Monday’s popular vote was for Dec 1 (Thu) at 7pm to meet with Richard Farley about the connection between physics and architecture / structures. Normal homework study session will start just after this discussion wraps up.

I made a mistake in solving problem 2 on homework 6. As nearly everyone figured out, the correct answer was 1.73 hours, not the 1.0 hours that I got. The problem is that my solution used (0,1) instead of (1,0) for “east” on the first step.

I will be at this Thursday’s 7pm homework session. Zoey will do tonight’s session.
Recap from Monday / last week’s reading, this week’s HW

For motion in a circle, acceleration has a \textit{centripetal} component that is perpendicular to velocity and points toward the center of rotation. If we put the center of rotation at the origin \((0, 0)\) then

\[ x = R \cos \theta \quad y = R \sin \theta \]

\[ \vec{r} = (R \cos \theta, R \sin \theta) = R (\cos \theta, \sin \theta) \]

The “angular velocity” \( \omega \) is the rate of change of the angle \( \theta \)

\[ \omega = \frac{d \theta}{dt} \]

The units for \( \omega \) are just \( s^{-1} \) (which is the same as radians/second, since radians are dimensionless). Revolutions per second are \( \omega/2\pi \), and the period (how long it takes to go around the circle) is \( 2\pi/\omega \). The velocity is

\[ \vec{v} = \frac{d\vec{r}}{dt} = \omega R (-\sin \theta, \cos \theta), \quad |\vec{v}| = \omega R \]
The magnitude of the centripetal acceleration (required rate of change of $\vec{v}$, to keep object on circular path) is

\[ a_c = \omega^2 R = \frac{v^2}{R} \]

and the centripetal force (directed toward center of rotation) is

\[ |\vec{F}_c| = ma_c = m\omega^2 R = \frac{mv^2}{R} \]

Moving in circle at constant speed (velocity changes, speed does not) is called *uniform circular motion*. For UCM, $\vec{a} \perp \vec{v}$, and $\omega = \text{constant}$. Then

\[ \vec{a} = \frac{d\vec{v}}{dt} = -\omega^2 R \left( \cos \theta, \sin \theta \right) = -\frac{v^2}{R} \left( \cos \theta, \sin \theta \right) \]

(For non-UCM case where speed is not constant, $\vec{a}$ has an additional component that is parallel to $\vec{v}$.)
We can also consider circular motion with non-constant speed, just as we considered linear motion with non-constant speed. Then we introduce the *angular acceleration*

\[ \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \]

and we can derive results that look familiar but with substitutions

\[ x \rightarrow \theta, \quad v \rightarrow \omega, \quad a \rightarrow \alpha, \quad m \rightarrow I, \quad p \rightarrow L \]

if \( \alpha \) is constant (which is a common case for constant torque), then:

\[ \theta_f = \theta_i + \omega t + \frac{1}{2} \alpha t^2 \]

\[ \omega_f = \omega_i + \alpha t \]

\[ \omega_f^2 = \omega_i^2 + 2\alpha (\theta_f - \theta_i) \]

(If you use these equations, be careful about cases where \( \theta \) wraps around at \( 2\pi \), or you can get an answer that makes no sense.)
Rotational inertia ("moment of inertia")

\[ I = \sum mr^2 \rightarrow \int r^2 \, dm \]
Rotational inertia ("moment of inertia")

\[ I = \sum m r^2 \rightarrow \int r^2 \, dm \]
Parallel axis theorem

If an object revolves about an axis that does not pass through the object’s center of mass (suppose axis has \( \perp \) distance \( \ell \) from c.o.m.), the rotational inertia is larger, because the object’s c.o.m. revolves around a circle of radius \( \ell \) and in addition the object rotates about its own center of mass. This larger rotational inertia is given by the parallel axis theorem:

\[
I = I_{cm} + M \ell^2
\]

where \( I_{cm} \) is the object’s rotational inertia about an axis (which must be parallel to the new axis of rotation) that passes through the object’s c.o.m.
I twirl a basketball of mass $M$, radius $R$, on a string of length $\ell$. The length $\ell$ is measured from my fingertips to the surface of the basketball, where the string is taped down. What is the rotational inertia for twirling the ball about my fingertips?

