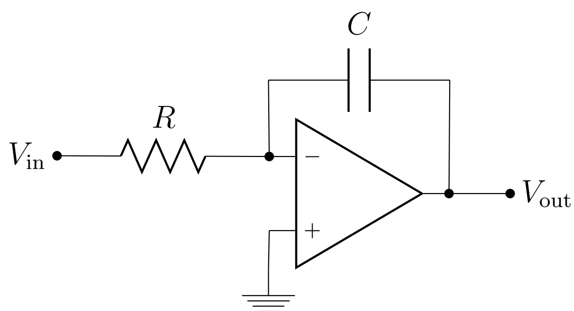
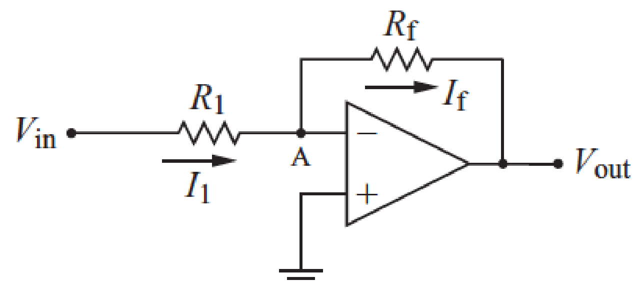
Physics 364, Fall 2014, reading due 2014-09-21.
Email your answers to ashmansk@hep.upenn.edu by 11pm on Sunday

Course materials and schedule are at http://positron.hep.upenn.edu/p364

Assignment: This week's textbook reading is only about 8 pages and is pretty clearly written, so there's no need to skim. (a) First read carefully Eggleston's sections 6.1–6.3. (b) Then read through my notes (starting on next page), which this week aren't so different from the book's approach. (c) Then email me your answers to the questions below. **Please look over this assignment early enough that you have time to ask me questions if anything is unclear.**

1. What is the below-left circuit intended to do? In other words, if this circuit is performing its usual function, how does V_{out} relate to V_{in}?





2. (a) If I choose $R_1 = 2$ kΩ and $R_f = 20$ kΩ in the above-right circuit, what is the relationship between V_{out} and V_{in}? (Try to get both the magnitude and the sign right.) (b) Assuming that the "golden rules" correctly describe this circuit's operation, what potential difference (voltage) is measured between point A and ground? (c) Again using the golden rules, what relationship can you write between I_1 and I_f?

3. Is there anything from this reading assignment that you found confusing and would like me to try to clarify? If you didn't find anything confusing, what topic did you find most interesting?

4. How much time did it take you to complete this assignment? Also, how many total hours did you spend on Physics 364 in the past week? If you feel that some of these hours were a poor use of your time (for most learning per hour invested), please suggest ways in which we might better align the course with your learning style.

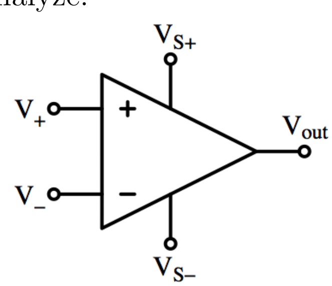
We will spend the next 2.5 weeks on *opamps* ("operational amplifiers"). An **opamp** is a high-gain differential amplifier with very high input resistance. It is nearly always used with negative feedback. Negative feedback means, for example, that if my car starts to veer toward the right, I turn the steering wheel to the left; and if the car starts to go off-course to the left, I turn the wheel to the right. Negative feedback has widespread use for making a desired outcome *stable*, such that a deviation from the desired outcome causes a correction of opposite sign to be applied. (Think of stable equilibrium in mechanics.)

