Please turn in Homework 6 now if you have not done so already. I will hand out solutions in a moment, while you’re voting.

Let’s choose either Nov 30 (Wed) or Dec 1 (Thu) at 7pm as time to meet with Richard Farley about the connection between physics and architecture / structures. We’ll vote once everyone is here:

(A) Nov 30 (Wednesday), 7pm
(B) Dec 1 (Thursday), 7pm
Figure 1 for HW7 is not very easy to photo-copy: poor choice of colors, it seems.
With 2+ dimensions, you can move in a circle. Circular motion lets us analyze several new situations, e.g.

- Your car rounding a turn, as on a highway offramp
- A centrifuge (or a salad spinner)
- **Twirling a ball on a string**

To move in a circle requires a component of acceleration that is perpendicular to velocity. This *centripetal* acceleration points toward the center of the circle.

So motion in a circle requires a *centripetal force* pointing toward the circle center.

- Friction of tires on pavement
- Wall of spinner keeping salad in but letting water out
- **Tension in string points toward my fingers**
Moving in a circle at constant speed (velocity changes but speed does not!) is called *uniform circular motion*. Then \( \vec{a} \perp \vec{v} \).

\[
\begin{align*}
x &= R \cos \theta \\
y &= R \sin \theta
\end{align*}
\]

\[
\vec{r} = (R \cos \theta, R \sin \theta) = R (\cos \theta, \sin \theta)
\]

Introduce “angular velocity” \( \omega \), which is constant for U.C.M.

\[
\omega = \frac{d\theta}{dt} = \text{constant (for u.c.m.)}
\]

\[
\vec{v} = \frac{d\vec{r}}{dt} = \omega R (-\sin \theta, \cos \theta)
\]

\[
\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2 R (\cos \theta, \sin \theta) = -\frac{v^2}{R} (\cos \theta, \sin \theta)
\]

Magnitude of centripetal acceleration is

\[
a_c = \omega^2 R = \frac{v^2}{R}
\]
Can an object be accelerated without changing its kinetic energy?

(A) No, because acceleration always changes an object’s speed, and kinetic energy is proportional to speed squared.

(B) Yes. For example, in uniform circular motion, the object’s velocity vector is constantly changing direction, but the object's speed is constant. Acceleration is the rate of change of the velocity vector. It is possible to change the velocity vector’s direction while leaving the magnitude (i.e. the speed) unchanged. So kinetic energy can also be unchanged.

(C) Neither of the above is correct.
Suppose that a highway offramp that I often use bends with a radius of 20 meters. I notice that my car tires allow me (in good weather) to take this offramp at 15 m/s without slipping. How large does the offramp’s bending radius need to be for me to be able to make the turn at 30 m/s instead?

(A) 10 m
(B) 20 m
(C) 40 m
(D) 80 m
Because the required centripetal force has magnitude

\[ |\vec{F}_{\text{centripetal}}| = ma_c = \omega^2 R = \frac{v^2}{R} \]

the string tension quadruples if I double \( v \) without changing \( R \), for this tennis ball on a string.

To keep tension unchanged while doubling \( v \), I need to quadruple \( R \). Same argument goes for friction of tires on highway offramp.
Chapter 11 review (last week’s reading, this week’s HW)

We can also consider circular motion with non-constant speed, just as we considered linear motion with non-constant speed. Then we introduce the *angular acceleration*

$$ \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} $$

and we can derive results that look familiar but with substitutions

- $x \rightarrow \theta$
- $v \rightarrow \omega$
- $a \rightarrow \alpha$
- $m \rightarrow I$
- $p \rightarrow L$

if $\alpha$ is constant, then:

$$ \theta_f = \theta_i + \omega t + \frac{1}{2} \alpha t^2 $$

$$ \omega_f^2 = \omega_i^2 + 2\alpha (\theta_f - \theta_i) $$
A point-like object has only a position. It has no orientation in space. An extended object (like this hammer) has both a position and an orientation. I can translate the hammer’s center of mass, and I can rotate the hammer about its center of mass.

The rotational component of motion must have some corresponding kinetic energy, since we know that the pieces of the hammer are moving even if the hammer is just spinning about its center.

The rotational component also has an “angular momentum” that is conserved, in analogy to the (linear) momentum we studied before.

And there is a “rotational inertia” that resists changes in angular momentum, just as inertia resists changes in linear momentum.
An ice cube and a rubber ball are both placed at one end of a warm cookie sheet, and the sheet is then tipped up. The ice cube slides down with virtually no friction, and the ball rolls down without slipping. If the ball and the ice cube have the same inertia, which one makes it to the bottom first?

(A) They reach the bottom at the same time.

(B) The ball arrives later because the frictional force is dissipating energy from the ball as it rolls.

(C) The ice cube slides with downhill acceleration $mg \sin \theta$, while the ball must have a smaller downhill acceleration, because the ball will have both translational and rotational motion in order to travel down the hill without slipping. For the ice cube, we will have $mgh = \frac{1}{2}mv^2$, while for the ball we will have $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, so the ball must move more slowly and therefore will arrive later.

(D) None of the above answers is correct.
Rotational inertia ("moment of inertia"): \[ I = \sum mr^2 \quad \rightarrow \quad \int r^2 \, dm \]

Kinetic energy has both translational and rotational parts:

\[ K = \frac{1}{2} mv_{cm}^2 + \frac{1}{2} I \omega^2 \]

Angular momentum:

\[ L = I \omega = m \, v_{\perp} \, r_{\perp} \]

(where \( \perp \) means the component that does not point toward the "reference" axis — which usually is the rotation axis)
Two spheres are placed on an inclined plane and roll downhill without slipping. The first sphere is hollow (with all of the mass concentrated on the outside), while the second sphere has uniform density. Which one reaches the bottom first?

(A) They reach the bottom at the same time.

(B) The hollow sphere has a larger fraction of its mass concentrated toward the outside, which means that its ratio of \( I/m \) will be larger. So more of its kinetic energy will be in the \( \frac{1}{2} I \omega^2 \) term and less will be in the \( \frac{1}{2} mv^2 \) term. So the uniform sphere will arrive sooner.

(C) Because the hollow sphere has more of its mass concentrated toward the outside, gravity will pull toward the outside of the hollow sphere, exerting a larger torque to get it moving. So the hollow sphere reaches the bottom first.

(D) None of the above answers is correct.
HW7, problem 11

In the figure below, two identical pucks B and C, each of inertia $m$, are connected by a rod of negligible inertia that is free to rotate about its center. Then puck A, of inertia $m/2$, comes in and hits B. After the collision, in which no energy is dissipated, what are (a) the rotational speed of the dumbbell, and (b) the linear velocity of A?