1. A rigid beam of length $L = 2.0\, \text{m}$, mass $M = 100\, \text{kg}$ rests on two narrow posts — one at the far left end and one at a distance $x \leq L$ from the left end. (a) If $x = L$, what is the force that each post exerts on the beam? (b) If $\frac{L}{2} < x < L$, what is the force exerted by each post on the beam? (c) What happens if $x < \frac{L}{2}$?

![Diagram of beam on posts]

2. Now I bolt the left end of the beam from problem (1) down to the post and let $x = \frac{L}{4}$. (a) What are the magnitudes and directions of the forces on the beam at $x = 0$ and at $x = \frac{L}{4}$? (b) Suppose we add a 50 kg person standing at $x = L$. Now what are the magnitudes and directions of the forces on the beam at the locations of the two posts?

3. In problem (1a) above, suppose that each post has Young’s modulus $E = 1.0 \times 10^9\, \text{N/m}^2$, height $h = 1.0\, \text{m}$, and a square cross-section of width $w = 0.10\, \text{m}$. How much does each post compress (i.e. what is the change in $h$) under the weight of the beam? (The number that I calculated was larger than $1\, \mu\text{m}$ but smaller than $1\, \text{mm}$.)

(More on other side of page!)
4. A rigid beam of mass $M$ and length $L$ is held in place symmetrically at its ends by two posts. The ends of the beam and the top end of each post are mitered so that the joining faces make an angle of $30^\circ$ with respect to the horizontal. The mitered surfaces are smooth, so friction is negligible. (a) What are the vertical and horizontal forces exerted on the beam by each post? (b) Now add the weight of a person of mass $m$ standing a distance $x$ to the right of the center (which breaks the left-right symmetry) and solve for the forces exerted by each post. (c) If you extend part (b) to include friction for the mitered surfaces, how many equations and how many unknowns will you have? Is there enough information to solve for the forces?

5. Picture a triangular arch like the one demonstrated in class. It consists of two identical rigid upright pieces, each of mass $M$ and length $L$, whose frictionless faces rest against one another at the top. Let $\theta$ be the angle of each upright with respect to the horizontal. (a) What horizontal buttressing force is needed on each side of the base to keep the arch in equilibrium? (b) By varying $\theta$ while leaving $L$ and $M$ unchanged, you are changing the height of the arch. Make a rough sketch of a graph showing how the buttressing force relates to the height of the arch.

6. (a) Look up the strength of steel, both for compression and for tension. (b) Look up the strength of concrete, both for compression and for tension. (c) Imagine making a long beam of concrete and supporting it from its ends. Where would you expect the concrete to fail first? (d) Look up “prestressed concrete” and explain in a sentence or two how it overcomes the limitations that you encounter when you put concrete in tension.