Physics 364, Fall 2014, reading due 2014-10-19.

Email your answers to `ashmansk@hep.upenn.edu` by 11pm on Sunday

Course materials and schedule are at `http://positron.hep.upenn.edu/p364`

Assignment: (a) First finish reading whatever portion of Eggleston's chapter 4 (Bipolar Junction Transistors, pp 104–130) you may not have read last weekend. (b) Then read through my notes (starting on next page), which directly relate to the coming week's labs. (c) Then watch the YouTube videos about PID controllers, whose URLs are listed at the end of my notes. (d) Finally, email me your answers to the questions below.

1. What alternative name does Eggleston's text give for the circuit that we call the emitter follower?

2. What is the name of the annoying feature of a push-pull buffer that occurs at the $V_{\text{in}} \approx 0$ portions of a sinusoidal input?

3. What does "common-mode rejection ratio" mean? Give some relevant context.

4. Is there anything from this reading that you would like me to try to clarify? If you didn't find anything confusing, what topic did you find most interesting?

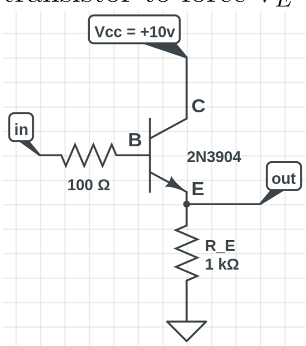
5. How much time did it take you to complete this assignment?

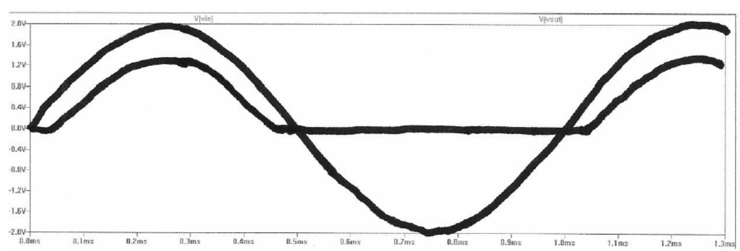
This is the second of two weeks we'll spend on Bipolar Junction Transistors. Our main goal for this second week is to introduce just enough additional BJT-based circuits to allow us to understand the internal workings of a (simplified) home-made opamp, so that the functioning of an opamp no longer needs to seem like magic.

First we will analyze three transistor circuits that are commonly found in opamp designs: the push-pull follower (nearly always used as an output stage); the current mirror (used as a current source, to obtain a large dynamic resistance $R_{\rm dyn} = \frac{{\rm d}V}{{\rm d}I}$); and the differential amplifier (used to produce an output voltage that is proportional to the difference in two input voltages). Finally, we will put all of these pieces together to form our own ad-hoc opamp. Actually, our home-made opamp will use the simple current source you studied in Lab 13, instead of a current mirror, but we'll include the current mirror in these notes so that you recognize one when you see it — amplifiers built as integrated circuits (e.g. the '741 opamp) often contain current mirrors.

Push-pull follower

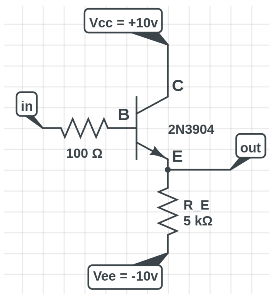
The first emitter follower that we studied last week had a serious limitation (shown below): it only followed positive input signals, because there was no way for the transistor to force V_E to go below V_{EE}, and in this first case, we used $V_{EE} = 0$.



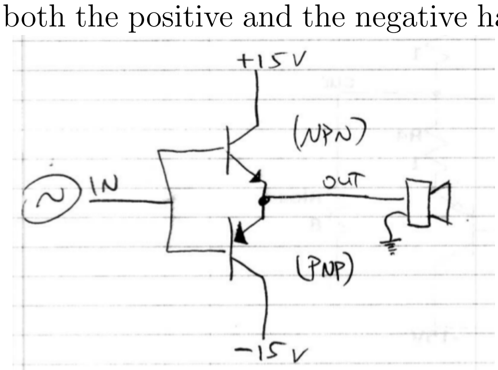


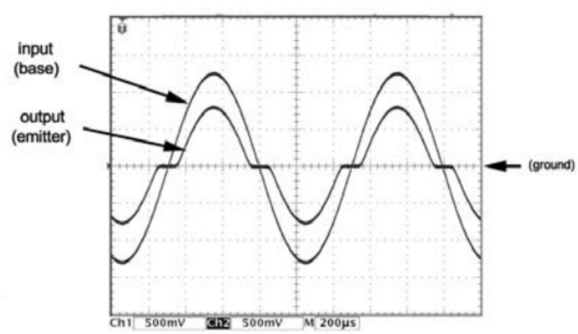
Fundamentally, the reason for this clipping is that the transistor has no way to force current to flow up through the emitter, which is what would be needed to get $V_E < 0$ when $V_{EE} = 0$. The transistor permits, to a varying degree (controlled by V_{BE}) current to flow downward, into the collector and out of the emitter.

The way that we worked around this problem was to use symmetric positive and negative power supplies, i.e. by moving V_{EE} to -10 V, as shown below. One problem with this modification is that the magnitude of the current I_E through the transistor even when $V_{\rm in} = 0$ must be larger than the current that flows to the downstream load when $|V_{\rm in}|$ is at its maximum amplitude, because to avoid clipping we need $I_E > 0$ when $V_{\rm in}$ reaches its most-negative value. (Notice that the original emitter follower, with $V_{EE} = 0$, didn't have this problem: it had $I_E = 0$ when $V_{\rm in} = 0$, but at the very annoying price of only being able to follow the positive half of the signal.) So the simple emitter follower wastes power: the power that it dissipates in the quiescent state (when there is no signal to amplify) is much larger than the useful power supplied to the load when a signal is present. If this were an audio amplifier intended to drive a 10 watt speaker, a simple emitter follower might dissipate $\mathcal{O}(100\text{ W})$ of power even when the speaker is silent. What is to be done?



The solution (shown below, left, with a speaker attached to its output[1]) is to use two emitter followers in tandem: one that follows only the positive half of the signal and one that follows only the negative half of the signal. For the positive half, we use a familiar NPN transistor; for the negative half, we use a PNP transistor. A PNP transistor follows the same rules as an NPN transistor, but with all of the polarities reversed. When $V_{\text{in}} \gtrsim 0.6$ V, the NPN transistor is active, and $V_{\text{out}} \approx V_{\text{in}} - 0.6$ V, with positive current flowing down into the NPN transistor's collector and out from its emitter and through the speaker; meanwhile, the PNP transistor is off (no current flows through it). When $V_{\text{in}} \lesssim -0.6$ V, the PNP transistor is active (because $V_{\text{BE}} < -0.6$ V), and $V_{\text{out}} \approx V_{\text{in}} + 0.6$ V, with positive current flowing down from the speaker into the PNP transistor's emitter and out from its collector; meanwhile, the NPN transistor is cut off. When $V_{\text{in}} \approx 0$, both transistors are cut off, and $V_{\text{out}} = 0$. This clever pairing of NPN and PNP followers — called a **push-pull** — allows us to follow both the positive and the negative halves of V_{in} without wasting power when $V_{\text{in}} = 0$.