What is the point of an amplifier, anyway? Circuits having only passive components have no power gain: the power in the signal coming out of the circuit cannot exceed the power in the signal going in. If you want to drive your massive stereo speakers from your tiny MP3 player, you need an intermediate circuit that will arrange for the power going into the large speakers to exceed the power coming out of the MP3 player. This can't be done with the light bulbs, resistors, capacitors, inductors, and diodes that we have seen so far. So one common use of amplifiers is e.g. to turn a voltage signal $V_{\rm in}$ into a signal $V_{\rm out}$ that is e.g. $10\times$ as large. While a transformer (another passive device) with turns-ratio $n_2/n_1 > 1$ could produce $V_{\rm out} = (n_2/n_1)V_{\rm in} > 1$ for a time-varying signal, it could do so only at the cost of lower current: $I_{\rm out} = (n_1/n_2)I_{\rm in}$. To boost the power in a signal, we need an *active* device such as an amplifier, which makes use of an external power source.

An amplifier can also turn a weak voltage source into a strong voltage source — for example serving as a go-between for a source whose relatively large $R_{\rm thevenin}$ would preclude it from driving a relatively small $R_{\rm load}$ without drooping. We'll see how this works when we study the *opamp follower* (a.k.a. *buffer amplifier*) circuit below.

Perhaps the most remarkable use of opamps is that they allow you without too much effort to perform all kinds of mathematical operations. You can make a weighted sum: $V_{\rm out} = \sum_i w_i V_{{\rm in},i}$. You can integrate (without the RC circuit's annoying $V_{\rm out} \ll V_{\rm in}$ restriction): $V_{\rm out} \propto \int V_{\rm in}\,{\rm d}t$. You can even compute logarithms: $V_{\rm out} \propto \log(V_{\rm in})$.

Opamps are building blocks that allow you to build useful circuits whose performance depends more on the chosen values of a few passive components than on the details of the opamp itself. They will become one of your favorite LEGO bricks — able to fit into your circuit in many different configurations, which you will soon learn to recognize and know how to analyze.



An opamp (shown above) has two inputs (labeled "+" and "−") and one output. The − input is called the *inverting* input, and the + input is called the *non-inverting* input. The + and − do not refer to the polarity of the input signal, but rather to the fact that the output is proportional to $V_+ - V_-$. An opamp also needs an external source of power, which we label $V_{S\pm}$ (letter S for "supply"): $V_{S\pm}$ are typically something like ±15 V, though the exact choice depends on the opamp model and what power-supply voltages you have available. Most of the time, $V_{S\pm}$ are omitted from the schematic diagram — their presence is implicit.

The basic function of an opamp is to produce an output proportional to the difference between its inputs, with a very large constant of proportionality:

$$V_{\text{out}} = A \cdot (V_+ - V_-)$$

with $A \gg 1$ (typically $A \sim 10^6$ or more). V_{out} cannot go past the power supply *rails,* i.e. $V_{S-} < V_{\text{out}} < V_{S+}$. In fact, most opamps do not permit V_{out} to get closer than 1 or 2 V from $V_{S\pm}$, so for $V_{S\pm} = \pm 15$ V, typically V_{out} cannot exceed $\approx \pm 13$ V.[1]

Most of what you need to know about analyzing opamp circuits can be summed up in two **Golden Rules**. We will see next week why the rules work. We will also see later some limitations of real-world opamps that cause deviations from the ideal behavior described by the golden rules. But for this week, we'll just try to get comfortable with using the golden rules as stated.

Rule #1. V_{out} takes on whatever value is needed to arrange that $V_+ - V_- = 0$.

Rule #2. The inputs draw negligible current.

Rule #0 (the fine print). (a) There must be negative feedback: i.e. a fraction of V_{out} must be fed back into V_-, the inverting input. (b) V_{out} must not be saturated (i.e. V_{out} can't be stuck at or near the $V_{S\pm}$ rails). (c) Negative feedback is especially important at DC, to prevent V_{out} from saturating at $V_{S\pm}$.

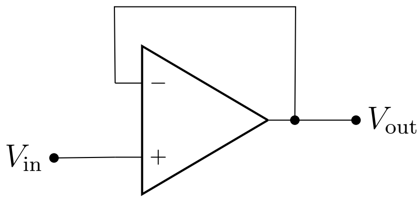
Rule #2 is a consequence of the opamp's very high input resistance, and does not depend on Rule #0. We will see next week that Rule #1 is just a consequence of the opamp's very high gain, as long as Rule #0 is satisfied.

