HW3 ½ median score was 57/60 — well done!!

midterm grade estimates (based on “comb_avg” thru Oct 8):
  - 95 – 99 : A+
  - 92 – 94 : A
  - 90 – 91 : A–
  - 80 – 89 : B+
  - 70 – 79 : B

I’m very happy with your work: about ¾ of class is ≥ A–

Study sessions this week: Weds=Zoey, Thurs=Bill

DRL power went out last night ⇒ www issues . . . sorry!
Atwood machine — discuss with your neighbors

A contraption something like this appears in HW 4.

- Why aren’t the two masses accelerating?
- What is the tension in the cable when the two masses are balanced as they are now?
- If I make one mass equal 5.0 kg and the other mass equal 5.1 kg, what will happen? Can you predict what the acceleration will be?
- If I make one mass equal 5.0 kg and the other mass equal 6.0 kg, will the acceleration be larger or smaller than in the previous case?
- Try drawing a free-body diagram for each of the two masses
- How much work do I do when I raise the 6 kg mass 1 m?
Atwood machine — appears in HW 4
Atwood machine: write masses’ equations of motion

\[ m_1 g - T = m_1 a_x \]
\[ T - m_2 g = m_2 a_x \]
Atwood machine: write masses’ equations of motion

\[ m_1 g - T = m_1 a_x \]

\[ T - m_2 g = m_2 a_x \]

Solve second equation for \( T \); plug \( T \) into first equation; solve for \( a_x \):

\[ T = m_2 a_x + m_2 g \quad \Rightarrow \quad m_1 g - (m_2 a_x + m_2 g) = m_1 a_x \quad \Rightarrow \]

For \( m_2 \approx m_1 \), \( a_x \approx \Delta m_2 m g \).
Atwood machine: write masses’ equations of motion

\[ m_1 g - T = m_1 a_x \]
\[ T - m_2 g = m_2 a_x \]

Solve second equation for \( T \); plug \( T \) into first equation; solve for \( a_x \):

\[ T = m_2 a_x + m_2 g \implies m_1 g - (m_2 a_x + m_2 g) = m_1 a_x \implies \]

\[ (m_1 - m_2)g = (m_1 + m_2) a_x \implies a_x = \frac{m_1 - m_2}{m_1 + m_2} g \]
Atwood machine: write masses’ equations of motion

\[ m_1 g - T = m_1 a_x \]

\[ T - m_2 g = m_2 a_x \]

Solve second equation for \( T \); plug \( T \) into first equation; solve for \( a_x \):

\[ T = m_2 a_x + m_2 g \quad \Rightarrow \quad m_1 g - (m_2 a_x + m_2 g) = m_1 a_x \quad \Rightarrow \]

\[ (m_1 - m_2)g = (m_1 + m_2)a_x \quad \Rightarrow \quad a_x = \frac{m_1 - m_2}{m_1 + m_2} g \]

For \( m_2 = 0 \), \( a_x = g \)  (just like picking up \( m_1 \) and dropping it)

For \( m_2 \approx m_1 \),

\[ a_x \approx \frac{\Delta m}{2m} g \]
For $m_2 \approx m_1$, i.e.

$$m_2 = m, \quad m_1 = m + \Delta m$$

$$a_x \approx \frac{\Delta m}{2m} g$$

For example, $m = 3.73 \text{ kg}$, $\Delta m = 0.05 \text{ kg}$:

$$a_x \approx \frac{\Delta m}{2m} g = \left( \frac{0.05 \text{ kg}}{7.5 \text{ kg}} \right) (9.8 \text{ m/s}^2) \approx 0.065 \text{ m/s}^2$$

How long does it take $m_1$ to fall 2 meters?

$$x = \frac{a_x t^2}{2} \quad \Rightarrow \quad t = \sqrt{\frac{2x}{a_x}} = \sqrt{\frac{2 \times (2 \text{ m})}{0.065 \text{ m/s}^2}} \approx 7.8 \text{ s}$$
You can also solve for \( T \) if you like (eliminate \( a_x \))

Start from masses' equations of motion:

\[
m_1 g - T = m_1 a_x, \quad T - m_2 g = m_2 a_x
\]

Eliminate \( a_x \):

\[
\frac{m_1 g - T}{m_1} = \frac{T - m_2 g}{m_2} \quad \Rightarrow \quad m_1 m_2 g - m_2 T = m_1 T - m_1 m_2 g
\]

\[
\Rightarrow 2m_1 m_2 g = (m_1 + m_2) T \quad \Rightarrow \quad T = 2 \left( \frac{m_1 m_2}{m_1 + m_2} \right) g = 2 \mu g
\]

from HW3\( \frac{1}{2} \): for \( m_1 = m_2 \), \( \mu = \frac{m_1}{2} \); for \( m_1 \ll m_2 \), \( \mu \approx m_1 \).