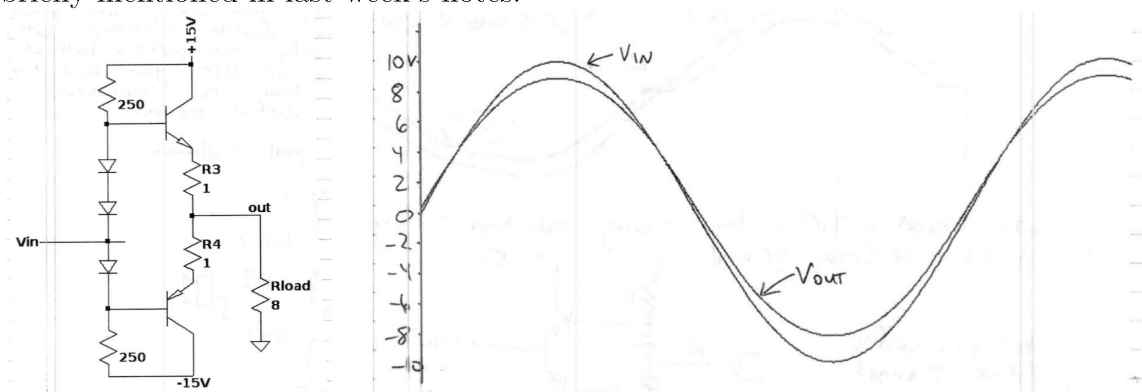


One well-known drawback (shown above, right) of this simple version of the push-pull is called *crossover distortion*: during the interval when $-0.6\text{ V} \lesssim V_{\text{in}} \lesssim +0.6$ V, neither the NPN nor the PNP transistor is active, so $V_{\text{out}}(t)$ shows a little flat spot where it crosses over between the positive and negative sides of the waveform. If $V_{\text{in}}(t)$ is a pure tone at frequency f and you play $V_{\text{out}}(t)$ into a speaker, you will hear

[1] In effect the speaker acts as an 8 Ω resistor between V_{out} and ground.

the original tone plus the buzzing-like sound of harmonics e.g. at $3f$, $5f$, ...

In practice, a real push-pull buffer normally uses several diodes (or the V_{BE} voltage drops of several additional transistors) to bias each base to stay about one diode drop away from ground when $V_{\text{in}} = 0$. This causes some power to be dissipated even when $V_{\text{in}} = 0$, but far less than in the case of a single emitter follower. The very small resistors between the emitters prevent the "thermal runaway" problem that I very briefly mentioned in last week's notes.



In the lab, we'll use the magical powers of opamp feedback as a different approach to eliminate crossover distortion. It is really fun to see how that works.

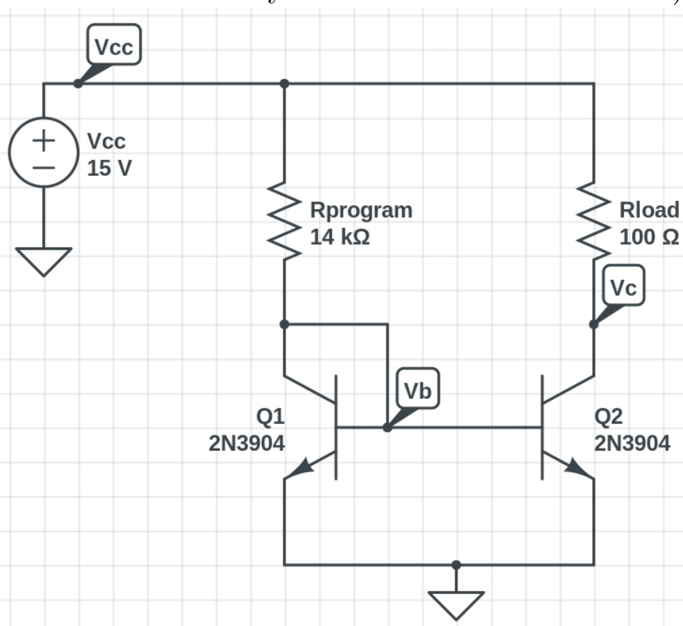
Current mirror

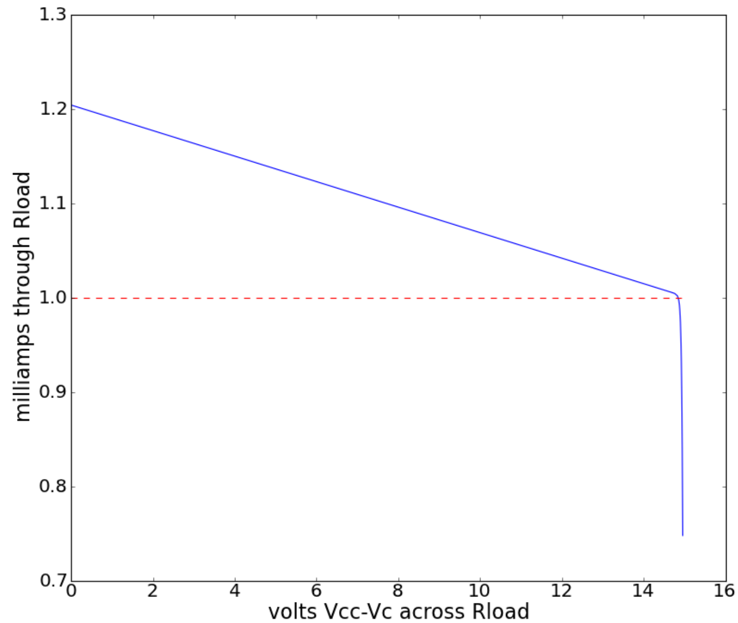
The second circuit that I want to describe is called a **current mirror** (shown below). It is one example of a current source. Voltage sources are far more common than current sources, but one occasionally finds a need for a current source. As you learned in Lab 13, you can actually build a much simpler current source using a single transistor, but I wanted to show you the current mirror because current mirrors are frequently found in the schematic diagrams of real-life opamps. So I want a current mirror to be something that at least looks familiar to you. In the figure below, the collector and base of transistor Q_1 are wired together to force $V_{CB} = 0$, which keeps Q_1 out of (but not very far away from) the $V_{CE} \approx 0$ saturation region: $V_{CE} = V_{CB} + V_{BE} \approx +0.6$ V in this case. Since Q_1 is active,[2] its base (and thus also Q_1's collector) is a diode drop above ground. So the current flowing through R_1 is $I_1 = (V_{\text{CC}} - V_{BE1})/R_1 \approx (V_{\text{CC}} - 0.6\text{ V})/R_1$. Because of the diode-like relationship between V_{BE} and I_C, we know that V_{BE1} will vary logarithmically with I_1 — remember that $I_C \approx I_0 \exp(V_{BE}/(25\text{ mV}))$.