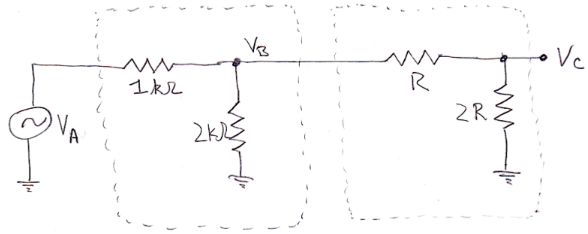
Let's draw out the schematic diagrams for several classic opamp configurations. As we go along, we will use the golden rules to analyze the operation of each circuit.

The first opamp circuit we'll study is called a **follower** (shown below, left), because $V_{\text{out}} = V_{\text{in}}$, i.e. V_{out} follows V_{in}. It is also called a **buffer**, because it can be placed in between two circuit fragments that normally would prefer not to talk directly to

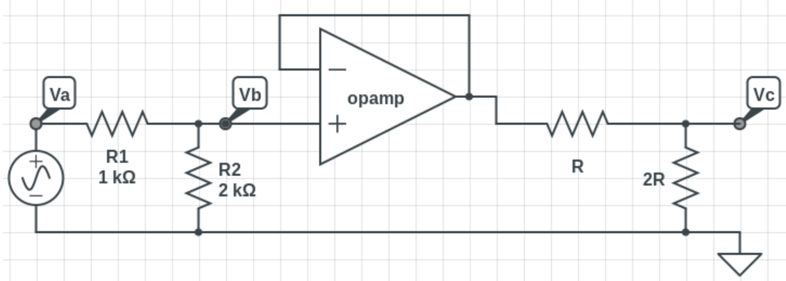
[1] Opamps that allow V_{out} to span the full $V_{S\pm}$ range will boast "rail-to-rail" operation on their data sheets.

one another.[2] To analyze this circuit, notice that Rule #1 implies that $V_{\text{out}} = V_{\text{in}}$ (so that $V_+ = V_-$). Then notice that Rule #2 implies that the input resistance of this circuit is $R_{\text{in}} = \infty$ (for an ideal opamp, anyway), since a change in V_{in} will not cause any change in current at the + terminal.





What use is a circuit for which $V_{\text{out}} = V_{\text{in}}$? Well, do you remember from Lab 3 what happened when we used the output of one voltage divider as the input for a second voltage divider, as shown above (right)? For the first voltage divider, the Thevenin resistance (a.k.a. source resistance, a.k.a. output resistance) is $R_{\text{thev},1} = 667\ \Omega$ (using V_B and ground as the two output terminals). Meanwhile, the input resistance of the second voltage divider is $R_{\text{in},2} = R + 2R = 3R$. As long as $3R \gg 667\ \Omega$, the analysis is simple: $V_B = \frac{2}{3}V_A$, and $V_C = \frac{2}{3}V_B = \frac{4}{9}V_A$. But if for example $R = 1\ \text{k}\Omega$, then V_B droops about 18% below its unloaded (or open-circuit) value, and the analysis is more complicated. By inserting an opamp follower between the output of the first voltage divider and the input of the second voltage divider (see figure below), we restore the simple $V_C = \frac{4}{9}V_A$ result, independent of the value of R. You can think of the opamp buffer circuit as "rose-colored glasses" for each of the circuits that it separates: the upstream circuit sees what it prefers to see (an ideal $R_{\text{in}} = \infty$ for its downstream circuit); and the downstream circuit sees what it prefers to see (an ideal $R_{\text{thev}} = 0$ for its upstream circuit).[3] This can be very handy for interfacing two circuits that normally would not be so compatible.