So for \( m_1 = m_2 \), \( T = mg \). For \( m_1 \ll m_2 \), \( T \approx 2m_1 g \).

And for \( m_1 \approx m_2 \), \( T = mg + m a_x \approx (m + \frac{\Delta m}{2}) g \).
Measuring your weight \((F = mg)\) with a spring scale

Most bathroom scales work something like this:

Now suppose I take my bathroom scale on an elevator . . .
Bathroom scale on moving elevator

A bathroom scale typically uses the compression of a spring to measure the force of Earth’s gravity \( F = mg \) on you, which we call your weight.

Suppose I am standing on such a scale while riding an elevator. With the elevator parked at the bottom floor, the scale reads 700 N. I push the button for the top floor. The door closes. The elevator begins moving upward. At the moment when I can feel that the elevator has begun moving upward, the scale reads

(A) a value smaller than 700 N.
(B) the same value: 700 N.
(C) a value larger than 700 N.

You might want to try drawing a free-body diagram for your body, showing the downward force of gravity, the upward force of the scale pushing on your feet, and your body’s acceleration.
What is impulse?

Chapter 4 defined impulse as a change in momentum caused by a (non-isolated) system’s interaction with something external:

\[ \vec{J} = \vec{p}_f - \vec{p}_i \]

Chapter 8 defines force as the rate of change of momentum:

\[ \vec{F} = \frac{d\vec{p}}{dt} \iff \Delta \vec{p} = \int \vec{F} \, dt \]

If you integrate an external force over time, the resulting change in momentum is called the impulse delivered by the external force:

\[ \vec{J} = \Delta \vec{p} = \int \vec{F}_{\text{external}} \, dt \]

Same definition as in Chapter 4, but now you can calculate \( \vec{J} \) from external forces instead of introducing it in an ad-hoc way.
Bridge abutments now sometimes have a row of sand-filled or water-filled plastic barrels in front of them to increase the survivability of a collision in a car that runs off the road. Which does the presence of the barrels change:

(A) the impulse delivered to the car
(B) the force exerted on the car
(C) both
(D) neither
Chapter 9 — Work

“The force exerted on the object would be the slope of a work vs. displacement graph, since inversely, the work is the area under the force vs. displacement graph.”

“The graph will be a line if the force is constant.”

“Work = force \times \text{displacement}, so the slope should be the force on an object, if we graph work vs. displacement.”

“The work done by a force on a system is positive when the force and the force displacement point in the same direction and negative when they point in opposite directions. There are two forces acting on my body as I stand up: the downward force of gravity and the upward contact force exerted by the floor surface. The contact force is greater than the gravity force, thus force is positive. My displacement is also positive. Because the force and force displacement vectors point in the same direction, the work done is positive.”
“The magnitude of work done (by my feet pushing on the floor) is zero because the point of application is on the floor, which does not move.”

“No, it is not negative work when you stand up because the force and the force displacement point are pointing in the same direction. Because the force in your legs is pushing you up and you are moving up as well, the work done is positive and the energy of the system is increased.”

“Does this depend on what you consider the system? Is it the body and the chair? If I think of the chair as a compressed spring the direction it wants to move is upwards, my legs also want to push me upwards, this makes me feel as though positive work has been done. Energy made my body displace and the direction of the movement of the chair and my legs go towards the same direction.”

“No. I’m not sure how to explain it, but I don’t think this involves negative work.”
Equation sheet additions (see “homework” on web page)
positron.hep.upenn.edu/wja/p8/equations.pdf

Work (external, nondissipative, 1D):
\[ W = \int F_x(x) \, dx \]
which for a constant force is
\[ W = F_x \Delta x \]

Power is rate of change of energy:
\[ P = \frac{dE}{dt} \]

G.P.E. near earth’s surface:
\[ U_{\text{gravity}} = mgh \]
Force of gravity near earth’s surface (force is \( -\frac{dU_{\text{gravity}}}{dx} \)):
\[ F_x = -mg \]

Potential energy of a spring:
\[ U_{\text{spring}} = \frac{1}{2} k(x - x_0)^2 \]
Hooke’s Law (force is \( -\frac{dU_{\text{spring}}}{dx} \)):
\[ F_{\text{by spring ON load}} = -k(x - x_0) \]
A few key ideas from Chapters 8 and 9

Impulse (i.e. momentum change) delivered by external force:

\[
\text{force} = \frac{d(\text{momentum})}{dt} \iff \vec{J} = \int \vec{F}_{\text{external}} \, dt
\]

External force exerted ON system:

\[
\text{force} = \frac{d(\text{work})}{dx} \iff W = \int F_x \, dx
\]