Now if transistors Q_1 and Q_2 are matched, so that they have the same physical properties, and if they are held at the same temperature (which you can arrange by

[2] If you start out assuming that Q_1 is off, then Q_1's $V_B \approx V_{CC}$ (because little or no current flows through R_1), which makes Q_1's $V_{BE} \approx V_{CC}$, contradicting the assumption that Q_1 is off. Since Q_1 is neither off nor saturated, it should be active.

making the two transistors sit side-by-side on the same piece of silicon), then the I_C vs. V_{BE} curve for Q_1 will match that of Q_2. And since V_{BE2} for Q_2 is the same as V_{BE1} for Q_1 (because the two bases are wired together), the current through R_2 is $I_2 \approx I_1$, even if R_2 is very different from R_1. To make the graph on the right below, I varied the resistance R_2 and then graphed I_2 as a function of the voltage drop across R_2. You can see that I_2 is a nearly constant 1 mA. (The dashed curve shows that I_1 is exactly constant at about 0.95 mA.)





https://www.circuitlab.com/circuit/ucc8ej/reading08-fig03/

The reason that I_2 is not perfectly flat is that a transistor's collector current I_C does vary slightly with V_{CE}, as you can see from the left-hand graph below. This feature of bipolar transistors, which we will mostly ignore in this course, is called the "Early Effect." For our purposes, the Early Effect (i.e. the nonzero slope of each I_C vs. V_{CE} curve in the active region below) puts a limit on how good a current source you can make with a bipolar junction transistor. For an ideal current source, $-\frac{1}{R_{\text{out}}} = \mathrm{d}I_{\text{out}}/\mathrm{d}V_{\text{out}} = 0$, which corresponds to $R_{\text{out}} = \infty$. For our current mirror, $R_{\text{out}} \approx (15\text{ V})/(0.2\text{ mA}) \approx 75\text{ k}\Omega$.

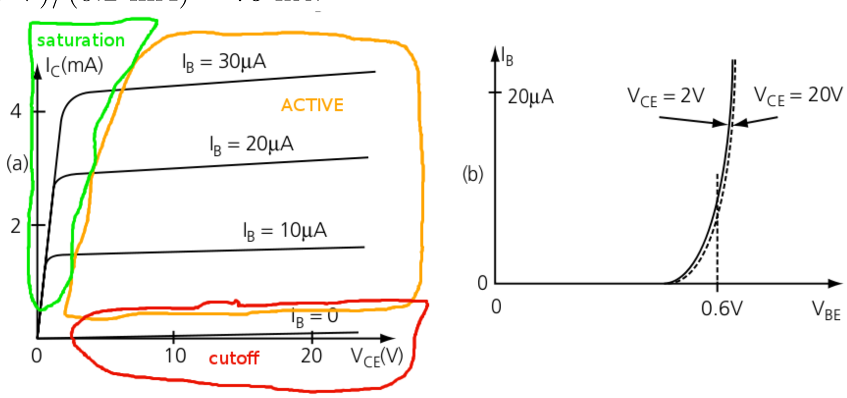
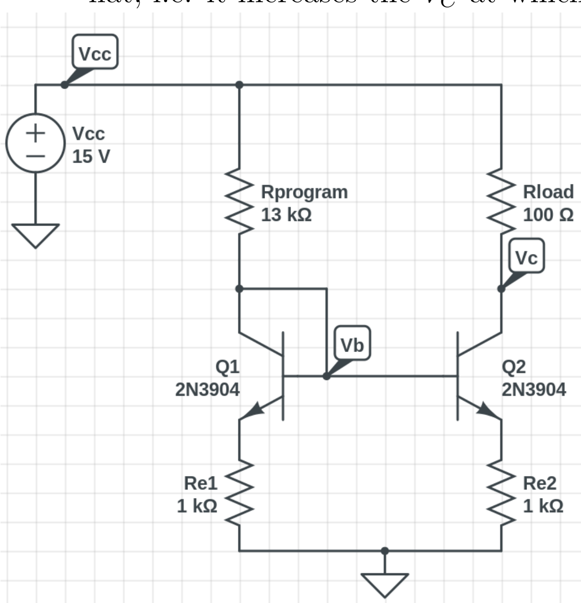
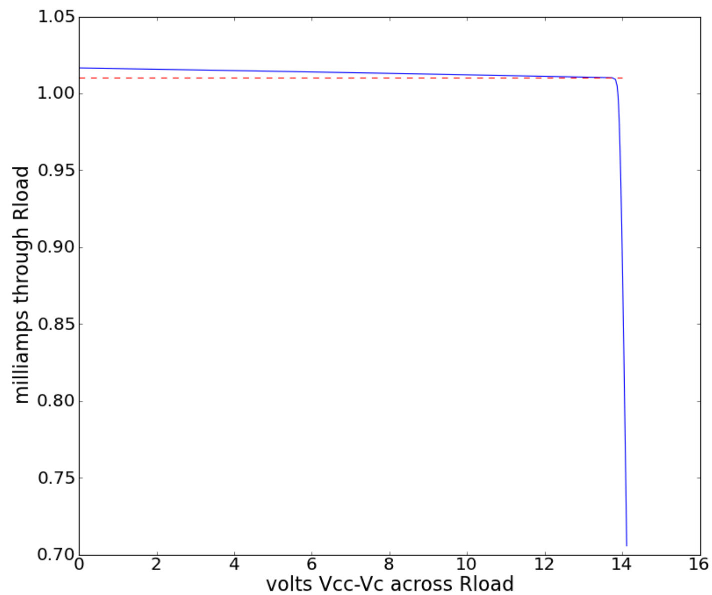


Fig. 9.26. Characteristics of the bipolar transistor.

[Short digression: As you saw in Lab 13, you can make $R_{\rm out}$ of a transistor-based current source (or current mirror) even larger than the reciprocal-slope of the transistor's I_C-vs.-V_{CE} curve by putting a resistor R_E at the emitter.[3] This creates a sort of negative feedback that reduces the change in I_C as V_C decreases. The easiest value of $R_{\rm load}$ for the current mirror to handle is $R_{\rm load} = 0\ \Omega$, a short circuit. As you increase $R_{\rm load}$, the IR drop across $R_{\rm load}$ causes V_C to decrease, which therefore decreases V_{CE}. Thus, because of the finite slope of the transistor's characteristic I_C-vs.-V_{CE} curve, I_C starts to decrease. **But** this decrease in I_C now causes V_E to decrease as well, because of the IR drop across R_E. Since the base current I_B changes much less than I_C changes (by a factor $1/\beta$), V_B stays relatively constant, so the slight drop in V_E actually *increases* V_{BE}. This in turn pushes I_C back up toward its original value. The end result is to increase the current mirror's $R_{\rm out}$ by roughly an order of magnitude. In the example circuit shown below, $R_{\rm out} = 2.3\ \mathrm{M}\Omega$. Larger R_E gives a larger improvement in the slope, but decreases the range of V_C over which I_C stays relatively flat, i.e. it increases the V_C at which the transistor reaches saturation.]