The next circuit (figure below, left) is the **inverting amplifier** configuration. Rule #2 implies $I_1 - I_2 = 0$, and Rule #1 implies $V_- = 0$. Using Ohm's law for R_1 gives

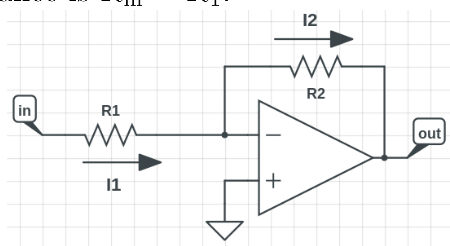
[2]Think of a "buffer state" that keeps the peace between two hostile nations.

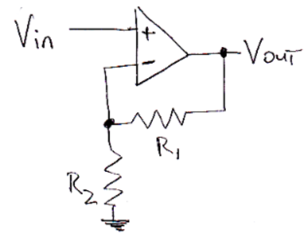
[3]Remember that an ideal voltage source has $R_{\text{thev}} = 0$, and that the ideal load for a voltage source is an open circuit ($R_{\text{load}} = \infty$). One more thing: I didn't make this up — I borrowed the metaphor from Tom Hayes's notes!

$I_1 = V_{\text{in}}/R_1$, and for R_2 gives $I_2 = -V_{\text{out}}/R_2$. Then $I_2 = I_1$ gives

$$V_{\text{out}} = -\frac{R_2}{R_1}\, V_{\text{in}}.$$

Since the upstream circuit (i.e. whatever is driving V_{in}) sees what looks like a resistor R_1 connected to ground (we call V_- a "virtual ground" here), the inverting amplifier's input resistance is $R_{\text{in}} = R_1$.





The next configuration (above, right) is the **non-inverting amplifier**. Rule #1 implies $V_- = V_{\text{in}}$, and Rule #2 (no current drawn by $-$ input) lets us use the voltage-divider equation to relate V_- to V_{out}: we find $V_{\text{out}}\,\frac{R_2}{R_1+R_2} = V_{\text{in}}$. Solving for V_{out},

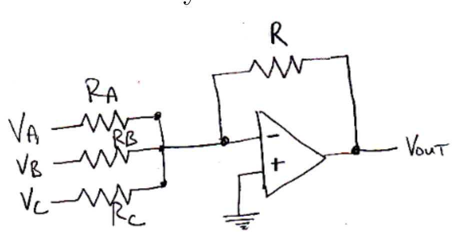
$$V_{\text{out}} = \left(1 + \frac{R_1}{R_2}\right)\, V_{\text{in}}.$$

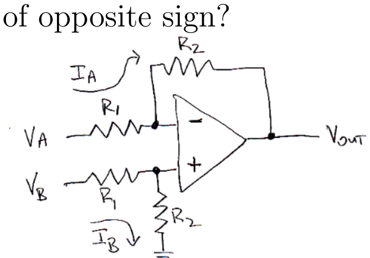
In this case, $R_{\text{in}} = \infty$ (for an ideal opamp), i.e. the input resistance is very high, because the + input draws negligible current.

Next, we have the **summing amplifier** configuration (below, left). Since $V_- = 0$ ("virtual ground"), the current flowing to the right through R_A, R_B, and R_C is, respectively, V_A/R_A, V_B/R_B, and V_C/R_C. By Rule #2, the sum of these three currents must flow to the right through R. So V_{out} is (minus) the weighted sum of the inputs:

$$V_{\text{out}} = -I_{\text{total}}R = -\left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)R = -\left(V_A\frac{R}{R_A} + V_B\frac{R}{R_B} + V_C\frac{R}{R_C}\right).$$

Can you think of a way to include a coefficient of opposite sign?