(A) $I = \frac{2}{3} MR^2$
(B) $I = M\ell^2$
(C) $I = \frac{2}{3} MR^2 + M\ell^2$
(D) $I = \frac{2}{3} MR^2 + M(\ell + R)^2$
(E) $I = M(\ell + R)^2$
Kinetic energy has both translational and rotational parts:

\[ K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2 \]

Angular momentum:

\[ L = I \omega = m v_\perp r_\perp \]

(where \( \perp \) means the component that does not point toward the “reference” axis — which usually is the rotation axis)
HW7, problem 11

In the figure below, two identical pucks B and C, each of inertia $m$, are connected by a rod of negligible inertia that is free to rotate about its center. Then puck A, of inertia $m/2$, comes in and hits B. After the collision, in which no energy is dissipated, what are (a) the rotational speed of the dumbbell, and (b) the linear velocity of A?
What is the rotational inertia of the B+C system about its axis of rotation?

(A) $I = \frac{1}{2} mR^2$
(B) $I = mR^2$
(C) $I = 2mR^2$
(D) $I = 4mR^2$
Which equation correctly represents conservation of angular momentum when puck A hits puck B?

(A) \[ mv_i R = mv_{Af} R + mR^2 \omega_f \]

(B) \[ \left( \frac{m}{2} \right) v_i R = \left( \frac{m}{2} \right) v_{Af} R + 2mR^2 \omega_f \]

(C) \[ mv_i R = mv_{Af} R + 2mR^2 \omega_f \]
Which equation correctly represents conservation of energy when puck A hits puck B?

(A) \[
\frac{1}{2} \left( \frac{m}{2} \right) v_i^2 = \frac{1}{2} \left( \frac{m}{2} \right) v_{Af}^2 + \frac{1}{2} (2mR^2) \omega_f^2
\]

(B) \[
\frac{1}{2} m v_i^2 = \frac{1}{2} m v_{Af}^2 + \frac{1}{2} (mR^2) \omega_f^2
\]

(C) \[
\frac{1}{2} m v_i^2 = \frac{1}{2} m v_{Af}^2 + \frac{1}{2} (2mR^2) \omega_f^2
\]
Conservation of energy implies (2nd term on right is $\frac{1}{2}I\omega^2$):

$$\frac{1}{2} \left( \frac{m}{2} \right) v_i^2 = \frac{1}{2} \left( \frac{m}{2} \right) v_{Af}^2 + \frac{1}{2}(2mR^2)\omega_f^2$$

Conservation of angular momentum implies:

$$\left( \frac{m}{2} \right) v_i R = \left( \frac{m}{2} \right) v_{Af} R + 2mR^2\omega_f$$

$$\Rightarrow \quad \frac{m}{2}(v_i - v_f) R = 2mR^2\omega_f$$

$$\Rightarrow \quad v_i - v_{Af} = 4R\omega_f$$
But this is a hassle to solve, so let’s try another approach. Treat problem as $\frac{m}{2}$ colliding elastically with $2m$. Momentum conservation implies:

$$\left( \frac{m}{2} \right) v_i = \left( \frac{m}{2} \right) v_{Af} + (2m)v_{Bf}$$

Elastic collision just negates relative velocity:

$$v_{Bf} - v_{Af} = v_i \Rightarrow v_{Bf} = v_i + v_{Af}$$

Solving 1st equation (and plugging in $v_{Bf}$ from 2nd):

$$v_i = v_{Af} + 4v_{Bf} = v_{Af} + 4(v_i + v_{Af}) = 4v_i + 5v_{Af}$$

$$v_{Af} = -\frac{3}{5}v_i$$

Use 2nd equation again to get $v_{Bf}$:

$$v_{Bf} = v_i + v_{Af} = v_i - \frac{3}{5}v_i = +\frac{2}{5}v_i$$

So the angular velocity after collision is

$$\omega_f = +\frac{2}{5}v_i/R$$
Now let’s check that this solution satisfies the two original equations written before we reformulated the problem:

\[ v_i - v_{Af} = 4v_Bf = 4R\omega_f \]

so angular momentum is conserved. Now energy:

\[
\frac{1}{2} \left( \frac{m}{2} \right) v_{Af}^2 + \frac{1}{2} (2mR^2)\omega_f^2 = \frac{1}{2} \left( \frac{m}{2} \right) \left( -\frac{3}{5} v_i \right)^2 + \frac{1}{2} (2mR^2) \left( \frac{2}{5} \frac{v_i}{R} \right)^2
\]

\[
= \frac{m}{4} \cdot \frac{9}{25} v_i^2 + m \cdot \frac{4}{25} v_i^2 = \left( \frac{m}{4} \right) v_i^2 = \frac{1}{2} \left( \frac{m}{2} \right) v_i^2
\]

so the energy-conservation equation is also satisfied.