`https://www.circuitlab.com/circuit/na6upg/reading08-current-mirror-improved/`

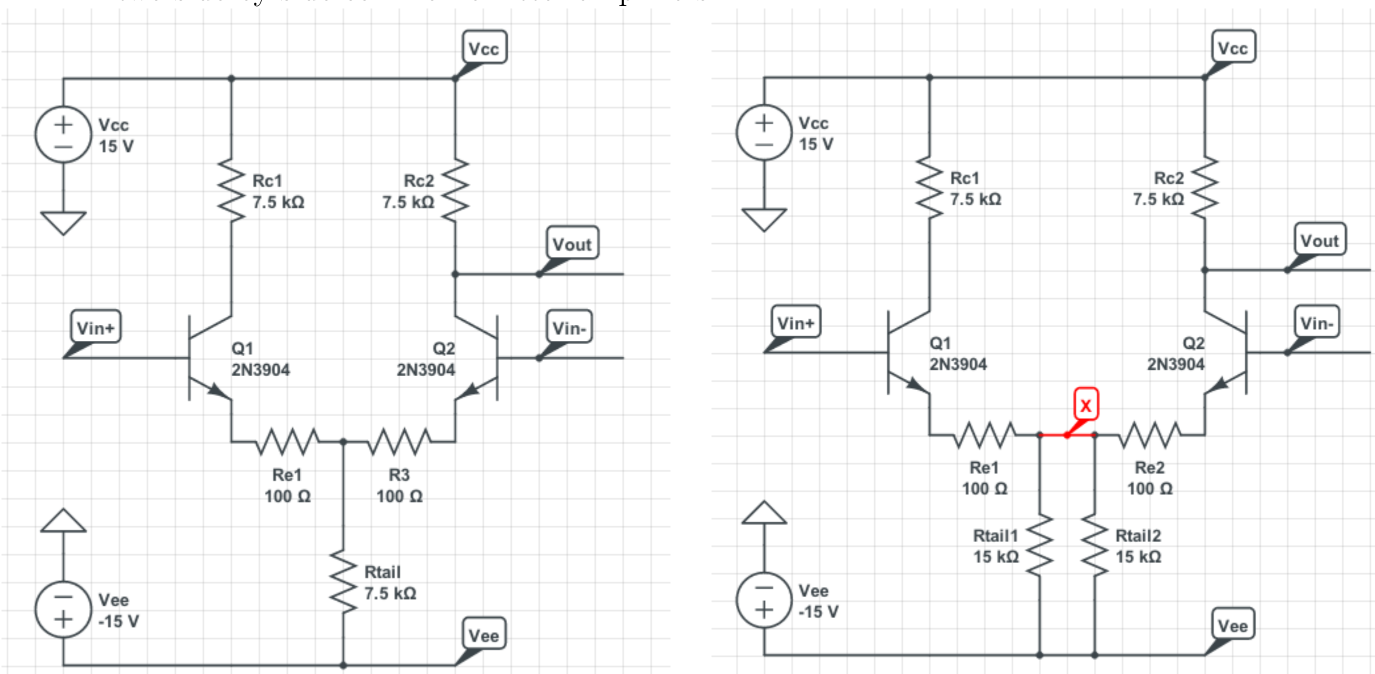
The amazing thing about a transistor-based current mirror (or current source) is not so much the slope of the I-V curve but rather the intercept. If you replaced the current source with a battery+resistor having the same $\mathrm{d}I_{\rm out}/\mathrm{d}V_{\rm out} = -(2\ \mathrm{M}\Omega)^{-1}$ and the same $I_{\rm out} \approx 1$ mA at $V_{\rm load} \approx 0$ across $R_{\rm load}$, the intercept would be ≈ 2000 V: you would need to use a 2000 V voltage source to get 1 mA to flow through a 2 MΩ resistor. A transistor-based current source is a much more convenient way to get a small $\mathrm{d}I_{\rm out}/\mathrm{d}V_{\rm out}$ — which corresponds to a large $R_{\rm out}$. Therefore, in amplifier circuits, we sometimes see a current source (or a current mirror) used in place of a large resistor: the current source can provide the same $\mathrm{d}I/\mathrm{d}V$ as a large resistor without the large $\Delta V = IR$ required by a large resistor. The jargon for this use of a

[3] Engineers call this "emitter degeneration."

current source in an amplifier is "active load."[4]

Differential amplifier

We next consider the circuit shown below (left), whose purpose is to amplify the difference between $V_{\text{in}+}$ and $V_{\text{in}-}$, so that $V_{\text{out}} = A \cdot (V_{\text{in}+} - V_{\text{in}-})$. The easiest way to understand this circuit is to redraw it as shown below (right), so that it looks (except for the mysterious wire joining the two 100 Ω resistors at the point marked **X**) like two side-by-side common emitter amplifiers.



`https://www.circuitlab.com/circuit/y7nr2f/differential-amplifier-phys-364-lab-6/`

Now define $V_{\text{cm}} = \frac{1}{2}(V_{\text{in}+} + V_{\text{in}-})$ and define $V_{\text{dif}} = V_{\text{in}+} - V_{\text{in}-}$. Then $V_{\text{in}+} = V_{\text{cm}} + \frac{1}{2}V_{\text{dif}}$ and $V_{\text{in}-} = V_{\text{cm}} - \frac{1}{2}V_{\text{dif}}$. We call V_{cm} the *common mode* input, and we call V_{dif} the *differential* input. We want V_{out} to amplify V_{dif} by some large factor A, while if possible making V_{out} completely insensitive to V_{cm}. We do not want the amplifier to respond to changes in the inputs that take the form $\Delta V_{\text{in}+} = \Delta V_{\text{in}-}$; we want the amplifier only to respond to the difference $V_{\text{in}+} - V_{\text{in}-}$. The ratio of an amplifier's *differential gain* (the factor by which it amplifies V_{dif} to its *common-mode gain* (the factor by which it amplifies V_{cm}) is called its **common mode rejection ratio**.

Suppose that $V_{\text{in}+}$ and $V_{\text{in}-}$ are both initially at ground. Then each transistor's emitter

[4]To quote the Wikipedia article for *active load:* "In circuit design, an active load is a circuit component made up of active devices, such as transistors, intended to present a high small-signal impedance yet not requiring a large DC voltage drop, as would occur if a large resistor were used instead. Such large AC load impedances may be desirable, for example, to increase the AC gain of some types of amplifier."

is one diode drop below ground, $V_{E1} = V_{E2} \approx -0.6$ V, and each emitter current is

$$I_{E2} \;=\; I_{E1} \;=\; \frac{V_{E1} - V_{EE}}{R_{E1} + R_{\text{tail1}}} \;\approx\; \frac{14.3 \text{ V}}{15.1 \text{ k}\Omega} \;\approx\; 0.95 \text{ mA}.$$

Notice that the current flowing through the short wire at point X is zero by symmetry. The collector currents are

$$I_{C2} \;=\; I_{C1} \;=\; \frac{\beta}{\beta+1} I_{E1} \;\approx\; I_{E1}$$

(since $\beta \gg 1$), so the collector voltages are

$$V_{C2} \;=\; V_{C1} \;=\; V_{CC} - I_{C1}R_{C1} \;\approx\; 15 \text{ V} - (0.95 \text{ mA})(7.5 \text{ k}\Omega) \;\approx\; +7.9 \text{ V}.$$