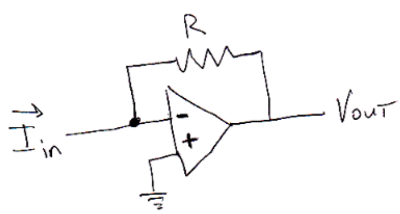


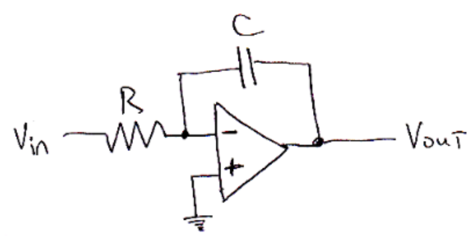
Next is the **differential amplifier** configuration (above, right). By Rule #2, the current I_A that flows through the upper R_1 continues through the upper R_2. Similarly, the current I_B that flows through the lower R_1 continues through the lower R_2. Ohm's law gives $V_{\text{out}} - I_A R_2 = V_-$ and $V_- - I_A R_1 = V_A$, which combine (eliminating I_A) to give $\frac{V_{\text{out}}-V_-}{R_2} = \frac{V_- - V_A}{R_1}$, which simplifies to $R_1 V_{\text{out}} = (R_1+R_2)[V_-] - R_2 V_A$. Meanwhile, the voltage-divider equation gives $V_+ = \frac{R_2}{R_1+R_2}\, V_B$, and Rule #1 gives $V_- = V_+$. Substituting for $[V_-]$, we get $R_1 V_{\text{out}} = (R_1+R_2)[\frac{R_2}{R_1+R_2}\, V_B] - R_2 V_A$, which gives $R_1 V_{\text{out}} = R_2 V_B - R_2 V_A$, which simplifies to

$$V_{\text{out}} = \frac{R_2}{R_1}\,(V_B - V_A).$$

Amplifying the difference between two inputs is often a helpful technique for eliminating interference from unwanted signals.

Another useful opamp circuit is the **current-to-voltage amplifier** (below, left). Sometimes your incoming signal is most easily described as a weak current source. For instance, many devices for photon detection emit an electric current in proportion to the incoming light intensity. Whereas the easiest load for a voltage source to drive is an open circuit ($R_{\text{load}} = \infty$), the easiest load for a current source to drive is a short circuit ($R_{\text{load}} = 0$): connecting too large a resistance to a weak current source will suppress the flow of current.[4] Because the opamp's + input is grounded, the − input is held at "virtual ground" by feedback (Rule #1). So the current flowing into the opamp current-to-voltage amplifier sees what looks like a short-circuit to ground, which is ideal for a current source. By Rule #2, I_{in} must flow to the right through resistor R, since the opamp inputs draw negligible current. So $I_{\text{in}} + V_{\text{out}}/R = 0$, which yields $V_{\text{out}} = -RI_{\text{in}}$.





The next application (above, right) is called the **opamp integrator.** By Rule #1, $V_- = 0$, so the current flowing to the right through R is just $I = V_{\text{in}}/R$. By Rule #2, current I must continue to the right through C, since negligible current flows into the opamp inputs. With the left side of the capacitor fixed at $V_- = 0$, the result of current I flowing through C from left to right is $C\, \mathrm{d}V_{\text{out}}/\mathrm{d}t = -I = -V_{\text{in}}/R$, so

$$V_{\text{out}}(t) = -\frac{1}{RC}\int_{-\infty}^{t} \mathrm{d}t'\; V_{\text{in}}(t').$$

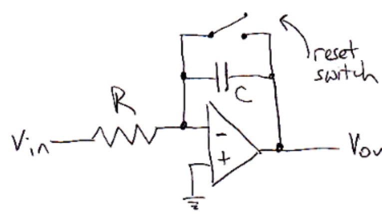
The opamp circuit makes a very nice integrator — there is no longer the annoying $V_{\text{out}} \ll V_{\text{in}}$ restriction that we had with the passive RC integrator of Lab 2. One serious problem with this circuit, however, is that if the time-averaged value of V_{in}

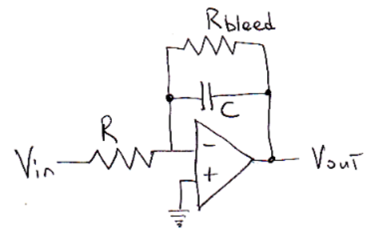
[4] For current sources, there is a theorem — called Norton's theorem — that is analogous to Thevenin's theorem. Calculating the Norton equivalent circuit allows you to quantify this loading effect for current sources.

