Notice that for the circuit considered in the previous paragraph, $R_{\rm thev} = R_1 \parallel R_2$, as you would compute by replacing $V_{\rm batt}$ with a short-circuit.

[5] Oops: please don't be confused by my using $V_{\rm batt}$ to mean the ideal voltage source inside the black box, while $V_{\rm out}$ is now what we are evaluating as if it were a non-ideal battery.

[6] We use "load" generically to refer to whatever downstream object is connected to the output of our upstream circuit—usually representing the useful purpose served by the upstream circuit. In our first example, the lightbulb serves as the load for the flashlight battery.

The concept of **Thevenin resistance**, which we also call **output resistance** or sometimes **source resistance** is extremely important. The reason is that we often build up a complex circuit out of separately designed fragments: a sequence of abstracted gadgets connected to one another. If each fragment in the chain doesn't place too heavy a burden on its neighbors, we can analyze the fragments independently, which considerably simplifies the task of understanding how the whole circuit works.[7] If you pencil in R_{load} on the above-right figure, you can see that R_{thev} and R_{load} form a voltage divider:

$$V_{\text{load}} = \frac{R_{\text{load}}}{R_{\text{load}} + R_{\text{thev}}} V_{OC}$$

If $R_{\text{load}} \gg R_{\text{thev}}$, then $V_{\text{load}} \approx V_{OC}$: the upstream circuit hardly notices the presence of the load. But if $R_{\text{load}} = R_{\text{thev}}$, then $V_{\text{load}} = V_{OC}/2$: the output voltage droops to half of its unloaded value. And if $R_{\text{load}} \ll R_{\text{thev}}$, then $V_{\text{load}} \approx (R_{\text{load}}/R_{\text{thev}})V_{OC}$: you get only a small fraction of the original output voltage.

A good rule of thumb for circuit fragments that want to deliver voltages is to keep $R_{\text{load}} \geq 10 \times R_{\text{thev}}$, so that the upstream circuit droops by no more than about 10%. A good voltage source has a small R_{thev} (a.k.a. R_{out}); and a voltage source wants to drive a large R_{load}. Another way to say this is that when you want fragment A to send a voltage signal to fragment B without drooping, make sure that the **input resistance** of fragment B is much larger than the **output resistance** of fragment A: $R_{\text{in}}(B) \geq 10 \times R_{\text{out}}(A)$ [maximize delivered voltage].

It turns out that the opposite is true for current sources: if you want to maximize the current delivered from fragment A to fragment B, then you need $R_{\text{in}}(B) \ll R_{\text{out}}(A)$ [maximize delivered current]. And if you want to maximize power delivered to the load (avoiding reflected power on a transmission line, for example), then you want $R_{\text{in}}(B) = R_{\text{out}}(A)$ [maximize delivered power].

A voltage source with a small (large) R_{out} [a.k.a. R_{thevenin}] is called a strong (weak) voltage source; an ideal voltage source has $R_{\text{out}} = 0$. A current source with a large (small) R_{out} [a.k.a. R_{norton}] is called a strong (weak) current source; an ideal current source has $R_{\text{out}} = \infty$. A voltage source's favorite load is an open circuit ($R_{\text{in}} = \infty$); a current source's favorite load is a short circuit ($R_{\text{in}} = 0$).

[7] If you're a computer scientist, "We don't break the abstraction barriers." If you're a physicist, "The off-diagonal terms vanish." Other nerdy metaphors?