So with no input, $V_{\text{out}} \approx +8$ V.[5]

Now let's try wiggling both inputs together by ΔV, so that $V_{\text{in}+} = V_{\text{in}-} = \Delta V$, or $V_{\text{cm}} = \Delta V$. Each emitter voltage $V_{E1,2}$ then also rises by ΔV, which increases each emitter current $I_{E1,2}$ by $\Delta V/(R_{E1,2} + R_{\text{tail1},2})$. (Notice that by symmetry, there is still no current through the wire at point X.) So the change in V_{out} is

$$\Delta V_{\text{out}} \;=\; -\,R_{C2}\,\Delta I_{C2} \;\approx\; -\,\frac{R_{C2}\,\Delta V}{R_{E2} + R_{\text{tail2}}} \;=\; -\,\frac{R_{C2}}{R_{E2} + R_{\text{tail2}}}\,V_{\text{cm}}.$$

The common-mode gain is $-R_{C2}/(R_{E2} + R_{\text{tail2}}) \approx -0.5$. That doesn't sound so impressively small yet, but let's see how it compares with the differential gain.

Let's next try wigging the two inputs in opposite directions by $\Delta V/2$, so that we have $V_{\text{in}+} = -V_{\text{in}-} = \frac{1}{2}\Delta V$, or $V_{\text{dif}} = \Delta V$, while $V_{\text{cm}} = 0$. Now emitter voltage V_{E2} rises by $\frac{1}{2}\Delta V$, while V_{E1} falls by $\frac{1}{2}\Delta V$. So there are equal and opposite changes in the two emitter currents: $\Delta I_{E1} = -\Delta I_{E2}$. This breaks the symmetry between the left and right sides of the circuit: in fact, since the changes in current are completely antisymmetric, this change in current flows entirely through the small wire marked X. So there is no change in current through $R_{\text{tail1},2}$, and $\Delta V_X = 0$. So then $\Delta I_{E1} = \Delta V_{E1}/R_{E1}$, and $\Delta I_{E2} = \Delta V_{E2}/R_{E2} = \Delta V_{\text{in}-}/R_{E2} = -\frac{1}{2}V_{\text{dif}}/R_{E2}$. Then

$$\Delta V_{\text{out}} \;=\; -\,R_{C2}\,\Delta I_{C2} \;\approx\; +\,\frac{R_{C2}\,V_{\text{dif}}}{2R_{E2}} \;=\; \frac{R_{C2}}{2R_{E2}}\,V_{\text{dif}}.$$

The differential gain is $R_{C2}/(2R_{E2}) \approx 38$, which is considerably larger than the common mode gain.

Before we go on, let's correct one important detail that I omitted above: we didn't consider $r_e = \mathrm{d}V_{BE}/\mathrm{d}I_C \approx 25 \text{ mV}/I_C$, also known as "little r_e."[6] Little r_e acts as if

[5] The fact that the quiescent value of V_{out} is not zero isn't a problem, because we will eventually put another amplifier stage downstream of this one. We're really only interested in how V_{out} changes as the inputs change.

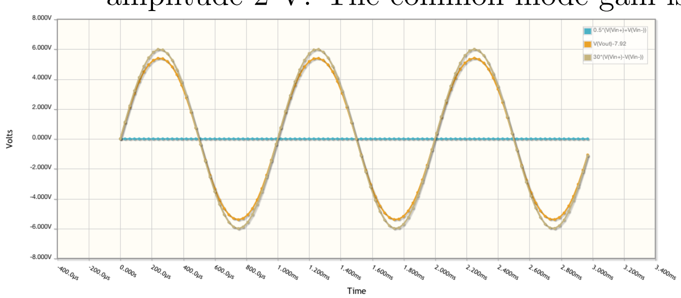
[6] To engineers, this is more commonly known as $1/g_m$.

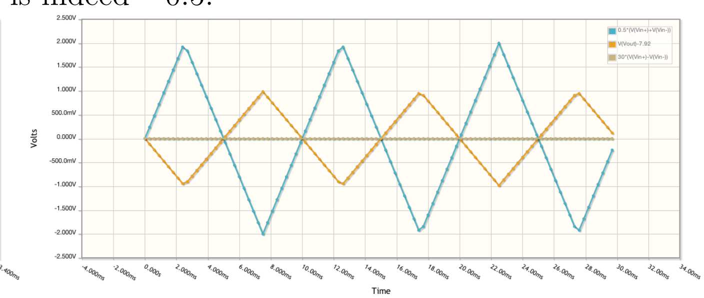
it were in series with R_E, so we should replace R_E with the expression $R_E + r_e$ in all of the results above. Thus, we find the differential gain to be $A = \frac{1}{2}R_{C2}/(R_{E2} + r_e)$, the common mode gain to be $A_{CM} = -R_{C2}/(R_{\text{tail2}} + R_{E2} + r_e)$, and common-mode rejection ratio to be $|A/A_{CM}| = \frac{1}{2}(R_{\text{tail2}} + R_{E2} + r_e)/(R_{E2} + r_e)$.

Next, notice that the circuit that we have been analyzing (above-right figure) is electrically identical to the circuit in the above-left figure: we simply replaced a single 7.5 kΩ resistor with two parallel 15 kΩ resistors. So we can directly write down the gain and CMRR for the differential amplifier in the above-right figure, using $R_{\text{tail2}} = 2R_{\text{tail}}$. We find

$$A = \frac{R_C}{2(R_E + r_e)}, \quad A_{CM} = -\frac{R_C}{2R_{\text{tail}} + R_E + r_e}, \quad \text{CMRR} = \frac{2R_{\text{tail}} + R_E + r_e}{2(R_E + r_e)}.$$

For the circuit drawn above, $I_C \approx 1$ mA gives $r_e \approx 25\ \Omega$, so we expect differential gain $A \approx 30$ and common-mode gain $A_{CM} \approx -0.5$. The left figure below shows the CircuitLab simulation results, where V_{dif} is a sine wave of amplitude 100 mV. The brown curve shows $V_{\text{out}} - 7.92$ V (so the simulator finds a quiescent $V_{\text{out}} = 7.92$ V), and the blue curve shows $30 \times (V_{\text{in+}} - V_{\text{in}-})$. (The simulator finds a differential gain of 27.) The right figure below simulates $V_{\text{dif}} = 0$, with V_{cm} being a triangle wave of amplitude 2 V. The common-mode gain is indeed -0.5.