differs at all from zero, $V_{\rm out}$ will saturate near $V_{S\pm}$. We have violated Rule #0c, that there should be feedback at DC. There are two common solutions to this problem. The first (shown below, left) is to use a switch to zero the charge on the capacitor at $t = 0$ (i.e. whenever the integration should begin). Then

$$V_{\rm out}(t) = -\frac{1}{RC}\int_0^t {\rm d}t'\ V_{\rm in}(t')$$

and you can choose RC such that $V_{\rm out}$ does not saturate until well after the desired integration time has elapsed. I remember finding this circuit in the analogue of Physics 414 that I took in college: to observe changes in magnetic flux during phase transitions of a superconductor, the experiment reported Φ_B by integrating the emf $\mathcal{E}$ induced in a wire coiled around the sample.[5]

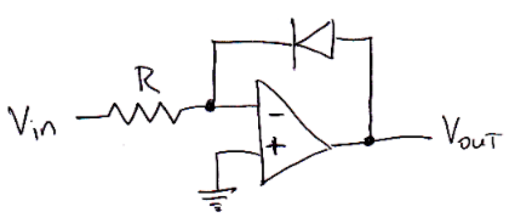




The second solution (shown above, right) is to include a "bleeder resistor" $R_{\rm bleed}$ to drain the capacitor's charge over some long time scale $R_{\rm bleed}C$. Now there is a feedback path at DC, and the circuit is doing a weighted integration, emphasizing the recent time interval $R_{\rm bleed}C$. I didn't try to work this out explicitly, but by intuition I think the result is something like

$$V_{\rm out}(t) = -\frac{1}{RC}\int_{-\infty}^t {\rm d}t'\ V_{\rm in}(t')\ e^{-(t-t')/(R_{\rm bleed}C)}.$$

Another way to look at this is as a generalization (using a complex impedance in place of the feedback resistor) of the inverting amplifier circuit. Without $R_{\rm bleed}$ the amplifier's gain at $f = 0$ is ∞; with $R_{\rm bleed}$, the DC gain is reduced to $-R_{\rm bleed}/R$.



One final application (above) is a **logarithmic amplifier**. Using the Shockley diode equation, $I_{\rm diode} = I_0\ e^{V_{\rm diode}/(25\ {\rm mV})}$, we use Rule #1 ($V_- = 0$ since + input is at

[5] Alas, when we did the experiment, the opamp chip was dead, and I had not yet taken an electronics course, so I didn't know how to fix it. We wound up collecting the $\mathcal{E}(t)$ data directly and then doing the integration numerically in a spreadsheet.

ground), and then Rule #2 (the currents flowing into the − input sum to zero) to find $I_0\ e^{V_{\rm out}/(25\ {\rm mV})} + V_{\rm in}/R = 0$. Rearranging,

$$V_{\rm out} = -25\ {\rm mV}\ \log\left(\frac{V_{\rm in}}{I_0 R}\right).$$

The quantity 25 mV is really kT/e, as we'll see once we study diodes and transistors in detail.[6] And the constant I_0 (usually written $I_{\rm sat}$) is a very small current (e.g. picoamps) that depends on the physical characteristics of the diode. It's quite a neat trick that you can get a circuit to take a logarithm! Notice that $V_{\rm out}$ figures out how to "undo" whatever is in the feedback loop. This technique can be used more generally.

CircuitLab

If you haven't already had a chance to do so in class, please point your web browser to `www.circuitlab.com` and sign up (using your `upenn.edu` email address) for a free Student Edition account. The Engineering School paid for a Penn site license through 3/2015, so you should be able to use all key CircuitLab features for free, as long as you sign up from a `upenn.edu` email address.

You can find a large number of Phys364-related circuits (mostly from the Fall 2012 course) in my CircuitLab workbench at

`www.circuitlab.com/user/ashmanskas/`

In future weeks, I will occasionally ask you to build and simulate a circuit at home in CircuitLab as part of the weekly reading assignment.

[6] You might remember that kT at room temperature is about $\frac{1}{40}$ eV.