Force exerted BY spring, gravity, etc.:

\[
\text{force} = -\frac{d(\text{potential energy})}{dx}
\]

\[\Delta E_{\text{system}} = \text{flow of energy into system} = \text{work done ON system}:
\]

\[
\text{work} = \Delta(\text{energy}) = \Delta K + \Delta U + \Delta E_{\text{source}} + \Delta E_{\text{thermal}}
\]

Notice that work : energy :: impulse : momentum
Reading question 2 had no really simple answer

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

Suppose “system” = me + earth + floor + chair
Reading question 2 had no really simple answer

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

Suppose “system” = me + earth + floor + chair

- $\Delta K = 0$
Reading question 2 had no really simple answer

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

Suppose “system” = me + earth + floor + chair

- $\Delta K = 0$
- $\Delta U = mg \ (\Delta x)_{\text{my c.o.m.}} > 0$
When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

Suppose “system” = me + earth + floor + chair

\[
\begin{align*}
\Delta K &= 0 \\
\Delta U &= mg \ (\Delta x)_{\text{my c.o.m.}} > 0 \\
\Delta E_{\text{thermal}} &= 0
\end{align*}
\]
Reading question 2 had no really simple answer

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

Suppose “system” = me + earth + floor + chair

- $\Delta K = 0$
- $\Delta U = mg \ (\Delta x)_{\text{my c.o.m.}} > 0$
- $\Delta E_{\text{thermal}} = 0$
- $\Delta E_{\text{source}} = -mg \ (\Delta x)_{\text{my c.o.m.}} < 0$
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- $\Delta E_{\text{thermal}} = 0$
- $\Delta E_{\text{source}} = -mg \ (\Delta x)_{\text{my c.o.m.}} < 0$
- $W = 0$

There are no external forces. Everything of interest is inside the system boundary.
Let’s try choosing a different “system.”

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

Suppose “system” = me + floor + chair
Let’s try choosing a different “system.”

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Suppose “system” = me + floor + chair

- $\Delta K = 0$
- $\Delta U = 0$
- $\Delta E_{\text{thermal}} = 0$
- $W = -mg(\Delta x) < 0$

External force of gravity does negative work on me. Point of application of this external force is my body’s center of mass.
Let’s try choosing a different “system.”

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

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- $\Delta E_{\text{thermal}} = 0$
- $\Delta E_{\text{source}} = -mg (\Delta x)_{\text{my c.o.m.}} < 0$
- $W = -mg (\Delta x)_{\text{my c.o.m.}} < 0$

External force of gravity does negative work on me. Point of application of this external force is my body’s center of mass.
Let’s try answering a slightly different question.

When a friend stands me up from a chair (e.g. my knees are weak today), does my friend do positive or negative work?

Suppose “system” = me + earth + floor + chair
Let’s try answering a slightly different question.

When a friend stands me up from a chair (e.g. my knees are weak today), does my friend do positive or negative work?

Suppose “system” = me + earth + floor + chair

► $\Delta K = 0$
Let’s try answering a slightly different question.

When a friend stands me up from a chair (e.g. my knees are weak today), does my friend do positive or negative work?

Suppose “system” = me + earth + floor + chair

- $\Delta K = 0$
- $\Delta U = mg (\Delta x)_{my \ c.o.m.} > 0$
Let’s try answering a slightly different question.

*When a friend stands me up from a chair (e.g. my knees are weak today), does my friend do positive or negative work?*

Suppose “system” = me + earth + floor + chair

- \(\Delta K = 0\)
- \(\Delta U = mg (\Delta x)_{\text{my c.o.m.}} > 0\)
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Let’s try answering a slightly different question.

When a friend stands me up from a chair (e.g. my knees are weak today), does my friend do positive or negative work?

Suppose “system” = me + earth + floor + chair

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- $\Delta E_{\text{source}} = 0$
- $W = mg \cdot (\Delta x)_{\text{my c.o.m.}} > 0$

My friend applies an upward force beneath my arms. The point of application of force is displaced upward.
Let’s include my friend as part of “the system.”

When a friend stands me up from a chair (e.g. my knees are weak today), does my friend do positive or negative work?

Suppose “system” = me + earth + floor + chair + friend
Let’s include my friend as part of “the system.”

When a friend stands me up from a chair (e.g. my knees are weak today), does my friend do positive or negative work?

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- $\Delta E_{\text{thermal}} = 0$
- $\Delta E_{\text{source}} = -mg \ (\Delta x)_{\text{my c.o.m.}} < 0$
- $W = 0$

There is no external force. Everything is within the system.
When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

I think the work done **ON the system BY external forces** is either positive (if my legs are considered “external” to the me+earth+floor system and are supplying the energy to lift me) or zero (if my legs are part of the system).