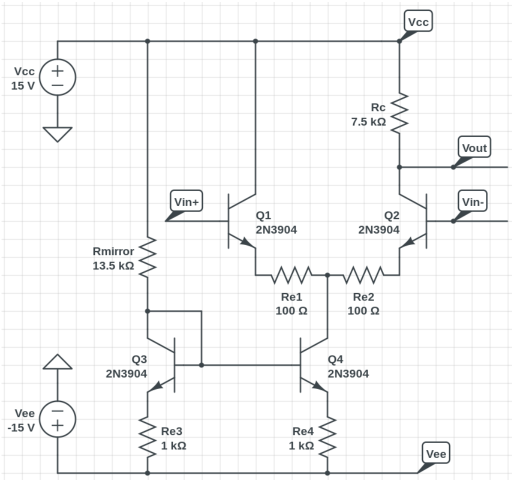
Now suppose that we want to make $R_{\text{tail}} = 2$ MΩ to reduce the common-mode gain to ≈ -0.002. To do that while keeping a 1 mA quiescent current through each of the two transistors would require $V_{EE} = V_X - 2 \times 1 \text{ mA} \times 2 \text{ M}\Omega \approx -4000$ V, which is absurd. Hmmm, what can we do? Let's try replacing R_{tail} with a 2 mA current source! If we adapt the "improved" current mirror from earlier in this reading to be a 2 mA current source, we know that its *I*-*V* curve will have a slope corresponding to about 2 MΩ,[7] but without the absurdity of a -4000 V power supply.[8]

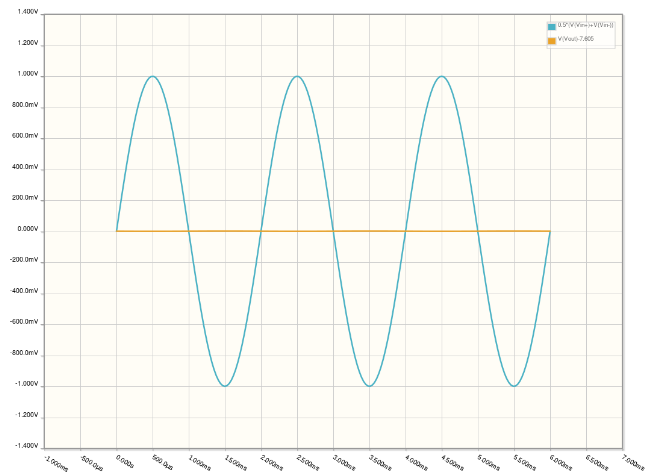
The circuit shown below (left) replaces R_{tail} with a 2 mA current mirror, whose

[7] This R_{out} value for the current mirror depends on the slope of the relevant I_C vs. V_{CE} curve of the particular transistor model we are using — in this case the 2N3904. In engineers' terms, it depends on the "Early Voltage" V_A of the 2N3904 transistor: $\mathrm{d}I_C/\mathrm{d}V_{CE} \approx I_C/V_A$, with $V_A \approx 100$ V for the 3904. The exact R_{out} value also depends on the β value of the transistor and on the emitter resistor R_E used in the current mirror.

[8] We could have used the simpler transistor current source from Lab 13, but I chose here to illustrate the use of a current mirror. In next week's lab, we will use the simpler current source.

dI/dV turns out to be $\approx$ (2 MΩ)$^{-1}$ when made with 2N3904 transistors, so my predicted common-mode gain is -0.002. In the simulation show below (right), I measure $A_{CM} = -0.001$, which is very small (as desired), but is puzzlingly about 2$\times$ smaller than I predicted.[9] The graph shows $\frac{1}{2}(V_+ + V_-)$ in blue and $V_{\rm out}$ in orange; in this simulation $V_{\rm dif} = 0$. I also verified that the differential gain is still around 27, so now the CMRR is over 10^4. One other modification shown in the schematic below was to remove the collector resistor above Q_1. Since Q_1's collector is not used as a voltage output, there is no need to put a resistor there: the only function of R_C above Q_2 is to make $V_{\rm out}$ wiggle in proportion to wiggles in I_{C2}. By the way, if we had left R_{C1} in place, we could have used V_{C1} as an opposite-sign output, i.e. 180° out of phase with $V_{\rm out}$.





https://www.circuitlab.com/circuit/x49463/reading08-diff-amp-mirror-tail/
https://www.circuitlab.com/circuit/j32k3p/reading08-current-mirror-2ma/

One final trick that you will see in the differential amplifiers used inside real opamps is to replace the two collector resistors $R_{C1,2}$ with a current mirror, to obtain even higher gain. I won't try to explain how this works right now, as you already have more than enough to digest from this reading, but I think that you can imagine that replacing R_C with some kind of transistor-based current source would affect the gain in the same way as using a very large R_C.

Three-stage home-made opamp

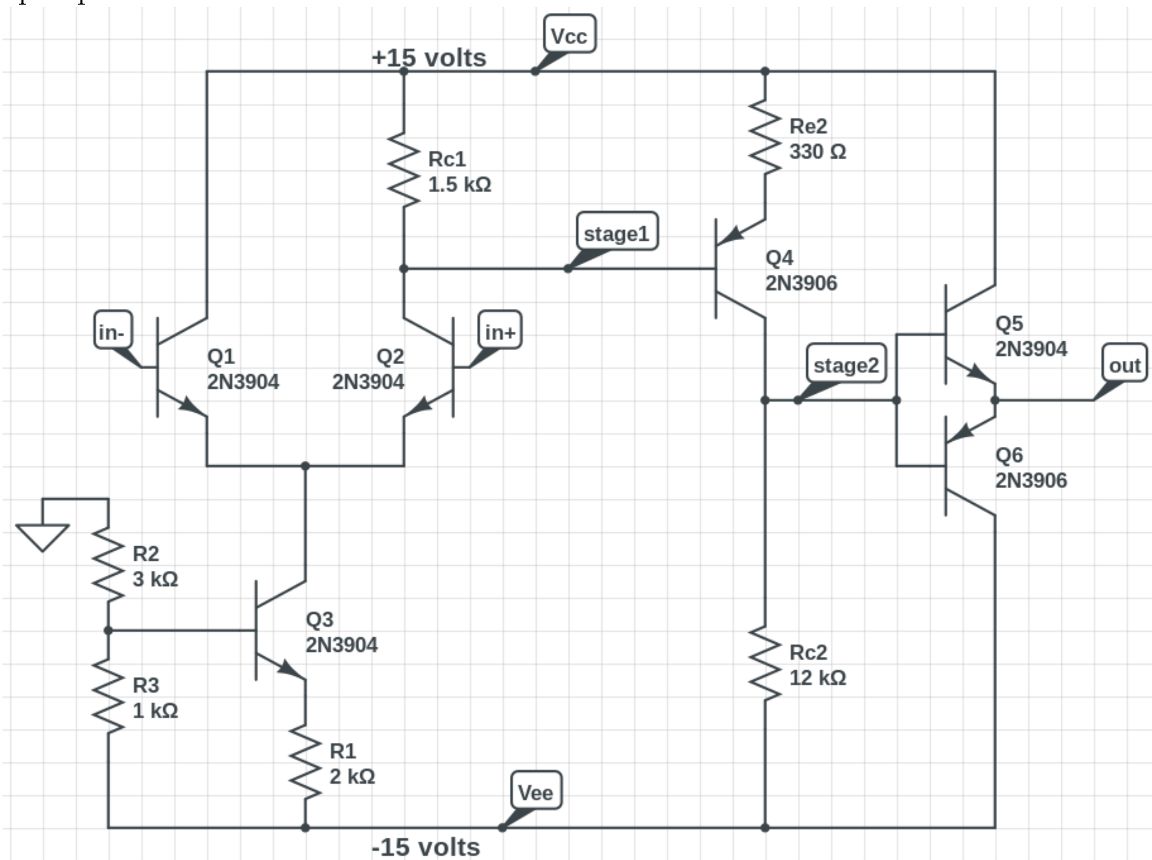
Now let's string together several recognizable circuit fragments into an ad-hoc opamp. The first stage is the differential amplifier that we just finished in the previous section. We have swapped the names of $V_{\rm in+}$ and $V_{\rm in-}$ because the gain of the second stage will be negative, so we need to get the overall sign right. I've replaced the current mirror with the simpler current source from Lab 13. I've changed the current supplied by the current source from 2 mA to 1.5 mA for a somewhat silly reason — that I am copying this home-made opamp from an example that is used in the Harvard course.