For the homework, you can just write down the answer and show that the answer conserves both angular momentum and energy.
Two skaters skate toward each other, each moving at 3.3 m/s. Their lines of motion are separated by a perpendicular distance of 2.0 m. Just as they pass each other (still 2.0 m apart), they link hands and spin about their common center of mass. What is the rotational speed of the couple about the center of mass? Treat each skater as a point particle, one with an inertia of 75 kg and the other with an inertia of 48 kg.
About what point will the two skaters rotate once they link hands?

(A) About the point halfway between the two skaters  
(B) About the center of mass of the two-skater system  
(C) About the center of the more massive skater
\[ I \omega = m_1 v_{1\perp} r_{1\perp} + m_2 v_{2\perp} r_{2\perp} \]

Let’s put skater 1 at \( x = +1.0 \) m, with \( m_1 = 75 \) kg, and skater 2 at \( x = -1.0 \) m, with \( m_2 = 48 \) kg. Center of mass is

\[
\chi_{cm} = \frac{(75 \text{ kg})(+1.0 \text{ m}) + (48 \text{ kg})(-1.0 \text{ m})}{75 \text{ kg} + 48 \text{ kg}} = +0.22 \text{ m}
\]

So perpendicular displacements from center of mass are \( r_{1\perp} = +0.78 \) m and \( r_{2\perp} = -1.22 \) m. Conserving angular momentum in “collision” implies:

\[
(m_1 r_1^2 + m_2 r_2^2) \omega = m_1 v_1 r_1 + m_2 v_2 r_2
\]

\[
\omega = \frac{m_1 v_1 r_1 + m_2 v_2 r_2}{m_1 r_1^2 + m_2 r_2^2}
\]

\[
\omega = \frac{(75 \text{ kg})(3.3 \text{ m/s})(0.78 \text{ m}) + (48 \text{ kg})(-3.3 \text{ m/s})(-1.22 \text{ m})}{(75 \text{ kg})(0.78 \text{ m})^2 + (48 \text{ kg})(-1.22 \text{ m})^2} = 3.3/\text{s}
\]
If you look at the face of a clock, whose hands are moving clockwise, do the rotational velocity vectors of the clock’s hands point toward you or toward the clock?

(A) Toward me
(B) Toward the clock
(C) Neither — when I curl the fingers of my right hand toward the clock, my thumb points to the left, in the 9 o’clock direction
When you are about to fall over the edge of a precipice, you instinctively wheel your fully extended arms rapidly in vertical circles. What does this accomplish?

(A) It accomplishes nothing because there is no external torque, since my arms are part of my body

(B) When I spin my arms counterclockwise (as drawn in picture), conservation of angular momentum pushes my body’s center of mass clockwise, with my feet as the pivot point, and this may help me not to fall

(C) I think argument (B) sounds good in theory, but I’m skeptical about how well it really works in practice
I want to tighten a bolt to a torque of 1.0 newton-meter, but I don’t have a torque wrench. I do have an ordinary wrench, a ruler, and a 1.0 kg mass tied to a string. How can I apply the correct torque to the bolt?

(A) Orient the wrench horizontally and hang the mass at a distance 0.1 m from the axis of the bolt

(B) Orient the wrench horizontally and hang the mass at a distance 1.0 m from the axis of the bolt
If the wrench is at 45° w.r.t. horizontal, will the 1.0 kg mass suspended at a distance 0.1 m along the wrench still exert a torque of 1.0 newton-meter on the bolt?

(A) Yes. The force of gravity has not changed, and the distance has not changed.

(B) No. The torque is now smaller — about 0.71 newton-meter.

(C) No. The torque is now larger — about 1.4 newton-meter.
The angular equivalent of $F = ma$

$$\tau = I \alpha$$

Let’s try it . . .