Homework #8 was too difficult! Mean was 5.8 hours (median 5.5), while I intended more like 4 hours. So I think I should keep the homework to 10–15 problems in the future (and half that for HW10). Thanks for the helpful feedback — that’s why I ask.
Forces and torques on an accelerating car (e.g. #5)?
Forces and torques on an accelerating car (e.g. #5)?

I was thinking more about homework problem #5, and I worked this out on the train. See if you agree:

Normal force on front wheels:

\[ F_{N,\text{front}} = \frac{mg(x_{rw} - x_{cm}) - may_{cm}}{x_{rw} - x_{fw}} \]

Normal force on rear wheels:

\[ F_{N,\text{rear}} = \frac{mg(x_{cm} - x_{fw}) + may_{cm}}{x_{rw} - x_{fw}} \]

where “rw” (“fw”) means rear (front) wheels, and \( y_{cm} \) is the COM height above the ground.
Many people found problem 11 (on HW8) confusing. In my opinion, #11 would be too tricky for a final exam question, but the easier version (#7) would be a reasonable final exam question.

If we have a few minutes left at the end of class, I will work through problem 11.
Here (in the next few slides) is what I want you to know about gravity. The rest of Chapter 13 is for your broader education.

\[
F = \frac{Gm_1m_2}{r^2}
\]

where \( \vec{F} \) points along the axis connecting \( m_1 \) to \( m_2 \).

\[ G = 6.67 \times 10^{-11} \frac{\text{N m}^2}{\text{kg}^2} \]

is a *universal* constant — the same on Earth, on Mars, in distant galaxies, etc. That is a remarkable fact.

\[ g = 9.8 \text{ m/s}^2 = \frac{GM_e}{R_e^2} \]

shows that an apple falling onto Newton’s head results from the same force that governs the motion of the Moon around Earth, Earth around the Sun, etc. That too is amazing.
(Digression)

What is the acceleration due to gravity at a distance of one earth radius above earth’s surface?

\[
a = \frac{GM_{\text{earth}}}{(2R_{\text{earth}})^2} = \frac{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(6.0 \times 10^{24} \text{ kg})}{(2 \times 6.4 \times 10^6 \text{ m})^2} = 2.44 \text{ m/s}^2
\]

or more simply

\[
a = g/4 = 2.45 \text{ m/s}^2
\]
Important stuff about gravity, continued

For an orbit, gravity provides the centripetal force, so

\[ \frac{mv^2}{R} = \frac{GMm}{R^2} \]

In general angular momentum \((mvR_\perp)\) is constant, but we’ll mainly study circular orbits, where \(R = \text{constant}\), thus \(v = \text{constant}\).
How fast would a space shuttle need to be going to release a satellite into a stable circular orbit 600 km above earth's surface?

$$\frac{mv^2}{R} = \frac{GM_e m}{R^2}$$

Solving for $v$,

$$v = \sqrt{\frac{GM_e}{R}} = \sqrt{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(6.0 \times 10^{24} \text{ kg})} = 7560 \text{ m/s}$$

check: period of revolution $T$ is

$$T = \frac{2\pi R}{v} = \frac{(6.28)(7.0 \times 10^6 \text{ m})}{7560 \text{ m/s}} = 97 \text{ minutes}$$

which is a familiar number for low-earth orbit, if you’ve watched space-shuttle launches on TV.
Things to know about gravity, continued

Gravitational potential energy for objects 1 and 2 is

\[ U = -\frac{Gm_1m_2}{r} \]  

(note the sign)

which \( \to 0 \) as \( r \to \infty \). The objects are bound if \( K + U < 0 \).

If \( K + U \geq 0 \), they escape each other. They just barely escape if \( K + U = 0 \)

\[ \frac{1}{2}mv_{\text{escape}}^2 = \frac{GmM}{R} \]

in which case \( K \to 0 \) when \( R \to \infty \).

G.P.E. of e.g. a spacecraft of mass \( m \) in the field of two large objects (e.g. earth and moon) of mass \( M_1 \) and \( M_2 \):

\[ U = -\left( \frac{GM_1m}{R_{M1,m}} + \frac{GM_2m}{R_{M2,m}} \right) \]

(needed for Problems 12 and (if you want to be precise) 10).
For a central force that goes like $F \propto 1/R^2$, the forces from a uniform spherical shell add (if you’re outside the shell) up to one force directed from the center of the shell. So a rigid sphere attracts you as if it were a point mass.

If you’re inside the shell, the sum of the forces adds up to zero.

→ That’s it ←
Question: “I’m still not entirely sure what the gravitational constant is. Also, I wish the reading were more clear about how far is too far from the Earth’s surface for regular mechanics laws to apply.”

If you’re standing on earth’s surface,

\[ F = \frac{GM_e m}{R_e^2} = m \left( \frac{GM_e}{R_e^2} \right) = mg \]

which points toward the center of earth, because outside a uniform spherical shell, the vector sum of all of the $1/r^2$ contributions acts just like a point mass at the center of the sphere. (If you’re inside the spherical shell, the forces add to zero.)

What if I’m a small distance $h$ above earth’s surface? (And how small is small?)
What if I’m a small distance \( h \) above earth’s surface? (And how small is small?)

\[
a = \frac{F}{m} = \frac{GM_e}{(R_e + h)^2} = \frac{GM_e}{R_e^2(1 + \frac{h}{R_e})^2} = \frac{GM_e}{R_e^2} \left( 1 + \frac{h}{R_e} \right)^2
\]

If \( h \ll R_e \), then (don’t write this down—it’s just for illustration)

\[
a \approx \frac{GM_e}{R_e^2} \left( 1 - \frac{2h}{R_e} + \mathcal{O}\left(\frac{h}{R_e}\right)^2 \right)
\]

At commercial airplane altitude (\( h \approx 10000 \text{ m} \)), \( 2h/R_e = 0.0031 \), so \( g \) is only 0.3% smaller at 30000 ft than on the ground.

In one HW9 problem, you will figure out (using \( F = GMm/r^2 \)) at what altitude gravity’s acceleration becomes smaller than \( g \) by 0.1%, by 1%, and by 10%.
What if earth were of completely uniform density (really it isn’t!) and you somehow managed to go down into a mine shaft 400 km beneath earth’s surface. What \( g \) would you measure? (Remember that \( R_{\text{earth}} = 6400 \text{ km} \).)

(A) The same \( g = 9.8 \text{ m/s}^2 \) that I find on earth’s surface.

(B) \( g = GM_{\text{earth}}/R^2 \), where \( R=6000 \text{ km} \) instead of 6400 km. So \( g \) would be bigger, because I am closer to earth’s center.

(C) \( g = GM/R^2 \), where \( R=6000 \text{ km} \) and \( M \) is smaller than \( M_{\text{earth}} \) by a factor \((6000/6400)^3 = 0.824\), because the force of gravity on me from the spherical shells from \( R = 6000 \text{ km} \) out to \( R = 6400 \text{ km} \) adds up to zero. So \( g \) would actually be smaller by a factor \((6000/6400) = 0.94\).
If we have a few minutes left at the end of class, I will work through problem 11.

If we have even a few more minutes, let’s talk about what a geostationary orbit is and where they are located.

A big part of next week’s reading is further application of the conditions for an object to be in equilibrium. This is the part of Physics 8 that most pertains to architecture.

So if you’re still confused by calculating the forces and torques and sines and cosines for the ladder and hanging-sign problems, etc., don’t worry. We will have a lot more opportunity to practice!