[9] If you can resolve the discrepancy, please let me know!

So I changed the current to match their opamp design. We also changed R_C of the differential stage from 7.5 kΩ to 1.5 kΩ in order to put the base of Q_5 (the common emitter amplifier) around 13.9 V, which turns out to be a convenient operating point. (More on this below.) Also, we removed the two 100 Ω emitter resistors: this keeps the differential gain high (about $R_C/2r_e \approx 1.5\ \text{k}\Omega/67\ \Omega \approx 20$), but at the price of some distortion. Since this amplifier will always be used with negative feedback, the distortion doesn't worry us, because it will be corrected by the feedback loop.

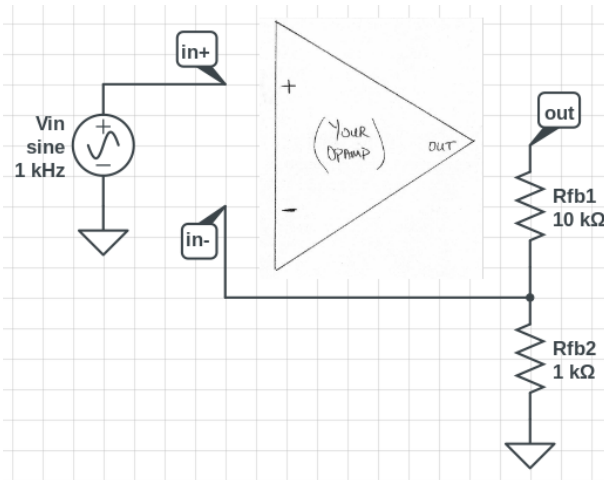
The second stage is just a common emitter amplifier, with a gain $-R_C/R_E = -(12\ \text{k}\Omega)/(330\ \Omega) = -36$. Notice that we used a PNP instead of an NPN transistor. This turned out to be convenient here, because the output of the differential stage stays much closer to +15 V than to −15 V. Since the quiescent value of Q_5's base voltage is about 13.9 V, the quiescent value of Q_5's emitter voltage is about 14.5 V, so the quiescent emitter current through Q_5 is about 1.5 mA.

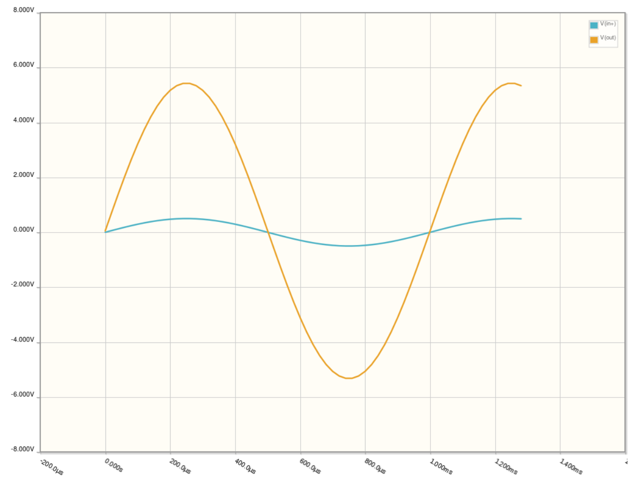
The third stage is the push-pull follower that we have seen before. The completed opamp is shown below.



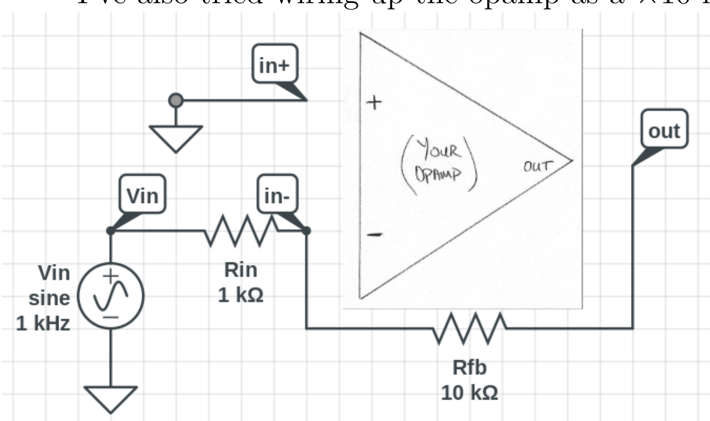
https://www.circuitlab.com/circuit/8u3369/lab15-homemade-opamp/

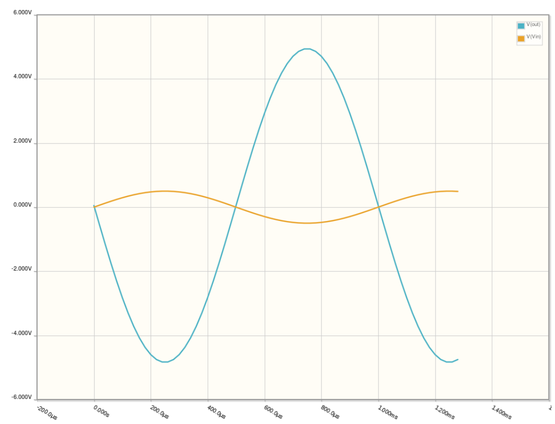
Now let's wire it up as a ×11 non-inverting amplifier and see what we get! The figure below shows how I connected our little opamp's input and output pins, and a trace of $V_{\text{in}}(t)$ and $V_{\text{out}}(t)$.





I've also tried wiring up the opamp as a ×10 inverting amplifier, as shown below.





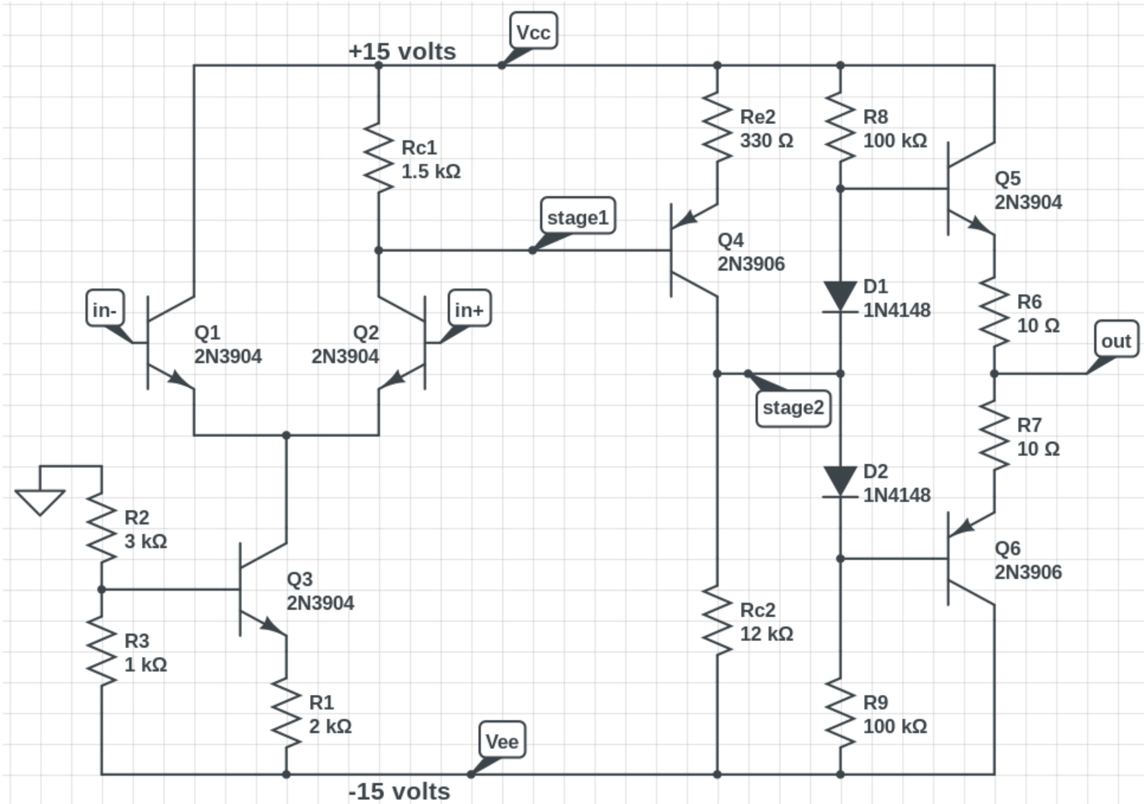
This is not a great opamp, but it illustrates the pattern that you will see in many opamp schematics: a differential input stage, with a current source in place of R_{tail} to provide good common-mode rejection; a second stage to increase differential gain; and finally a push-pull output stage. The open-loop gain of this opamp is only about 700, compared with $\mathcal{O}(10^5)$ for real opamps. We'll build and test this opamp in class this week.

An OK description of the inner workings of a 741 opamp can be found in the Wikipedia at `en.wikipedia.org/wiki/Operational_amplifier`. The textbook by Sedra & Smith includes a nice description of the 741 schematic, but it takes a quite some effort to go through it. A more understandable description (lecture slides by C.K. Tse of Hong Kong Polytechnic University) can be found at

`positron.hep.upenn.edu/wja/p364/2014/files/cktse_opamp.pdf`

When Prof. Tse writes g_m, it means $1/r_e = \mathrm{d}I_C/\mathrm{d}V_{BE}$. When he writes r_o, it means $(\mathrm{d}I_C/\mathrm{d}V_{CE})^{-1}$, the finite resistance due to the nonzero slope of the transistor's I_C vs. V_{CE} curve ("Early effect").

I find that the CircuitLab simulation model for the home-made opamp is prone to oscillation when $V_{\mathrm{out}} \approx 0$, because the output of stage 2 has to change very rapidly in order to undo the crossover distortion from the stage 3 push-pull buffer. The simulation behaves much better when I "bias" the push-pull with a pair of diodes to keep the NPN and PNP transistors' bases two diode drops apart from one another. I am not sure yet whether in the lab we will build the simpler circuit or the circuit with the improved output stage. I show below the updated circuit. The update is nothing more than what we discussed at the very end of the push-pull section of these notes.



`https://www.circuitlab.com/circuit/6758tv/lab15-homemade-opamp-biased-pushpull/`

Circuit simulation

You can find CircuitLab models for a decent number of the circuits we study in Physics 364 on my CircuitLab public page:

`www.circuitlab.com/user/ashmanskas/`

I usually try out new circuits in CircuitLab before trying them in the lab.

PID controllers

Just over a week from now (after next weekend), we will spend one lab building and trying out an opamp-based P-I-D controller (Proportional, Integral, Derivative). PID controllers are useful in a wide range of control applications (e.g. I want to output a voltage that controls the speed of a motor such that an elevator stops at the desired floor) that involve feedback (e.g. one of my circuit inputs tells me how far away the elevator currently is from the desired position). The key idea is that there is an error function $E(t)$ that tells you at a given time how far away you are from the desired state (e.g. how many millimeters your elevator is from the 3rd floor, where you want it to go). The output that you send to the motor at time t has three terms: one that is **proportional** to the present $E(t)$, one that is proportional to the present **derivative** $\mathrm{d}E(t)/\mathrm{d}t$, and one that is proportional to the recent **integral** $\int_{t-\Delta t}^{t} E(t')\mathrm{d}t'$.

The PID controller can be a useful addition to your toolkit, if you work in a research lab. It is also a pretty sophisticated application of opamps, combining the opamp difference amplifier, the opamp inverting amplifier, the opamp differentiator, the opamp integrator, and a push-pull transistor follower into one big circuit. So it provides an opportunity to review several circuits that you will have seen in earlier labs.[10] PID controllers are often implemented using computer programs instead of opamps. While it is not surprising to see a computer perform a sophisticated control task, it is fun to see this job done by hardware as "unintelligent" as a few opamps.

So please watch these two video lectures. First part of video (about 8 minutes):

`https://www.youtube.com/watch?v=UR0hOmjaHp0`

Second part of video (about 13 minutes):

`https://www.youtube.com/watch?v=XfAt6hNV8XM`

One important tip that will make these videos easier to follow: Engineers solve differential equations using Laplace transforms, in which a time-domain problem is transformed to the frequency domain. Whereas the Fourier transform of a function $F(t)$ is a related function $\tilde{F}(\omega)$, the Laplace transform of a function $G(t)$ is a related function $\tilde{G}(s)$, where $s = j\omega$. For solutions of the form $Ae^{j\omega t}$, differentiating w.r.t. time is the same as multiplying by $j\omega$, and integrating w.r.t. time is the same as dividing by $j\omega$. So in the video you will see the presenter write the expression "s" to represent a circuit fragment that outputs the derivative of its input, and you will see him write the expression "$1/s$" to represent a circuit fragment that outputs the integral of its input. You can just interpret s and $1/s$ as a shorthand notation for these two operations (derivative and integral, respectively), without actually knowing anything about Laplace transforms.

[10] Alas, the opamp difference amplifier appeared in `reading04` but not in the opamp labs.